THE EFFECTS OF A MATHEMATICS COURSE ON CANDIDATES' BELIEFS ABOUT LEARNING AND TEACHING MATHEMATICS

by

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ABSTRACT

This qualitative case study examined a cohort of prospective teachers who completed a university mathematics course during which they were given the opportunity to relearn geometry conceptually and gain experience teaching using contemporary approaches. The course embedded mathematics content using the context of (a) problem solving in university-based seminar classes and (b) a concurrent Grade 7 and 8 teaching placement. Questionnaires, administered at the beginning and end of the semester, and individual interviews were used to obtain data about the prospective teachers' beliefs about the nature of mathematics and its teaching, and about their pedagogical content knowledge.

The pre-course profile was that of a group who held comparable views about mathematics teaching as passive, learning as procedural, and mathematics itself as instrumental. The post-course profile, however, was of a radically different group who once again claimed to hold comparable beliefs about reform-based teaching methods, conceptual learning, and a dynamic view of the utility of mathematics. There are five findings from this study with import for preservice mathematics teacher education. First, learning best occurs within a community. Second, an inquiry-based approach to mathematics provides a rich learning opportunity for students. Third, the instructor, who must have expertise in both mathematics and mathematics pedagogy, must embody reform-based teaching practice. Fourth, to enhance their competence and confidence, teacher candidates must have opportunities to emulate the instructor in a safe environment. Fifth, reflecting critically about course experiences is an essential part of learning for prospective educators.

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CHAPTER 1

INTRODUCTION

Since mathematics teachers' beliefs about, and knowledge of, mathematics and its teaching are important factors in characterizing individual teachers' instructional practices (Boaler, 1999), it would follow that there is a need for teacher education courses to provide opportunities for beginning teachers to acknowledge, articulate, and reflect on their existing beliefs, and simultaneously stimulate knowledge development about mathematics and content-specific pedagogy. This study examines such a course. A cohort of prospective teachers completed a university mathematics course during which they were given the opportunity to relearn geometry conceptually and gain experience teaching geometry using contemporary, research-based approaches. The course embedded mathematics content using the context of problem solving in (a) university-based seminar classes, and (b) a concurrent semester-long Grade 7 and 8 classroom teaching placement.

Autobiographical Signature

As a secondary school student, I always achieved high grades in mathematics. The fact that mathematics was a subject that I enjoyed and in which I was successful led me to a career as a secondary school mathematics teacher. My career began in Calgary, Alberta, a province in which examinations for core subjects are administered at the end of Grade 12 by the Ministry of Education: a situation that, from my perspective as a teacher, induced anxiety in students, not only because of the requirement to write the test, but because of the external pressures to score well (e.g., parental pressures). My perception was that the school curriculum focused on "teaching to" the provincial examination, with a heavy emphasis on test-specific content. Having experienced this academic culture for two years, I found myself questioning why teachers were put into situations in which they were required to teach a wide breadth of topics, instead of focusing on fewer concepts, taught to a greater depth.

After graduating from the Bachelor of Education program at Queen's University, I was able to articulate my own beliefs about the utility of mathematics and to describe classroom scenarios that aligned with my own professional conceptions. In my view, mathematics programs would challenge students to use mathematics conceptually to solve authentic problems instead of completing exercises that required the application of repetitive rules and procedures. In spite of my own acknowledged beliefs, once in the classroom, I found myself falling into the pattern of teaching mechanically, structuring my lessons to match the provincial curricular expectations and directly teaching the standard rules and procedures necessary to solve typical problems that would likely be replicated on the provincial test. I felt conflicted because these behaviours belied my fundamental beliefs; that is, that a teacher's role should be to prepare students to think critically about authentic and novel mathematical problems, not to coach them to be proficient in providing formulaic answers to routine questions.

This professional dissonance led me to pursue a Master of Education degree and provided an opportunity to explore my own beliefs and knowledge about teaching mathematics. At the graduate level, I believed that I would have the opportunity to familiarize myself with scholarly research and professional literature about mathematics educators' beliefs about what mathematics is and how it should be taught. At the same time, I believed that I would gain practical knowledge about the research-based instructional strategies that would correlate with successful classroom practice.

Thus, as a mathematics teacher, I began this research with an insider perspective and approached the study from a curriculum stance. Now that it is completed, I believe that I have increased my understanding of successful classroom practice in mathematics education. I also believe that by disseminating its results, this study will contribute to the scholarly and professional literature by describing a course whose goal was two-fold: (a) to develop in-depth knowledge of both mathematics and mathematics teaching; and (b) to support candidates' reflection on their own beliefs about what mathematics is and how it could be taught.

Rationale

The field of mathematics education has undergone sweeping reform to content and pedagogy over the last six decades. The overall goal of the reform has been to improve the teaching and learning of mathematics (Grouws, 1992). According to Grouws, mathematics curriculum at the beginning of the twentieth century focused on mundane procedures and concepts that were not easily applicable to the workplace and lacked clear goals for educational outcomes. Reform to mathematics education in North America began in the 1950s in response to workplace and military demands for better basic mathematics skills. In response to research-based findings, the National Council of Teachers of Mathematics (NCTM) started working towards a new approach to learning mathematics to address the workplace and military demands, where discovery and active learning were fundamental. The most radical changes to mathematics curriculum were in response to research-based recommendations and expert opinions in 1989 with the publication of the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics*, which redefined school mathematics content across North America. The *Standards* (NCTM, 1989) advocated that mathematics be taught through a problem-based curriculum, relevant to the workplace and society, and that mathematics teachers become both facilitators of knowledge discovery and development, as well as expert resource-providers of rich and illuminative cases. To this end, the NCTM promoted a view of learning mathematics as a subject that is continually growing, changing, and being revised as solutions to new problems are explored (Suurtamm & Roulet, 2007).

Since the implementation of the *Standards* (which is the basis for current mathematics curricula across Canada and the United States), research has focused on teacher knowledge of mathematics to improve mathematics teaching and learning to align with the new vision (Begle, 1979; Ernest, 1989; Grouws, 1992). The measure of a teacher's mathematical content knowledge, however, is not definitive enough to predict student success (Begle, 1979). Many researchers believed that changes in response to the new directions described by the NCTM *Standards* would be slow and difficult because of teachers' firmly-held beliefs about the nature of mathematics and mathematics pedagogy (Cooney, 1987; Cross, 2009). Cooney contended that these beliefs would be formidable barriers to the acceptance by teachers of the need for large-scale reform to content, instructional practices, and assessment. Barriers to the reform, such as comfortable practices like rote teaching (often attributed to beliefs about the nature of mathematics), have been well-documented in teachers who simultaneously expressed beliefs that mathematics is a rigid set of concepts and who have adopted classroom practices that focused on formulae, procedures, and rules (Cross, 2009).

Ernest (1988) argued that, because inservice teachers hold beliefs that are less likely to change, it might be less challenging and more probable to impact *prospective* teachers' beliefs. Thus, the professional education of teachers in response to the expanded content and facilitative approach to instruction has been a focus in preservice mathematics education research. Empirical research to date, however, has suggested that preservice teacher education has, in fact, a weak impact on both beliefs and practice, including, (a) teacher knowledge of mathematics and pedagogy and beliefs about what mathematics is and (b) how it should be taught (Boaler, 1999). This, in turn, has led to a 25-year focus on identifying the factors that do have a positive influence on teachers' knowledge of mathematical content and help to align their beliefs about the nature of mathematics and mathematics teaching with those constructs upon which the contemporary mathematics curriculum is built (Boaler, 1999).

Purpose of the Study

The purpose of this study was to examine the ways in which prospective teachers perceived a change to their personal and professional beliefs about mathematics and mathematics teaching, and, content knowledge after experiencing a university mathematics course that purports to focus on both. This study was developed around a mathematics course with an education focus, MATH 010, entitled *Fundamental Concepts* *in Mathematics for Elementary Teachers*. The course was designed in response to the *Standards*-influenced Ontario Mathematics Curriculum, launched in 1997. The Ontario Mathematics Curriculum is based on a problem solving approach, and places great value on reasoning, communicating mathematically, and the application of concepts to authentic situations (Ontario Ministry of Education, 2005). The university course under study reflects the fundamental tenets on which the corresponding school-based curriculum was designed, in that prospective teachers are introduced to the problem-solving paradigm. In addition, the course includes formal opportunities for the course candidates to obtain in-school teaching experience and to reflect on their own experiences as preservice teachers and learners in the context of a reform-based culture. The course also affords prospective teachers the opportunity to examine the degree to which their own beliefs about the nature of mathematics and its teaching align with the teaching philosophy underlying the Ontario curriculum. Thus the subject of this study is an intensive mathematics education experience that focuses concomitantly on (a) the development of mathematics content knowledge, and (b) reflection on personally-held beliefs about mathematics, and its instruction.

The study seeks to answer the following research questions:

1. What are prospective teachers' beliefs about the nature of mathematics and mathematics teaching before the course experience?
2. How do prospective teachers perceive that their beliefs about the nature of mathematics and mathematics teaching change in response to the course experience?

3. To what do the prospective teachers attribute any changes to their beliefs about mathematics and mathematics teaching?

Definition of Key Terms

The key terms used for the purpose of this study are defined in this section.

"Beliefs" are defined as the individually validated confidence in the truth or existence of something not immediately susceptible to rigorous proof (McLeod, 1992). Personal beliefs are those that refer to topics of self-efficacy. The professional beliefs of teachers include ideas about teaching, learning, and subject content.

"Constructivism" is defined as a view of learning in which learners construct and design knowledge through discovery and reflection (Suurtamm & Roulet, 2007). The constructivist view of mathematics is that of a domain continually growing, changing, and being revised as solutions to new problems are explored (Suurtamm & Roulet).

"Discovery learning" is a method of inquiry-based instruction in which the learner uses his or her own experience and prior knowledge to explore questions and possible solutions in a problem-solving context (Thompson, 1992).

"Enrichment" refers to an enhanced mathematics program that complements and supplements regular classroom curriculum using challenging problems and activities as the context for instruction. The enrichment program in this study is called *StepAhead* and was designed for Grade 7 and 8 students who have been informally identified by their teachers as being mathematically talented, based on their performance in their regular mathematics class. The *StepAhead* program is further described in the Method chapter of this thesis.

“Inquiry-based mathematics” actively engages a student in classroom activities that are designed to promote the construction of mathematical concepts (Thompson, 1992; Wood, Cobb, & Yackel, 1991).

“Knowledge” is defined as the familiarity with facts, truths, or principles in agreement with members of a community that a certain position is true (Op’ T Eynde & De Corte, 2003).

“The nature of mathematics” refers to conceptions of the ways in which individuals define mathematics as a subject. For the purpose of this study, the nature of mathematics is not viewed from a philosophical perspective but rather through a practical lens. The definition includes perceptions of what it means to *do*, or perform, mathematics, which is the meaning used by the National Council of Teachers of Mathematics (2000).

“Preservice teachers” have begun, but have not completed, a Bachelor of Education program and intend to become certified professional educators.

“Prospective teachers” are university students who may consider completion of a Bachelor of Education and a career in teaching. A prospective teacher may or may not be enrolled in a Concurrent Education program as an undergraduate student.

“Reform-based” mathematics teaching refers to contemporary, research-based teaching strategies that include promoting classroom discourse, active learning through manipulatives, and a focus on multiple solution paths (NCTM, 1991). Reform-based mathematics is often equated with the National Council of Teachers of Mathematics’ vision of a mathematics classroom environment that is built on the goal of improving students’ experience with and achievement in mathematics.

“Rich learning tasks” refer to mathematics problems that require conceptual understanding to generate a solution. The task can be solved in a multiplicity of increasingly sophisticated ways.

“Traditional” mathematics teaching refers to instructional strategies that include lecture and the sole reliance of unidirectional knowledge transmission from the teacher to the student (NCTM, 1991). Traditional teaching also includes the rote application of algorithms by both the teacher and student to solve routine, textbook problems.

Conceptual Framework

Pedagogical Content Knowledge

Over the last several decades of empirical studies in mathematics education, researchers have debated the ways in which “knowledge” is best described and measured (Boaler, 1999). Shulman (1986) first proposed the idea of pedagogical content knowledge: expertise in the intersection of content and pedagogy that should improve a teacher’s understanding of “big idea” concepts which examine the whole picture, or the interconnectedness of topics, before analyzing the details. Shulman proposed a number ofcategories of knowledge, but only those most relevant to this study will be described: (a) content knowledge, a set of domain-specific, fundamental assumptions, definitions, and procedures to be learned; (b) pedagogical content knowledge, the representation of content knowledge through analogies, explanations, and insights, with a focus on teaching and teaching processes; and, (c) curricular knowledge, the understanding of how topics are sequenced and presented over time in a school setting. These categories were a major step in describing the knowledge required by educators for teaching mathematics

(Boaler, 1999). Pedagogical content knowledge has been more recently defined by Hill and Ball (2009), and will be further examined in the Literature Review chapter.

Beliefs and Knowledge about Mathematics

Beliefs and knowledge are not always clearly defined, nor is there agreement on the extent to which they are distinct (Thompson, 1992). For the purposes of this study, the distinction between beliefs and knowledge will be conceptualized using the framework developed by the *Teacher Educational and Development Study in Mathematics* (TEDS-M) study (Tatto, Schwille, Senk, Ingvarson, Peck, & Rowley, 2008). TEDS-M is a large-scale international study currently examining teacher education policies, practices, and outcomes in 17 participating countries. The study was organized by research coordinators from various participating countries and was funded primarily by the International Association for the Evaluation of Educational Achievement. The purpose of the study is described as follows: "to provide a better understanding of what qualified teachers of mathematics are able to learn about the content and the pedagogy of mathematics and the conditions these teachers need to acquire this knowledge" (Tatto et al., 2008, p. 15). The study is focused on how teachers are prepared to teach mathematics in elementary and early secondary school and thus is in line with the research questions driving this study.

TEDS-M was designed to examine preservice and inservice teachers' beliefs and knowledge and was conducted partially using a questionnaire based on several beliefs and knowledge categories, as delineated in Table 1. These categories will be used in this study to assess (a) perceived areas of change in prospective teachers' beliefs about what mathematics is and how it should be taught; and (b) knowledge of mathematics and research-based strategies for and approaches to classroom instruction.

The categories of knowledge presented in the TEDS-M study correspond to Shulman's (1986) seminal ideas that knowledge required for teaching requires expertise in both mathematics content and mathematics pedagogy (Tatto et al., 2008). TEDS-M is investigating three distinct categories of preservice teachers' knowledge: (a) content knowledge, which includes specific content expectations (e.g., numbers and geometry) and cognitive process expectations (e.g., knowing, applying, and reasoning); (b) pedagogical knowledge, which includes curricular, planning, and enacting knowledge; and (c) knowledge of teaching, which includes knowledge of teaching strategies and classroom organization models.

In the category of preservice teacher beliefs, the nature of mathematics refers to how novice teachers perceive mathematics as a subject (instrumental, procedural, or applied). According to the TEDS-M framework, the nature of mathematics refers to the ways in which preservice teachers perceive the act of performing mathematics tasks. Preservice teachers' beliefs about the nature of teaching and learning mathematics would reflect opinions about the appropriateness of choices made regarding certain instructional activities and questions related to the purposes of mathematics as a school subject. Self-efficacy would involve an individual's self-assessment of her ability to teach mathematics. A preservice teacher's self-assessment of her preparedness to teach would involve the degree to which the preservice teacher agrees that her preparation has given her the capacity to carry out central teaching tasks in the first few years of her career, and

Table 1

Beliefs and Knowledge Categories

Preservice teachers' beliefs	Preservice teachers' knowledge
Nature of mathematics	Mathematical Content Knowledge (MCK)
Nature of teaching mathematics	Content (numbers, geometry)
Nature of learning mathematics	Cognitive (knowing, applying, reasoning)
Self-efficacy	
Preparedness to teach	Mathematical Pedagogy Knowledge (MPK)
	Curricular
	Planning (pre-active)
	Enacting knowledge (interactive)
	Knowledge of teaching (pedagogy)

has provided rich opportunities to learn about the mathematics teaching profession.

Summary

The categories of beliefs and knowledge outlined in TEDS-M will be used in this study as a framework for classifying participants' beliefs and knowledge. The TEDS-M framework was based partly on Shulman's categories of knowledge. In the next chapter, other ideas, by various scholars and researchers, are discussed in order to examine seminal ideas in the literature surrounding beliefs and knowledge.

Thesis Structure

This thesis is written in five chapters. In the first chapter, I introduced the research context and purpose of the study by articulating specific research questions. In Chapter 2, which follows, I present literature related to categories of beliefs and knowledge used in this research. I conclude the second chapter with a description of beliefs and knowledge in relation to preservice teacher education and the influence of the literature on the method of this study. In the third chapter, I describe the study's methodology with attention to participant selection, data collection methods, organization and analysis of the data, and the trustworthiness of the study. I also describe the contextual framework of the study, MATH 010, in detail. I then present, in Chapter 4, the data collected from study participants and the course instructor in order to describe the MATH 010 experience, as well as describe participants' beliefs about the nature of mathematics and its teaching before and after the course. Finally, in Chapter 5, I discuss the data of the semester-long combination of course work and a practicum experience. The discussion probes the key findings of this study, which focus on the transformative elements of the course and the power of an expert instructor (Berliner, 1986; Miller, 2007). I close the thesis with limitations of the study and implications for future research.

CHAPTER 2

LITERATURE REVIEW

This chapter reviews the research and theoretical frameworks that guided the development of the method to collect and analyze the type of data necessary to achieve the goal of the study. The literature is approached from a curriculum perspective, as opposed to a cognitive perspective, to present a more discrete idea of beliefs and knowledge in order to later examine the data in this study. The chapter will explore three main areas of literature: (a) the relationship between teacher beliefs and knowledge, (b) how beliefs correspond with practice, and (c) how knowledge corresponds with practice.

The Relationship Between Teacher Beliefs and Knowledge

In the 1970s, researchers started studying mathematics education by focusing on teachers' thinking and decision-making processes (Thompson, 1992). The study of teacher cognition stimulated an interest in examining the beliefs and knowledge of teachers; however, as Thompson contends in a review of research on mathematics teaching and learning, distinguishing between beliefs and knowledge is difficult and the distinctions are often "fuzzy." One reason for the lack of clarity in distinguishing beliefs from knowledge is that many teachers treat their beliefs as knowledge (Thompson). Some research suggests that a belief can become knowledge if it is communally validated or that knowledge may become a belief if it is no longer consensually agreed upon (Kagan, 1992). This debate is complex and is fuelled by arguments from many disciplines including philosophy, cognitive science and neuropsychology. In order to establish the author's use of the terms "beliefs" and "knowledge" for the purpose of this thesis, several researchers' work will be described.

In this section of the literature review, several researchers' ideas about what constitutes beliefs and knowledge will be described. Although a definitive discussion about the ways in which beliefs differ from knowledge may not be possible within the scope of the present study, several distinctions are presented.

Defining Beliefs

Early definitions of beliefs were taken to mean different constructs for different researchers. Nespor (1987) noted that beliefs, unlike knowledge, are personally held mental constructs that do not require community consensus or agreement to establish their validity. Others argued that beliefs are informed by experiences. For example, Pajares (1992), in his review of research on the topic, defined beliefs as personal principles constructed from experiences, often unconsciously, to interpret new experiences and information and to guide action. Similarly, Thompson (1992) described beliefs as dynamic entities that undergo change as one evaluates beliefs against experiences.

Some authors have argued that beliefs influence practice (Kagan, 1992; Munby, 1982; Nespor, 1987) and that beliefs have the power not only to filter new information but to influence a person's interpretation of events (Nespor, 1987). This idea suggests that, once beliefs are established, they are less likely to change even if contradictory evidence is presented because of their deep-seated nature.

Examining beliefs, in the context of teaching mathematics, leads to another dimension of internal constructs. In a seminal paper that used both empirical findings and arguments from the philosophy of mathematics, Ernest (1989) proposed a hierarchical model of the key belief components for the mathematics teacher. Ernest argued that a teacher's beliefs about the nature of mathematics are the basis for a system of beliefs about teaching and curriculum. A teacher's beliefs are separated into three main categories, based on Thompson's (1984) earlier ideas:

> First of all, there is the instrumentalist view that mathematics is an accumulation of facts, rules and skills to be used in the pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts. Secondly, there is the Platonist view of mathematics as a static but unified body of certain knowledge. Mathematics is discovered, not created. Thirdly, there is the problem solving view of mathematics as a dynamic, continually expanding field of human creation and invention, a cultural product. Mathematics is a process of enquiry and coming to know, not a finished product, for its results remain open to revision. (p. 251)

Ernest's views on teachers' beliefs about the nature of mathematics broadly fall into two categories: (a) mathematics is a static or fixed body of knowledge, and (b) mathematics is a dynamic subject which involves continual change. These views are thought to influence two other key belief components: namely, the view of the nature of mathematics teaching and the view of the process of learning mathematics. The static versus dynamic stance for mathematics beliefs are still categorized in this way by current researchers (Tatto et al., 2008). The positions posited by Pajares, Thompson, and Ernest resulted in some of the early but persistent definitions of “beliefs,” which will now be contrasted with scholarly and empirical considerations of the term “knowledge.”

Defining Knowledge

Over the last several decades, mathematics education researchers have argued about what constitutes knowledge (Boaler, 1999). In contrast to beliefs, knowledge has been defined by some researchers to mean the acquaintance with facts, truths, or principles in agreement with members of a community that a certain position is true (Green 1971; McLeod, 1992; Op’ T Eynde & De Corte, 2003). Knowledge is thought to be a general agreement about procedures derived from evaluating and judging their validity (Thompson, 1992). Knowledge has also been argued to include practical skills, strategies, and information acquired through experience or education (Ernest, 1989). Nespor (1987) contended that knowledge is constructed in a variety of ways, including experimentation, reasoning, and exploration, all processes through which propositions are formed and can be further evaluated. The public validation of knowledge is a view that proposes a distinction between beliefs and knowledge; however, similarly to beliefs, scholars do not agree on whether experience influences knowledge, whether knowledge informs experience, or whether both are true (Hill & Ball, 2009).

Knowledge, in the context of teaching mathematics, can be further explored. A teacher’s domain-specific content knowledge is undoubtedly necessary for mathematics instruction; mathematical content knowledge, however, is not the only form of knowledge thought to be required by the teacher for effective classroom practice (Hill & Ball, 2009). Hill and Ball suggest that while content knowledge is important for teaching mathematics, conventional content knowledge alone appears to be insufficient. According to Delaney, Ball, Hill, Schilling, and Zopf (2008), another integral facet of the body of knowledge required by teachers is mathematical knowledge for teaching. It is suggested that mathematical knowledge for teaching requires unique insight and analysis of teaching scenarios. It is posited that this category of knowledge develops through familiarity with facts about mathematics in combination with a teaching application in which the explanation of why and how to use certain facts and strategies are necessary. For example, the authors state that teachers "need to have deep understanding of algorithms used for arithmetic operations, to be able to manipulate commonly used materials and to know terms that appear in the curriculum" (p. 192). More recently, knowledge was classified by Hill and Ball (2009) into two sub-categories of Shulman's (1986) earlier taxonomy: (a) subject matter knowledge and (b) pedagogical content knowledge. Subject matter knowledge includes common and specialized content knowledge. Pedagogical content knowledge includes knowledge of students, teaching, and curriculum.

Parallels Between Beliefs and Knowledge

There are parallels between the definitions of belief and knowledge because there is no universal consensus to establish one definition as absolute among researchers. The fact that there is not consensual agreement by experts in the field means that both terms lack an absolute definition in the literature. Nonetheless, there does appear to be tacit agreement that beliefs are best classified as an element of the nature of mathematics, which can be static or dynamic, and correlated to beliefs about learning and teaching mathematics. Knowledge is classified in terms of content-specific facts and strategies and may be related to knowledge of teaching (pedagogy), as well as knowledge of learning (cognition). Although not absolutely defined, the distinctions presented lead to further clarification between beliefs and knowledge in the context of teaching mathematics.

The Correspondence Between Beliefs and Practice

Due to the complexity of defining and measuring beliefs, there does not appear to be agreement among researchers about the relationship between beliefs and practice or the degree to which a teacher's beliefs influence his practice in the classroom, or vice versa (Ernest, 1989; Munby, 1982; Thompson, 1992). In the next section of this chapter, the influence of a teacher's beliefs on his practice will be presented, along with current research and a critical examination of the strongest relationship between beliefs and practice.

In a review of research, Pajares (1992) hypothesized that preservice teachers' perceptions of knowledge are shaped by systems of beliefs. Pajares argued that these perceptions are very difficult to predict. He suggested that it would also be difficult to (a) predict the development of beliefs over time in a teaching career, (b) control or (c) influence teacher beliefs. Pajares suggested that what might be needed to examine the belief system of teachers is either to examine how teaching influences beliefs or to try and change teacher beliefs by making teachers and prospective teachers conscious of their beliefs.

Beliefs Inform Practice

Beliefs and practice are deeply ingrained and affect the practice of teaching (Ernest, 1989; Kagan, 1992; Munby, 1982; Thompson, 1984). Thompson examined the practice and beliefs of three intermediate division teachers. Through interviews and classroom observations, Thompson described the relationship between beliefs and practice as complex but argued that teachers' views, whether consciously or unconsciously held, play a significant role in instruction. Thompson cautioned that failure to recognize the role of teachers' beliefs on instructional practice likely will result in misguided efforts to improve mathematics instruction in schools.

In his study of teacher education, Ernest (1988) examined 30 undergraduate students enrolled in a mathematics course for prospective elementary teachers. The 18 month course involved a weekly practicum. Ernest was able to collect interview and questionnaire data to suggest that prospective teachers' practice informed their instructional choices. For example, a teacher with a precise, structured view of mathematics used a narrow, computational approach to instruction. Ernest later (1989) suggested that the beliefs of the mathematics teacher represent the impact of curriculum and the degree to which mathematics teacher education has experienced successful reform. Ernest contended that beliefs account for most differences among mathematics teachers. For example, two teachers with very similar knowledge might teach mathematics very differently. One might teach with a problem solving approach, while the other adopts a more rote instructional method. These approaches, in Ernest's view, reflect the teachers' beliefs about the nature of mathematics (i.e., as dynamic or static, respectively). The dynamic view might lead to the acceptance of multiple perspectives about problem solving. Conversely, a static belief about mathematics as a subject might lead to the teacher's insistence on a single, prescribed method for solving a particular problem.

Kagan (1992) reviewed research in order to argue for the need for a better examination of teachers' beliefs and practice. Kagan found that preservice teachers tend to leave university with the same ingrained beliefs held on entry, likely because of the many hours spent in elementary, secondary, and undergraduate classrooms. Students are thought to internalize models of good and bad teaching from previous experiences and grow increasingly comfortable with these beliefs throughout their own teacher education. Kagan criticized university education programs for lacking cognitive engagement and not allowing for a critical examination of personal beliefs.

More recently, Stipek, Givven, Salmon, and MacGyvers (2001) examined teachers' beliefs about the nature of mathematics, in combination with categories of beliefs about teaching and learning mathematics. Using various instruments designed to collect qualitative data (videotape, questionnaires, and student interviews), the researchers studied 21 fourth to sixth Grade teachers by comparing their stated beliefs to their classroom instructional strategies. The results of the year-long study concluded that more traditional beliefs were associated with more traditional practices, including less student autonomy, while beliefs about mathematics as a subject involving creativity yielded a more inquiry-oriented instructional focus. These findings suggest that teachers' beliefs inform and influence instructional choices.

Stuart and Thurlow (2000) studied a university mathematics curriculum methods class, by examining the relationship between teacher beliefs and practice. The class was aimed at (a) evoking reflection on prior experiences in mathematics, (b) examining the effect of these experiences on teacher candidates' beliefs about teaching and learning mathematics, and (c) understanding how these beliefs might influence future instructional strategies. Twenty-six preservice teachers participated in the 13 week course, which met for 12 hours per week. Students initially met in the university classroom, but then started observations in a local elementary school, teaching in the local classroom during the final weeks of the semester. Interviews with preservice teachers were conducted, and written responses to journal prompts and written submissions about teaching and learning of mathematics were collected. The researchers found that many preservice teachers increased their understanding of the role that beliefs may play in instructional choices after explicating their personal beliefs about teaching and learning that, by self-admission, were heavily influenced by their childhood experiences. This study required preservice teachers to confront and articulate their own beliefs about mathematics teaching and learning by reflecting on their own actions during a course related to a practicum experience. Although the researchers attributed preservice teachers' change in beliefs to conscious recognition addressed in the course, it is likely that the change in beliefs also related to the practical experience of a teaching placement.

There are, however, factors that impact the degree to which beliefs will be evident in a teacher's instructional behaviours. Ernest (1989) suggested that social context and level of consciousness are two important factors in determining whether or not teachers' beliefs will be reflected in their practice. For Ernest, the social context encompasses the influence of parents, peers, students, and supervisors, and includes parameters set by the curriculum. Social influence is suggested to be powerful when a teacher conforms to the teaching practices of the institution, despite having differing beliefs. A teacher's level of consciousness about her own beliefs involves reflection about teaching practices, including justification of views, awareness of viable alternatives, and reciprocity between classroom practices and beliefs. Thus, if a teacher is not explicitly aware of her beliefs regarding the nature of teaching mathematics, she may not reflect on her teaching practice nor consider the effectiveness of other potential teaching alternatives. Similarly, Thompson (1984) suggested that differences in the awareness of the relationship between beliefs and practice varied among teachers based on their level of reflection and posited that such factors may cause a gap to appear between beliefs and practice. It is possible that these factors give rise to the significantly lower volume of research studies conducted to examine the ways in which beliefs may alter practice. This is likely because social context and level of consciousness constrain beliefs, making them difficult to observe and measure.

A recent study by Cross (2009) provides empirical evidence of Ernest's (1989) claims about the influence of a teacher's beliefs on his instructional practice. Cross investigated the ways in which five Grade 9 Algebra teachers organized their classroom activities and interacted with and assessed their students, and related these to the ways in which they expressed their beliefs about the nature of mathematics and teaching. Using data collected through individual interviews, observations, and field notes, Cross was able to match teachers' articulated beliefs that mathematics was a rigid set of concepts to observed classroom practices that focused on formulae, procedures, and rules. These teachers believed that their role was to hold facts and information which they transmitted to students. They also believed that students would acquire this knowledge through repetition and memorization. Although the majority of teachers in the study held beliefs aligned with their practice, Cross reported that some of the teachers' beliefs were not in line with their instructional practices. For example, one teacher stated that, for her, teaching mathematics necessitated a problem-solving approach; however, the same teacher employed lecture and procedural activities as instructional methods. The teacher attributed this disconnect to the task at hand, saying that lecture and procedure were appropriate for certain topics she was teaching.

Practice Informs Beliefs

There appears to be another group of researchers who believe that beliefs inform practice. This school of thought falls in line with the notion that beliefs are dynamic and responsive to evaluation through new experiences (Thompson, 1992). Kagan (1992) argued that because many preservice teachers leave university with the same beliefs with which they arrived, teachers need an opportunity to make knowledge their own before there can be a change in beliefs. Her claim suggests that the act of educating preservice teachers about their beliefs may not be enough to cause a change in beliefs. For meaningful, long-term change to occur within a belief system, teachers must engage in practical inquiry to learn about new instructional approaches and reflect collaboratively on what they learned (Kagan, 1992; Stipek et al., 2001; Wood, Cobb, & Yackel, 1991).

Wood, Cobb, and Yackel (1991) conducted a case study of a mid-career Grade 2 elementary mathematics teacher who was faced with the challenge of revising her mathematics program to reflect the NCTM's *Standards*. This shift required the teacher to implement open-ended activities that allowed for a variety of solution methods and were in sharp contrast to her previous practices in teaching mathematics. The instructional strategies provided a learning context for the teacher and her students. Using data collected over one year from field notes, videotapes, and individual interviews, researchers found that the teacher adopted practices that were compatible with constructivist tenets and her beliefs about the nature of teaching and the nature of mathematics changed drastically. For example, the teacher initially stated that her role was to resolve conflict and confusion about mathematics through the implementation of procedural methodology. By the end of the study, researchers found that the teacher viewed her role as a creator of situations through which students could resolve their confusion through exploration. Although this study involved only one teacher and cannot be generalized to all teacher interventions related to the implementation of the NCTM *Standards*, it does illustrate the potential for change to occur in teacher beliefs about the nature of mathematics and its teaching and learning when reform-based teaching strategies are used.

In a more recent study, Warfield, Wood, and Lehman (2005) investigated the relationship among teachers' beliefs, their reflection, and their learning. The study examined seven elementary mathematics teachers in their first or second year of teaching. Teachers were videotaped during classroom instruction and researchers used stimulatedrecall to prompt teachers to examine and reflect on their practice. Teachers were also interviewed individually by researchers and kept a journal throughout the two-year research project. Results of the study indicated that three teachers' reflection about their classroom instruction yielded a change in their thinking about their role in developing children's thinking. The study suggested that teachers' beliefs about student learning were influenced by their reflection upon instructional methods. The other four teachers' beliefs about their role in influencing learning, including the passive transmission of knowledge, were not influenced by reflection on instructional strategies. The researchers speculated that this may have been attributable to the fact that the teachers' beliefs were deeply ingrained and not amenable to change.

Guskey and Gates (1986) suggested that change in beliefs follow, rather than precede, change in behavior. Thus, when a teacher changes an instructional approach and recognizes that the new approach helps students' learning outcomes, Guskey and Gates would argue that the teacher is more likely both to adopt this method and change her beliefs. Cohen and Ball (1990) described a teacher who had used traditional teaching methods but was then taught to implement an inquiry-focused lesson. After seeing his students' success following the lesson, the teacher reflected that he was amazed that his students used such advanced thought and reasoning. The authors speculated that if this teacher was to continue to use and reflect upon inquiry-based lessons, he might change his beliefs about the nature of mathematics and its teaching and learning because of the impact on student learning.

Deeply held beliefs may be difficult to change, but may be malleable given the right context. Liljedahl (2005) examined preservice elementary teachers who self-identified as having phobias or limited capabilities in mathematics while enrolled in a mandatory mathematics education course. The semester-long (13 week) course was aimed at facilitating a deeper understanding of various topics by building connections among strands. Data was collected for this study from a final project in which students were required to write about a mathematical "AHA!" moment: suddenly realizing, for example, the answer to a problem and feeling pride. Sixty-one of the 76 participants discussed a change in their beliefs through the experience of their AHA! moment, including suddenly believing problem solving requires prolonged time and deep thought, not "intelligence." Although this research examined preservice teachers' beliefs about their abilities in mathematics, beliefs about the nature of mathematics were also addressed by participants. The context of this study is in contrast to previously discussed studies, which focused on classroom experiences to change beliefs; however, the problem-solving process provided an appropriate context for several teachers to express a change in their beliefs about the nature of mathematics, as well as their own abilities.

Summary of Beliefs and Practice

Evidence for the influence of beliefs on practice and the influence of practice on beliefs has been presented. Some evidence shows that beliefs are deeply ingrained and resistant to change through education, likely due to the "apprenticeship of observation" (Lortie, 1975, p. 61) that students acquire in the approximately 2000 hours spent as learners in elementary and high school mathematics classrooms (Ball, 1988). Thus, these deeply held beliefs are likely to inform practice. Other evidence shows that beliefs are dynamic and are susceptible to change in response to teachers' practical experiences, including teaching or positive interactions with mathematics, and reflecting critically about experience. In both cases, it is clear that beliefs and practice are linked and that it is important to emphasize both beliefs and practice in teacher education.

The Correspondence Between Knowledge and Practice

"Knowledge" is another complex construct whose definition is not consensual among researchers. Neither do researchers agree about the type of knowledge required for teaching. Does a teacher require theoretical knowledge acquired through a formal education about content or practical knowledge about pedagogy for classroom instruction? In this section, I examine both views, presenting research studies that examine the relationship between knowledge and, specifically, the types of knowledge that are considered fundamental for mathematics classroom instruction.

A mathematics teacher needs knowledge about mathematics content as well as knowledge about the specific strategies unique to teaching mathematics, in order to be successful in the mathematics classroom. Ernest (1989) suggested that mathematics subject knowledge is built up from childhood. According to Ernest, mathematics subject knowledge includes knowledge of procedures and strategies, links to other subjects, and knowledge of mathematics as a multi-faceted domain, including, among other strands, geometry, numbers, and probability. Knowledge of mathematics is thought to play a key role in the representations of the content in the classroom. This type of knowledge is posited to be the foundation for teaching mathematics because a teacher needs knowledge of the core content in order to plan meaningful opportunities for students to learn and to interpret learners' responses. Ernest also suggested that mathematics pedagogical knowledge is another important component of knowledge that contributes to students' learning and achievement. It is this category of knowledge that, it is argued, allows teachers to transform subject matter knowledge into representations of content knowledge for use in the classroom.

Conceptual knowledge about mathematics for teaching is also suggested as influential in the choice and approach to tasks used by teachers. For example, teachers design explanations, respond to questions, explain concepts, and show why procedures work (Delaney, Ball, Hill, Schilling, & Zopf, 2008). This knowledge for teaching comes not from the mastery with facts about mathematics, but from insight into and analysis about why, for example, integer arithmetic is practical and rich, and makes it possible for the teachers to explain why and how integer arithmetic is necessary. Hill and Ball (2009) suggest that mathematical knowledge for teaching is strengthened by extensive mathematics-focused professional development. With targeted professional development focused on mathematical knowledge for teaching, Hill and Ball suggest that teachers could learn how to explain concepts and show why procedures work and, therefore, improve their explanations to students.

Researchers still do not agree, however, on the type of knowledge required to teach mathematics (Ball, Lubienski, & Mewborn, 2001). Most notably, pedagogical knowledge, which is knowledge of teaching processes, and specialized content knowledge, which is conceptual knowledge of domain-specific content, appear to be key components of knowledge and have been shown to have a strong relationship with facilitating student learning (Ball, Lubienski, & Mewborn, 2001). Specialized content knowledge is considered to be a theoretical component of knowledge, in contrast to pedagogical knowledge which is more practical: that is, “knowing that” versus “knowing how” (Ryle, 1949). Few studies have been successful in pinpointing the kinds of knowledge that can provide teachers with the appropriate mathematics to help students learn, whether it be purely mathematical, grounded in practice, or both (Ball, Hill, & Bass, 2005).

Mathematical Content Knowledge

Begle (1979) argued that teachers can acquire the types of knowledge that are required for teaching through traditional approaches to teacher education, including course instruction. More recently, the type of content knowledge thought to be necessary for teaching mathematics includes common content knowledge, which is a breadth of knowledge about a subject including knowledge of procedures and algorithms, as well as a specialized content knowledge, which refers to deeper conceptual understanding of the subject (Delaney, Ball, Hill, Schilling, & Zopf, 2008).

Ma (1999) examined the mathematical knowledge of 23 American elementary school teachers and 72 Chinese elementary school teachers. Ma interviewed these teachers about their knowledge of mathematics content and led discussions about teaching scenarios involving student misconceptions. For example, she asked them how they would teach a certain topic, such as division by a fraction and then asked them to do a diagnostic assessment of student work that demonstrated common errors. Ma’s study had a profound impact on mathematics teacher education by highlighting the weak mathematical content knowledge of American elementary school teachers. Ma described the Chinese teachers as having a "Profound Understanding of Fundamental Mathematics" (PUFM). She further described PUFM in this way:

> A teacher with PUFM is aware of the 'simple but powerful' basic ideas of mathematics and tends to revisit and reinforce them. He or she has a fundamental understanding of the whole elementary mathematics curriculum, thus is ready to exploit an opportunity to review concepts that students have previously studied or to lay the groundwork for a concept to be studied later. (p. 124)

Ma used her data to argue that North American teachers need opportunities to develop a deep understanding of fundamental mathematics. Ma also argued for a school curriculum focused on coherent mathematics based on core ideas, connected longitudinally over time. Similarly, in the report, *The Mathematical Education of Teachers* (Conference Board of the Mathematical Sciences, 2001), it is argued that preservice teachers need to take courses that allow for the development of a deeper understanding of the mathematics they will teach, as well as develop a mastery of several grades above and below the grade that they will teach.

In an early study to investigate the relationship between teachers' mathematical knowledge and students' achievement, Begle (1979) conducted a meta-analysis about teacher effects, analyzing the number of mathematics courses teachers had taken correlated to student achievement. Begle's study indicated that teachers who had taken advanced mathematics courses produced positive main effects on student achievement in only 10% of cases and found negative main effects in 8% of cases. Ball (1988) was able to refute Begle's claims that more content knowledge correlated with student achievement with data to support that teachers with more mathematics coursework did not correlate with the effectiveness of students' learning. Ball (2002) later attributed these negative effects to the idea that more advanced domain-specific education for teachers may interfere with explanation and simplification of content. Ball further hypothesizes that these negative main effects might have to do with the advanced coursework using conventional approaches to mathematics education which may not contribute to the effectiveness of teaching students using research-based contemporary instructional methods. Ball's study was evidence against the claim that "the more a teacher knows about his subject matter, the more effective he will be as a teacher" (Begle, 1979, p. 51).

Pedagogical Content Knowledge

It is argued by another group of mathematics education researchers that practical experiences of teaching are necessary for the types of knowledge required for student learning in the classroom, namely, pedagogical content knowledge (Kagan, 1992; Stipek et al., 2001; Wood, Cobb, & Yackel, 1991). The definition of pedagogical content knowledge, originally articulated by Shulman (1986), has been more recently broadened to include knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum (Hill & Ball, 2009). Hill and Ball argue that it is not only through formal education that a teacher develops the knowledge for teaching, but it is also through experience in classroom instruction and reflection on that experience that professional growth is achieved. Even though the relationship between knowledge and practice may not be agreed upon, it is thought that practical knowledge about how to teach is important and should be a focus of education (Ball, Lubienski, & Mewborn, 2001). Based on their observations about teacher education and student learning, Hill and Ball (2009) suggested that teacher education should include familiarization with mathematical ideas and procedures in a way that affords opportunities to consider other explanations or representations; such as professional development opportunities that explore rich problems from multiple perspectives.

Ball, Hill, and Bass (2005) suggested that what is needed to improve mathematical knowledge for teaching is a program for teachers that demands depth and detail about knowing mathematics *for* teaching, with a focus on important skills for effective teaching of mathematics. One such skill involves the ability to pose mathematical problems having alternate solution methods, for example, using several contemporary methods for solving multiplication problems (e.g., manipulatives). The need for such a program was suggested based on a study of 700 Grades 1 and 3 teachers (Hill & Ball, 2004), who were given a questionnaire about the knowledge required for teaching, including common and specialized content knowledge. For example, one question asked teachers to choose the correct value of an exponent that would result in a solution of one (i.e., $10^0 = 1$). A correct response of “zero” would indicate that teachers had an understanding of common content knowledge about number theories. Student achievement was also measured, over two years, using questionnaires. The results showed that teachers who scored higher on measures of mathematical pedagogical content knowledge produced better gains in student achievement. This finding suggested that teachers should complete courses that focus on the kinds of mathematical knowledge needed for teaching.

To examine whether teachers learn about mathematical knowledge for teaching through formal coursework, Hill and Ball (2004) examined a professional development setting. The researchers examined a summer workshop program in California, the Mathematics Professional Development Institute, which served 398 teachers in 15 different workshop groupings. The study used questionnaires, administered before and after the program, examining areas of mathematical content and pedagogical knowledge required for teaching. For example, one item asked teachers a true-false question about the number zero (whether zero was a number and if so, whether it was an even number). Researchers found that teachers increased their knowledge for teaching mathematics as a result of participating in the workshops. In addition, greater gains (2−3 increase in the raw number correct) were measured related to the length of the program and experience with the specific curriculum, focused on proof, analysis, exploration, and representations. Although this study was conducted over a short time, the results suggest that teachers may be able to learn mathematical knowledge for teaching through course instruction. Researchers did not consider, however, whether this increase in knowledge was reflected in the instructional methods of the teacher and whether this newly acquired knowledge positively informed practice.

Summary of Knowledge and Practice

It appears that both mathematical content knowledge and pedagogical content knowledge are important factors when considering a teacher's knowledge of the mathematics required for teaching. Perhaps an appropriate balance between mathematical content knowledge and pedagogical content knowledge is necessary for contributing to the knowledge needed for teachers to instruct in ways that promote student learning. It is important now to consider teacher knowledge in concert with teacher beliefs to examine the ways in which preservice teacher education could be structured to develop knowledge of content and pedagogy and challenge beliefs about the ways in which a teacher's conception about the nature of mathematics and its instruction support or impede student learning.

Beliefs, Knowledge, and Prospective Teachers

Preservice teacher education has been criticized for not introducing the essential principles on which preservice teachers are thought to build a foundation for a successful teaching career (Ball, Hill, & Bass, 2005). Such criticisms include not challenging preservice teachers' beliefs about mathematics and its teaching and not achieving an optimal balance of theory and practice for education about knowledge of content and pedagogy. Three recent studies that examine teacher education programs focused on educating preservice teachers about beliefs and knowledge through a combination of theory and practice are presented in this section.

Swars, Smith, Smith, and Hart (2009) explored a teacher preparation program that supported teachers' change in beliefs by introducing preservice teachers to issues such as mathematics efficacy and anxiety, instruction in pedagogy, and specialized content knowledge for teaching mathematics. The two-year study included a course designed to change teachers' beliefs about mathematics teaching methods through a constructivist approach to teaching mathematics. The program also featured a developmental field placement at the same time as in-class course work which provided the context for the change in beliefs. Transformations in beliefs related to personal efficacy and teaching efficacy beliefs were described in response to questionnaire items administered throughout the course, suggesting that beliefs and content knowledge can be changed by providing a context such as a well-designed university course.

Similarly, Charalambous, Panaoura, and Philippou (2009) examined a preservice teacher education program aimed at aligning beliefs and attitudes about the nature of mathematics with the NCTM *Standards* vision of an exemplary mathematics program. Participants in the study were 94 preservice teachers. The program included instruction about the history of mathematics, with special focus on the theories through which mathematics was developed. The goal of this program was to enhance preservice teachers' understanding of the content and equip them with several methods and techniques needed to teach the concepts in novel and contextual ways. Changes in preservice teachers beliefs about the nature of mathematics were assessed using a questionnaire administered four times over the two-year period. The findings suggested that preservice teachers' beliefs and attitudes varied depending on mathematical background. This study suggested that if a preservice teacher had strong content knowledge of mathematics at the start of the course, she was unlikely to change her beliefs about the nature of mathematics.

Dinsmore and Wenger (2006) examined 12 preservice teachers' perceptions about their own learning during the first year of a teacher preparation program. The year-long course consisted of in-class instruction, as well as a field placement within a local elementary school. The goal of the course was to enhance participants' learning experiences by building a community of learners. Collaborative activities were used to build community and included group projects, presentations, and cooperative-learning strategies. Three sets of individual and group interviews, in addition to four different surveys, teaching observations, and written assignments were collected as data to support the study. The results suggested that participants valued learning within a community and positive relationships with teachers and students enhanced their learning. The study followed six of the participants into their first year of teaching and found that they had adopted many of the community-building activities in their own classrooms. The study suggests that teacher education programs that foster learning within a community will influence preservice teachers to build communities of learners within their own classrooms in order to enhance learning.

Summary

The arguments surrounding teacher beliefs and knowledge necessary for student learning have been examined. Beliefs about the nature of mathematics and its teaching and learning appear to either be deeply ingrained and influence practice, or to be dynamic in nature and amenable to changes in response to first-hand teaching experiences. Beliefs appear to be a very complex construct and being able to articulate one's beliefs may be difficult due to level of consciousness or curricular constraints. Knowledge appears to have the same complexity. Mathematical content knowledge (knowledge about the subject of mathematics) and pedagogical content knowledge (knowledge of teaching processes) contribute to the kind of instruction that promotes student learning; however, it is less clear what other precise kinds of knowledge are required for the teaching of mathematics.

Examining the complexity of beliefs and knowledge for preservice teacher education is important. Preservice teacher education programs are intended to equip teacher candidates with the necessary skills, attitudes, dispositions, and behaviours to succeed in the classroom. Opportunities to learn that combine theory and practice have been shown to provide a rich context for preservice teachers to examine their beliefs and consider the consequences of those beliefs. Through a combination of reflective courses and practical field placements, beliefs can be addressed, whether they are static or dynamic. The combination of theory and practice also appears appropriate for instruction about both mathematical content knowledge and pedagogical content knowledge. It then seems appropriate to suggest that preservice teacher education programs should include courses aimed at challenging candidates' beliefs about the nature of mathematics and its teaching and support the development of knowledge about content and pedagogy, in concert with practical classroom experiences. This type of education allows for alternate views, justification of views and assumptions, and hopefully a depth of knowledge that is built on a positive foundation of beliefs that support student learning.

Influence on Methodology

When designing the methodology for the current study about the beliefs and knowledge of prospective teachers enrolled in MATH 010, the literature presented in this review was instrumental in decision-making. From the literature, it is apparent that teacher perceptions must be articulated in order to describe an individual's beliefs about the nature of mathematics. It is also clear that measures about knowledge of content and pedagogy are key. MATH 010 provides a rich context which combines many elements found in the literature to support changes to beliefs and knowledge. Most notably, MATH 010 combines an in-class university experience with group discussion, problem sets, and peer problem solving as well as a practicum experience where prospective teachers are provided the opportunity to put in-class theories into practice, while reflecting on both throughout the course. It is clear in the literature that articulating a teacher's beliefs is difficult and measuring knowledge for teaching is not a simple process. For this reason, simply observing prospective teachers throughout a practicum experience seems insufficient to describe their beliefs and knowledge about mathematics and teaching. The literature does support, however, the need for personal reflection in examining one's beliefs about the nature of mathematics and knowledge of content and pedagogy. For this reason, the current study is framed with pre- and post-course questionnaires seeking prospective teachers' perceptions of their own beliefs and knowledge and how they may have changed over the duration of the course. Individual interviews were also necessary to explore these perceptions, using reflective prompts. In the next chapter, the complete methodology and methods for data collection and analysis for this study is explored in detail.

CHAPTER 3

METHOD

Methodology

In this chapter, I outline the methodology used to address purpose and answer the research questions. This includes the rationale for qualitative inquiry, the contextual framework of the study, as well as a description of the methods used to select participants and collect and analyze data. The chapter ends with a description of the process of obtaining ethics clearance and methods used for trustworthiness.

Research Design

This study was an empirical examination of a mathematics course experience that focused concomitantly on (a) the development of mathematics content knowledge, (b) reflection on personally-held beliefs about mathematics, and (c) practice using reform-based instructional strategies to facilitate student learning. The purpose of this study was to provide a comprehensive profile of the cohort of candidates in the 12-week MATH 010 course and describe the transformations that the prospective teachers reported with respect to their mathematical beliefs and knowledge.

Qualitative inquiry was appropriate for this study because the research was primarily concerned with understanding a social phenomenon from the participants' perspectives (McMillan & Schumacher, 2006). Given the internal nature of beliefs, a description of the unique characteristics of beliefs about the nature of mathematics and its teaching was best obtained through qualitative approaches. Qualitative analysis was both necessary and appropriate because one of its strengths is to look at a program holistically (Patton, 2002).

This research was conducted as a case-study, an interactive method that focuses on individual experiences and examines a *bounded system*, or a case, over time in detail, employing multiple sources of data found in the setting (McMillan & Schumacher, 2006). The bounded system that was studied in this research was the MATH 010 course, the class participants, and the instructor.

Method

The research design included: (a) one questionnaire at the beginning of the course, (b) one questionnaire at the end of the course, (c) individual interviews with four information-rich informants following the final questionnaire, and, (d) an interview with the course instructor. The combined methods of data collection allowed for the examination of the semester long experience of MATH 010.

Contextual Framework

Course description. This inquiry took place within the boundaries of a university mathematics course. MATH 010, entitled *Fundamental Concepts in Mathematics for Elementary Teachers*, is currently a half-course targeted at, but not limited to, prospective elementary teachers. It is offered through the Department of Mathematics and Statistics in the Faculty of Arts and Science at Queen's University, Kingston, Ontario in the fall semester (September to December). The course syllabus outlines the intended purpose of MATH 010:

The course considers elementary school mathematics from an enriched point of view.

> Theoretical and pedagogical questions are raised throughout the course, and students are required to teach a one-hour enrichment class, once a week for 10 weeks, to Grade 7 or 8 students in a local elementary school. (Jonker, 2011)

The goal of the course, a non-traditional mathematics course, focuses on opportunities for undergraduate students interested in education to learn about and reflect upon the impact of teacher-held beliefs and knowledge on teaching and learning in the mathematics classroom. MATH 010 is aimed at providing first-hand experiences for prospective teachers in which mathematics content, as well as the teaching and learning activities designed to address the specific concepts under study, are fun, imaginative, and open-ended.

The instructor, Dr. Leo Jonker, anticipates that in response to the mathematics problems that he routinely poses, students enrolled in the course will be challenged to "think outside-of-the-box" and seek multiple solution paths. The course instructor describes that it is "significant that, since the university students have to present enrichment classes based on the material they are learning in class, and since many of the students they will face are very quick, it is not possible to get away with [a] superficial understanding" (Jonker, 2008, p. 332). It is also intended that prospective teachers will have opportunities to construct connections among mathematical concepts in a reflective and empowering way that may positively impact their ability to make better informed judgments in subsequent elementary mathematics teaching situations.

One important focus of the course is the provision of instructional opportunities with an enriched mathematics curriculum for intermediate division (Grades 7−9) elementary school students. Course candidates are provided with opportunities to experience and reflect on content and pedagogy that may differ significantly from the traditional approaches they themselves likely experienced as elementary students. MATH 010 candidates complete 36 hours of classroom instruction per semester over 12 weeks. In addition, they complete a practicum experience in a local elementary school designed so that they can gain experience teaching the topics discussed during in-class sessions. The practicum involves one hour per week throughout the semester in local Grade 7 and 8 classrooms, beginning in the first or second week of the semester. During the practicum, course candidates team-teach with one or two of their MATH 010 peers.

The MATH 010 curriculum. The enrichment program that is the basis of both the university seminar classes and the enrichment activities for Grade 7 and 8 students is titled *StepAhead*. *StepAhead* was developed and has been continuously revised by the MATH 010 instructor, Dr. Leo Jonker, over the 10 years that he has taught the course. Dr. Jonker defines *StepAhead* as a collection of coherent mathematics and science enrichment problems for Grade 7 and 8 students (Jonker, 2008). When designing and developing the curriculum for *StepAhead*, Dr. Jonker's goal was to develop a syllabus that would be used by university students as a component of MATH 010, but also could be left in an elementary classroom as a permanent resource to be used for instructional purposes by the host teacher. *StepAhead* problems are considered rich learning tasks because they can be solved in a multiplicity of increasingly sophisticated ways, and require a conceptual understanding to generate a solution.

The *StepAhead* curriculum is organized as a two-volume manual that can span Grades 7 and 8. Volume 1 is a collection of number activities and problems; Volume 2 features geometry concepts. Volumes 1 and 2 are rotated on a two-year cycle: Geometry is taught in even years and Numbers in odd years. The participants in this study used Volume 2 (Geometry) as the course fell on an even year (2010). An example of one *StepAhead* enrichment lesson is provided in Appendix A. This lesson explains how to guide learners to explore the thickness of toilet paper. The non-standard problem requires a student to apply what she has learned about the calculation of area and the circumference of a circle. Similarly, another *StepAhead* problem challenges learners to find the maximum distance that a fly can be from a spider in a room that has a floor that measures 2 metres by 2 metres and has a wall height of 2 metres. The problem tests students' understanding of the theorem of Pythagoras and introduces nets, which are a way of "unfolding" a three-dimensional object.

The *StepAhead* curriculum is based on a problem solving approach and emphasizes reasoning and application of concepts. At its core, the *StepAhead* program is designed to facilitate conceptual development and support the transferability of mathematics skills and processes among strands, thereby reflecting the fundamental philosophy of the Ontario Curriculum for Grades 7 and 8 (Ontario Ministry of Education, 2005).

MATH 010 structure. Mathematical content and pedagogical questions are explored during MATH 010 in-class sessions. During the year of the study, candidates discussed the content of Volume 2 of the *StepAhead* resource (topics in geometry), as well as pedagogical considerations related to each lesson, in one-hour sessions, three times weekly, with fellow prospective teachers and a mathematics professor.

MATH 010 candidates are required to team-teach a one-hour weekly enrichment class, for 10 weeks, to Grade 7 and 8 students in a local elementary school. The practicum experiences took place in various local Kingston schools, organized by Dr. Jonker. Teaching teams were assembled based on candidates' (a) university timetable and (b) access to transportation. Each enrichment lesson is taught by a team of two or three course candidates to a small group, of three to eight, Grade 7 and 8 students. Each week, MATH 010 candidates lead the students through an enrichment lesson that they developed collaboratively with their peers and with input from the course instructor. The topic of the lesson changes every week and includes concepts such as Pythagorean theorem and volume, which are loosely correlated to the Ontario mathematics curriculum (Appendix A).

Each week, MATH 010 candidates are required to review the *StepAhead* content in problem sets, or homework questions, related to the topic of that week and maintain a self-reflection journal about their teaching experiences. The self-reflection journals are intended to summarize course candidates' immediate experiences in conducting the enrichment class, as well as any reflections that arose from the experience. Examples of questions to be addressed in the self-reflection journals include answering what would be done differently if the session could be repeated and what techniques were used to engage students (Appendix B). Reflection journals were completed by the members of the teaching teams. Given that the researcher did not have permission to conduct research about all group members, the journals were not collected as data for this study,

Participants

The participants were selected based on convenience sampling. McMillan and Schumacher (2006) define convenience samples as "groups of subjects that are selected on the basis of being accessible or expedient" (p. 137). The convenience sample in this study consisted of 10 prospective teachers enrolled in MATH 010 between September and December 2010. The sample was convenient because both the researcher and the participants lived in Kingston, the location of the course. The prospective teachers provided the data for the examination of a course that embeds instruction about reflection, pedagogy, and mathematics content using the context of problem solving in both seminar classes and a semester-long Grade 7 and 8 classroom teaching placement. Permission to conduct research in the course was obtained from the course instructor following an email request. All university students enrolled in MATH 010 (approximately 30 course members) were then contacted by email prior to the start of the course (August 2010), to explain the research objectives and to invite participation in the study. All willing MATH 010 students who expressed interest in participation and completed the appropriate *Consent Forms* were accepted. The participants in the study were 10 course candidates enrolled in MATH 010 who consented to being a part of the study. Those MATH 010 candidates who declined to enroll in the study were not included in the data collection procedures. See Appendices E–G for *Recruitment Email*, *Letter of Information*, and *Consent Form*.

Data Collection Methods

Data was collected using four methods: (a) an initial pre-course questionnaire, (b) a final post-course questionnaire, (c) individual interviews with four course candidates, and (d) an individual interview with the course instructor.

Questionnaires. Two different questionnaires were administered online: (a) the first questionnaire, which contained 32 questions, was administered in the first week of the course (*Questionnaire 1*), and (b) the second questionnaire, which contained nine questions, was administered in the last week of the course (*Questionnaire 2*).

Questionnaire 1 was used to gain information about existing beliefs and knowledge in the categories outlined in Table 1, in order to create a class profile about the course candidates' background, beliefs, and knowledge at the start of the program. Questionnaire 1 (Appendix H) consisted of two parts and was administered online, using surveymonkey.com, to MATH 010 study participants during the first week of the semester (September 2010). The questionnaire provided a method that was economical, asked the same questions of all participants, and ensured anonymity amongst peers (McMillan & Schumacher, 2006). The first part of Questionnaire 1 (questions 1−11) sought biographical and contact information, as well as mathematical background (e.g., courses taken, current degree program). Questions also asked about students' participation in mathematics clubs or contests, tutoring experience, as well as intended career paths. The second part of Questionnaire 1 (questions 12−32) was divided into categories of beliefs and knowledge outlined by Tatto et al. (2008), and contained between five to 10 questions in each category. Questions sought participants' (a) beliefs about the nature of mathematics and the nature of teaching mathematics and (b) knowledge of content and pedagogy. The questionnaire took approximately one hour to complete and was first trialed on two Master of Education students at Queen's University who have a mathematics background. The questionnaire was trialed to approximate timing and was then followed by a read-through with the researcher and one of the Master of Education student volunteers to ensure clarity of questions.

Questionnaire 1 included material from four sources. First, the released items from the *Teacher Education and Development Study in Mathematics* (TEDS-M) questionnaire, which draws on the categories of beliefs and knowledge, were the source of most questions. TEDS-M was used as a framework for this study because it focuses on how teachers are prepared to teach mathematics in elementary and early secondary school and thus is in line with the research questions driving this study. Prior to its use in the TEDS-M study (Tatto et al., 2008), the questionnaire was trialed on a small scale with promising results. It was then revised and each section was expert reviewed (Tatto et al., 2008). Many of the questions reflected classroom teaching scenarios (pedagogical content knowledge) integrated with particular subject matter topics (content knowledge) and used open- and closed-forms of questions. Many closed-form questions used a 5-point Likert scale, which asked respondents to answer based on an agree–disagree scale. Secondly, questions were adapted from Voytsekhevoska and Li's (2010) instrument, which was designed to survey teachers' conceptions of mathematics and mathematics teaching and learning in the field of Grade 7 and 8 mathematics and reading. Voytsekhevoska and Li's survey was adapted from a variety of expert-reviewed instruments (Bandura, 1986; Beswick, 2006, 2007; Ernest, 1989; Lester, 2000; Pajares, 1992; Quinn & Wilson, 1997; Thompson, 1984; Tschannen-Moran & Woolfolk, 2001; and Tschannen-Moran, Woolfolk, & Hoy, 1998) and was used as a source of closed-form, Likert-scale items to obtain data regarding prospective teachers' beliefs about the nature of mathematics. Thirdly, mathematics content knowledge questions were adapted from the School Achievement Indicators Program (SAIP) *Report on Mathematics III Assessment* (2001). The SAIP is a Canadian assessment of mathematics achievement examining mathematics content and problem solving for a range of topics. The mathematics achievement of 41,000 students, aged 13 to 16 years, from 10 provinces was measured. Content questions from the SAIP report were developed based on the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* (1989). Finally, Questionnaire 2 was supplemented with released items from the instrument developed by Hill, Schilling, and Ball (2004) (specifically in the category of pedagogical content knowledge). Hill, Schilling, and Ball designed an instrument to measure teachers' content knowledge for teaching elementary mathematics. This quantitative questionnaire was large-scale field trialed with 1552 elementary school teachers enrolled in California's Mathematics Professional Development Institute (Hill, Schilling, & Ball, 2004).

Questionnaire 2 sought to investigate any changes in prospective teachers' perceptions of their beliefs about the nature of mathematics, teaching and learning, and, knowledge about content and pedagogy. For example, one question asked what factors had contributed to any changes in participants' beliefs. Sample questions can be found in

Appendix I. Questionnaire 2 was completed anonymously, online, again using surveymonkey.com by all study participants, during the final week of the course (December 2010). The questions were composed by the researcher using the initial class profile that described participants' beliefs and knowledge. The questionnaire also sought to identify course factors that were thought to be influential in changing knowledge, from the perspectives of the candidates themselves. Questionnaire 2 was also read by the same two volunteers as Questionnaire 1 to ensure clarity of questions. The final version of the questionnaire contained 10 open-ended questions and took approximately 45 minutes to complete.

Interviews. Following the completion of Questionnaire 2, one-on-one interviews were conducted to explore and expand on students' responses. Given the deeply personal nature of beliefs and knowledge, individual interviews were a suitable means of data collection to protect individual perceptions that might be influenced in a group setting, such as a focus group (McMillan & Schumacher, 2006). A semi-structured interview was conducted because this method allows for open-ended questions with a fairly specific intent (McMillan & Schumacher).

Interviews were conducted with four "information-rich" individuals (Patton, 2002, p. 46) in order to provide valuable data for the case study. Participants were selected on the basis of their ability to (a) articulate their experiences in the course, and (b) express connections between the course and intermediate division mathematics teaching in response to items on Questionnaire 2. Selected participants were reminded of the purpose of the study and the interview. Their right to decline to answer any objectionable questions and withdraw from the study at any time was reiterated. Semi-structured interviews were conducted on Monday December 6, 2010, in the foyer of Jeffrey Hall at Queen's University. Interviews lasted approximately 45 minutes and each interview was audio recorded using a Sony™ Digital Voice Recorder to capture, in full, participant responses and ensure transcription accuracy. Questions used for the interviews are provided in Appendix J.

An individual interview was also conducted with the course instructor in his office on Monday December 6, 2010. The purpose of this interview was to examine the instructor's intended goals for the course and personal beliefs about the nature of mathematics. The interview was audio recorded and lasted 45 minutes in total. Questions from the interview with the course instructor are provided in Appendix K and the email permission from the course instructor is provided in Appendix L.

Organization of Data

Transcription. After listening carefully to each recording from start to finish on two separate occasions, the recorded interview data was transcribed verbatim by the researcher. Each transcription was documented in a Microsoft Word™ file with continuous pagination. Interview transcripts totaled 86 pages: 22 pages for the interview with the course instructor and 64 pages, in total, from four student participants. Each file was titled using a pseudonym assigned to the participant and saved on the researcher's password protected computer. The course instructor was not given a pseudonym because the researcher was granted permission, by email, from the course instructor to use his real name. After transcribing each interview, the researcher then re-listened to each audio-recording to correct errors within the transcript and to enhance trustworthiness by maximizing accuracy. Next, the electronic version of the transcript was given to the individual participant for review, in order to make additions and corrections where appropriate (McMillan & Schumacher, 2006). Speech disfluencies, such as “umm” and “uh” have been omitted from the quotations presented in this thesis to provide a more readable text.

Coding. Responses to items in Questionnaire 2 and in the interview data were examined using standard qualitative analysis with coding techniques (McMillan & Schumacher, 2006). In coding the verbatim transcripts, etic and emic terms were used to develop a coding scheme. McMillan and Schumacher describe *in vivo* coding as labels that use participants’ wording. First, etic codes about areas of participants’ beliefs and knowledge were derived from existing literature. Emic codes emerged from the reading of transcripts and corresponded to themes introduced by participants.

Analysis of Data

All qualitative data were analyzed inductively to determine emergent themes as well as deductively to compare major themes and key ideas in relation to the conceptual framework.

Participants’ responses to items on Questionnaire 1 were analyzed using descriptive statistics in order to ascertain patterns of responses and to facilitate the development of a composite profile of the prospective teachers’ beliefs and knowledge at the start of the course. Several tools were used to guide the interpretation of the questionnaire results: (a) The TEDS-M analysis and scale development frameworks; (b) the Hill, Schilling, and

Ball (2004) item analysis and protocol; and (c) the Student Achievement Indicators Program (SAIP) (2001) item analysis protocol.

The conceptual framework was consulted to describe the ways in which the participants experienced changes within categories of beliefs and knowledge based on the core meanings that reflected these categories. Questionnaire 2 and interview data were coded using guidelines suggested by Patton (2002) to determine relevant passages or terminology that appeared within the data. After developing the coding scheme, each transcript was re-read. Table 2 lists both etic and emic codes with their corresponding mnemonic code as well as a definition. Segments that corresponded to a specific etic code were highlighted and the code was written in the margin of the passage using a chosen colour. Similarly, segments that corresponded to a specific emic code were also highlighted and the emic code was written in the margin of the passage with a specific colour.

Patterns were established by comparing individual responses and codes to determine the ways in which the course contributed to changes in beliefs and knowledge. The coding process was repeated several times and was verified, using inter-rater agreement (McMillan & Schumacher, 2006). Prior to coding, a training session occurred with a fellow Master of Education student. The student was provided with an outline of the study and its objectives, was informed of the meaning of all codes in relation to the literature, and trained how to code the data. Following the coding of two pages of transcript, a comparison was made between the researcher and her colleague's ways of

Table 2

Codes and Definitions Used in the Coding Process

Mnemonic	Code	Definition
	Etic Codes	
Al	Algorithmic	Using predetermined formulae to solve problems
Col	Collaboration	Learning by collaborating with others
Conn	Connected Topics	Making connections among topics within mathematics
CA	Contextual Applications	Linking concepts to other applications
CT	Critical Thinking	Learning by asking “how” and “why” concepts and solutions come about
ET	Engaging Tasks	Using activities to engage learners
JE	Joint Enterprise	Having a common goal of understanding problem solving
L	Lecture	Presenting information orally
MAF	Mechanistic Answer-Finding	Using repetitious methods to solve problems
MP	Multiple Perspectives	Having many ways to explain and solve a concept or problem
MR	Multiple Representations	Presenting information multiple ways (visually, physically)
Reif	Reification	Understanding abstract concepts and applying them
Ref	Reflection	Being reflective to improve practice
RR	Rich Resources	Using rich tasks for critical thinking
	Emic Codes	
An	Anxious	Feeling anxious about performing mathematics tasks
Conf	Confidence	Gaining confidence in teaching abilities
D	Disengaged	Being disconnected from the subject
Ind	Individual	Learning individually
Int	Intimidation	Feeling intimidated by a lack of teaching abilities
M	Memorization	Learning through memorizing rules and procedures
NT	Note-Taking	Supplementing oral information with written notes
OSP	One Solution Path	Considering only one way to solve a problem
P	Participation	Taking part in the learning process

coding. A consensus was reached as to the coding of the data on the two pages, making the inter-rater agreement 95% for the sample used.

Ethics Clearance

Prior to beginning the study, ethics clearance was sought and obtained from the General Research Ethics Board (GREB) at Queen's University. Permission to conduct research in the MATH 010 course was then sought from the course instructor (Appendices C and D). Following this permission, all prospective teachers in the course were given a Letter of Information (Appendix F) and two copies of a Consent Form (Appendix G). The Letter of Information included information about the study as well as the intended use of the data. The Consent Form corroborated willingness to participate and outlined that information would be kept confidential to the extent possible, and that participants' identity would be protected by the use of pseudonyms. As well, the form outlined that participants were not expected to answer questions that made them uncomfortable; they could withdraw from the study at any time without consequence or request the removal of part or all of their data.

Trustworthiness

In an attempt to enhance trustworthiness, this study used multiple methods of data collection. Patton (2002) stated that, "studies that use only one method are more vulnerable to errors linked to that particular method" (p. 248). Patton also stated that triangulation strengthens a study by combining methods while testing for consistency of different data sources. This study employed methodological triangulation which is "the use of multiple methods to study a single problem or program" (Patton, 2002, p. 247).

The data collection methods employed in this study were two questionnaires and individual interviews with four participants and the course instructor. The variety of sources of data allowed for different types of insights into the aspects of the course that contributed to changes in prospective teacher beliefs and knowledge. As well, audio recording with verbatim transcription of interviews in combination with member checking and continuous review from the thesis committee reduced bias in representing coding categories.

Summary

This chapter outlined the methodology used for this study, including the rationale for choosing a qualitative study. The context of the study, MATH 010, was described to present a thorough description of the structure of the course. The specific data collection methods employed were then detailed: data was collected using two online questionnaires (before and after the course), individual interviews with four participants, and an interview with the course instructor. The semi-structured interviews were audio-recorded and later transcribed verbatim. The method of data analysis for each type of data collected was also described: data was transcribed and coded using etic and emic terms to develop patterns and themes within the data. In the next chapter, the data is presented.

CHAPTER 4

PRESENTATION OF THE DATA

This study examined the ways in which prospective teachers perceived a change to their personal and professional beliefs about mathematics and mathematics teaching, as well as to their content knowledge after experiencing a university course that purported to focus on both. The course, MATH 010 (*Fundamental Concepts in Mathematics for Elementary Teachers*), was designed to allow candidates to experience (as a learner and as a teacher) an enriched program of intermediate division mathematics (i.e., Grades 7−9) called *StepAhead*; therefore, in addition to class sessions, students enrolled in MATH 010 also completed a practicum experience, for one hour per week for the duration of the course, in a local Grade 7 and 8 classroom in order to acquire experience teaching the enrichment topics discussed during in-class sessions.

The study was guided by three research questions: (a) What are prospective teachers' beliefs about the nature of mathematics and teaching before the course experience? (b) How do prospective teachers perceive that their beliefs about the nature of mathematics and teaching have changed in response to the course experience? and (c) To what do prospective teachers attribute any changes to their beliefs about mathematics and its instruction?

The data for this study, which were collected using standard qualitative methods described previously, are presented in this chapter. The data is presented chronologically so as to reflect participants' beliefs about the nature of mathematics and its teaching before and after the MATH 010 course experience. In the first section of the chapter, a general participant profile is presented, which details the course candidates' beliefs about the nature of mathematics and its teaching. Data in this section are based on responses to Questionnaire 1, completed during the first week of the course. In the second section, the MATH 010 course is described in detail through two lenses: the perspective of the instructor and the perspective of student participants. Perceptions highlight those specific aspects of the course that were identified by participants as new experiences related to mathematics or to pedagogy. In the third section of this chapter, participant perceptions of how their beliefs about the nature of mathematics and its teaching changed in response to the course experience are presented. Data are drawn from a post-course questionnaire and individual interviews, both of which took place during the last week of MATH 010. Participant perceptions of changes to their mathematical content knowledge are also presented. In the final section, the analysis of the data is presented.

Before the Course Experience

Data from Questionnaire 1, administered at the beginning of the course, were examined in order to answer the first research question, which sought prospective teachers' beliefs about the nature of mathematics and teaching before the course experience. Questionnaire 1 asked 32 questions and was separated into two parts: (a) Part 1 asked 11 questions about the 10 participants' mathematical background and (b) Part 2 asked 21 questions regarding participants' beliefs and knowledge, as outlined by Tatto et al. (2008) (Appendix H). Part 1 is presented in the next section, followed by categories of beliefs developed from Part 2.

Mathematical Background

Part 1 of Questionnaire 1 asked participants to describe their background in mathematics prior to the start of MATH 010 in September 2010. Table 3 shows a summary of participants' responses. All 10 participants had completed the required Grades 9–11 mathematics courses in high school. Seven students had completed various courses in the Grade 12 Ontario mathematics curriculum, including *Advanced Functions, Calculus and Vectors,* and *The Mathematics of Data Management*. Five of the 10 participants had previously competed in mathematics competitions, including University of Waterloo Mathematics Competitions. Specifically, one student was the winner of the Gauss Mathematics Contest for Grade 8 and two other students completed the Pascal Mathematics Contest for Grade 9 students.

Of the 10 participants, eight had no teaching experience other than one-on-one tutoring. Two participants had completed a teaching placement as part of their Concurrent Education university degree requirements. All participants stated that their reason for taking MATH 010 was to gain the "experience of teaching." Seven candidates expressed a desire for classroom experience to help them learn how to become a teacher, while two said they wanted the teaching experience to see if they would like teaching mathematics. All 10 participants stated that their goal, after graduation, was to seek employment as a teacher. Of the 10 participants, six were enrolled in the Concurrent Education program, which ensures them a place in the Bachelor of Education program contingent upon the successful completion of their undergraduate degree. For eight participants, the undergraduate degree being sought was a Bachelor of Arts. Two

Table 3

Summary of Participants' Mathematical Background

	Participant Number and Response									
	1	2	3	4	5	6	7	8	9	10
Degree Program	BA	BA	BSc, BEd	BEd	BEd	BA	BA, BEd	BA, BEd	BA	BSc, BEd
Year of Program	1	3	2	2	1	4	1	1	2	2
Previous Degrees	None	None	None	None	None	None	None	None	None	None
High School Courses	Grade 9-11	Grade 9-11	Grade 9-12	Grade 9-12	Grade 9-12	Grade 9-11	Grade 9-12	Grade 9-12	Grade 9-12	Grade 9-12
Math Club	No	No	No	No	No	No	No	No	No	No
Math Contests	No	Gauss	No	UofW Pascal	Yes	UofW Pascal	No	UofW	No	No
Teaching Experience	No	No	Prac	Tutor	Tutor	No	Prac	No	Tutor	Tutor
Why MATH010	Teach	Teach	Teach	Teach	Teach	Teach	Teach	Teach	Teach	Teach
Job	Teach	Teach	Teach	Teach	Teach	Teach	Teach	Teach	Teach	Teach
Math for fun	No	Sudoku	Sudoku	No	No	Sudoku	No	Sudoku	No	Sudoku
Math Conference	No	No	No	No	No	No	No	No	No	No

participants were studying mathematics as a major and one student was studying mathematics as a minor. Based on their responses to the first part of Questionnaire 1, the 10 participants enrolled in this study appeared to have a fairly homogeneous mathematics background based on their secondary school courses and teaching experiences. Data from categories of beliefs and knowledge from Part 2 of the questionnaire are presented next. General patterns of beliefs about the nature of mathematics and its teaching at the start of MATH 010 are described in order to have a baseline against which to describe perceived change.

Beliefs about the Nature of Mathematics

Beliefs about the nature of mathematics relate to participants' perceptions about what it means to do mathematics. In this section, data from Questionnaire 1 is presented in order to provide information about students' perceptions of their beliefs about the nature of mathematics in the first week of MATH 010.

In response to items 12A and J, respectively, on Questionnaire 1 (Appendix H), all 10 participants agreed that "mathematics is a set of rules and procedures that prescribe how to solve a problem" and that "mathematics involves being taught, remembering, and applying." Eight participants also agreed that when solving mathematical tasks one needs to know the correct procedure (12B). One participant described that mathematics is about "plugging numbers into formulas" while another commented that there is "one right way to solve a problem" (responses to item 28).

Participants described beliefs that appear to contradict those previously reported. For example, all 10 participants also agreed that mathematics problems can be "solved in many ways" (item 12G). All course candidates agreed that mathematics helps to solve everyday problems and tasks. Eight of the participants agreed with the belief that mathematics involves "creativity and new ideas" (item 12C) and that one can "discover and try out many things by oneself" (item 12D) in mathematics.

Beliefs about the Nature of Mathematics Teaching

Beliefs about the nature of mathematics teaching relate to participants' perceptions of how mathematics is best taught. In this section, course candidates' responses to questionnaire items are detailed to convey their ideas about best strategies for (a) teaching and (b) learning, at the beginning of the course.

Strategies for teaching. In response to items 13A−I on Questionnaire 1, all 10 candidates agreed that "showing connections between mathematics and other areas of knowledge" is important when teaching mathematics (item 13F). Ten participants agreed that teachers should spend time in mathematics classes improving students' problem solving skills. All participants agreed with the statement "teaching students to defend their solution process in their own words is an important part of teaching mathematics" (item 13E). One participant agreed with the statement that mathematics is "only about explaining rules, principles, and procedures," while another participant agreed with the statement that "focusing on the problem solving process tends to reduce mastery of computations," while teaching mathematics (items 13D and 13I). Four course candidates also commented, in response to item 29, that their previous mathematics teachers' typical teaching strategy involved the demonstration of procedures to solve particular problems, "on the board" or through "lecture." One participant noted that "the teachers would always write out the steps of how to solve the problem." Students were expected to "take notes," in order to record such procedures.

Strategies for learning. Several candidates described their experiences as secondary school students: being expected to copy and repeat the teachers' procedures were how they learned how to solve mathematics problems. One participant best articulated this view in response to item 29: “In high school you memorize what the teacher did, essentially, and the equations that they used and the steps they followed and just did that in the homework and on the math test.” Memorizing the teachers’ problem-solving method was a common learning strategy described by four course candidates.

Beliefs about Competency as a Developing Teacher

At the beginning of the course, participants undertook a self-assessment of their competence in their abilities to teach mathematics. Data were drawn from items 15A-J and 16A-H on Questionnaire 1 in reporting participants’ perceptions of their competence as a prospective teacher.

When asked to answer questions about their preparedness to teach mathematics, many participants responded that they did not feel that their education thus far had prepared them for the demands of the mathematics classroom. For example, five of the participants did not agree that their education had prepared them to “ask questions during mathematics instruction in order to promote higher-order thinking” (item 15C). Five of the participants also stated that they did not feel competent to “set up mathematics learning activities to help students achieve learning goals” (item 15B), nor did they feel sure about their ability to communicate ideas and information about mathematics clearly to students. Two participants’ self-assessments concluded that they had “all of the types of knowledge needed to be an excellent mathematics teacher” (item 16A). Two participants stated that they could handle any situation, whether content or management related, that arose in the mathematics classroom. All 10 participants did, however, express confidence in their abilities to gain respect from students, parents, and teachers, while also expressing confidence in their employability.

Beliefs about Competency as a Student Learner

This section addresses participants' self-assessment of their competence in problem-solving within various strands of mathematics content. Data were drawn from items 28–32 on Questionnaire 1.

Initially, participants commented that mathematics was a subject that was intimidating. Four course candidates described feeling "intimidated" when trying to solve problems which required going beyond the mere substitution of numbers into a formula. One participant described such an experience: "I really hated solving challenging math problems. I never liked them in high school." Another candidate recalled: "I wouldn't have [approached a problem] thinking I could solve it" and "dreaded" such problems. Another participant described that mathematics lacked relevance in secondary school: "I remember always asking why I need to know this."

Knowledge of Mathematical and Pedagogical Content

Items 17–27 on Questionnaire 1 asked participants to answer content and pedagogical content knowledge questions across a variety of topics, including probability, geometry, and number theory. Content questions (items 23–27) consisted of Grade 7 and 8 level problems related to the Ontario mathematics curriculum and were designed to elicit high level responses. For example, one question required participants to choose the most correct solution method (by multiple choice) for determining whether or not "371" is a prime number. Pedagogical content knowledge questions (items 17–22) asked participants about assessing and explaining Grade 7 and 8 level problems or student solutions, from the perspective of a teacher (Appendix H). For example, one question asked that participants choose, from a list of multiple choice statements, the best way to explain how to divide fractions.

A summary of correct and incorrect responses for specific topics can be found in Table 4. On average, each pedagogical and mathematics content question was answered correctly by seven out of 10 participants. This indicated that the average course candidate in MATH 010 could correctly answer the administered questions related to mathematics content and pedagogical content.

Table 4

Summary of Participant Responses to Pedagogical and Mathematical Content Questions

	Number of Correct Responses	Number of Incorrect Responses	Number of Unanswered Responses
Math Pedagogy Questions by Topic			
Q1. Probability	8	2	0
Q2. Decimals	6	4	0
Q3. Fractions	6	4	0
Q4. Multiplication	6	4	0
Q5. Reasoning	7	3	0
Q6. Angles	6	4	0
Math Content Questions by Topic			
Q1. Perimeter	7	2	1
Q2. Substitution	8	1	1
Q3. Exponents	6	4	0
Q4. Number Sequence	8	2	0
Q5. Prime Numbers	8	2	0
Average	7	3	0

Summary

Examining the 10 participants' mathematical background allows a descriptive class profile to be generated about MATH 010 students at the beginning of the course. In general, participants' mathematical backgrounds included high school mathematics courses, followed by Undergraduate courses in a subject area other than mathematics. MATH 010 participants had little experience teaching but intended to complete a Bachelor of Education and, following baccalaureate graduation, become classroom teachers. It appears that participants simultaneously viewed mathematics as a set of rules, procedures, and formulae that need to be followed strictly and that problem-solving should be an important focus in teaching mathematics. Although participants did not express competence in their abilities to perform mathematics tasks, the majority of participants could correctly answer representative questions related to the intermediate mathematics curriculum. They could also correctly answer the majority of questions related to classroom scenarios. The next section describes the course experience from the perspective of the instructor and the student participants.

The Course Experience

In this section, the course experience is described in order to draw attention to specific aspects of the course related to mathematics or pedagogy that were identified as new experiences for participants. Characteristics of the course experience included (a) a focus on problem solving, (b) modelling effective practice, (c) the practicum experience, (d) a comfortable classroom setting, and (e) the development of reflective practice. Each section is described from the perspective of (a) the instructor, and (b) the class participants. Data were drawn from the verbatim transcripts of interviews with the instructor and course participants. Data from the interview with the course instructor will be referred to as C1 (course instructor, interview one). When reference is made to the course instructor's interview, the citation includes C1 and the page number of the transcript. For example "C1, p. 5" indicates an interview with the course instructor with the quotation being found on page 5 of the transcript. Similarly, data from verbatim transcripts with the class participants are referred to as M1 (male student one) and F1-F3 (female students one, two, and three), used in citations with the page number of the corresponding transcript.

A Focus on Problem Solving

It was anticipated by the course instructor, Dr. Jonker, that students would develop their own solution paths to answer problems posed during MATH 010, given the challenging nature of the *StepAhead* enrichment problems (Jonker, 2008). The *StepAhead* curriculum is based on a problem solving approach, with value placed on reasoning and on the application of concepts. In this section, the course instructor's beliefs are described to outline the intended purpose of solving such problems, followed by participants' descriptions of their experiences with problem solving.

The course instructor's intended purpose. The course instructor central to this study, Dr. Jonker, held very specific ideas about mathematics, describing it quite pragmatically: "It's either about numbers or it's about shapes" (C1, p. 12). Dr. Jonker discussed these constructs as the "two different things that connect in mathematics" (C1, p. 12). More specifically, he explained that mathematics is either discrete or continuous, with connections between visual and spatial mathematics to symbolic or numeric mathematics. Dr. Jonker also described the relationship between experiencing the connections among the many facets of mathematics and understanding a broader vision of what mathematics is as a subject: “So, for me a proof is basically a kind of laying out [of] a set of connections, [a] kind of a vision into the thing of why it works the way it does” (C1, p. 16).

Dr. Jonker identified many students in his course as getting “reasonable” marks, but expressed concern that they are not seeing the bigger picture of mathematics as a subject: “They have a fairly, sort of, narrow and focused view of what mathematics is and they have not experienced much playfulness with it” (C1, p. 6). He described teaching through making connections within the content to better facilitate students’ understanding:

> The understanding should be a matter of many, many connections . . . for example, when you divide by a fraction you should multiply by its reciprocal. Your understanding of that would be based on a number of different ways of explaining it, not just one . . . web of connections between ideas. (C1, p. 15)

Through various explanations, Dr. Jonker hopes that MATH 010 students will see the content from a number of perspectives, thereby expanding their views about the nature of mathematics.

The course instructor’s ideas about mathematics influenced the design of the *StepAhead* problems covered in class. For example, one problem required students to calculate the dimensions of a cube that held an inscribed sphere with a given radius. This kind of problem does not have a common algorithmic solution and requires an understanding of the relationship between the dimensions of a sphere and the dimensions of a cube. Given the nature of this type of question (which has a number of solution paths), course candidates were required to persevere to solve this novel problem. Dr. Jonker explained how he approached problems for homework:

> The weekly homework, I believe, is there to get the students to experience a sustained . . . effort at a problem . . . they haven't seen before, though it relates to what they've been doing. Not just a problem that applies something they've just learned but a problem that is different, that is accessible to them . . . sometimes it's a little too easy, sometimes it's a little too hard, but I try to find one that's . . . more or less just right . . . that extends their thinking. (C1, p. 8)

Dr. Jonker also believes that he supports students with weak mathematical content knowledge through his efforts to display various connections among topics and his focus on multiple methods for solving problems:

> When it comes to the theorem of Pythagoras we go over it so long and [in] so many different ways that definitely at the end they will know the theorem; not just know it in terms of formally but they *know it* [emphasis added]. It's become somewhat embodied. (C1, p. 10)

Participants' experiences with problem solving. Participants experienced a rich variety of mathematical topics and they developed and practiced their problem solving skills through weekly in-class sessions and homework problems. One participant discussed how she would create a physical model in order to better understand a problem:

> I learned a lot in this course. A lot of the times for the homework problems I would try and actually make it. If it's about, like, a net or something, I would cut out the net and put it together because that would really help. (F1, p. 9)

Another participant noted that, "the assignments in MATH 010 . . . made you apply everything you knew" (F3, p. 10), while another participant commented that the assignments and their discussion provided, "really interesting new ways to think about math" (F1, p. 14). One participant recognized that effort and perseverance for logical thinking influenced his problem-solving process:

> I certainly got stumped and I had to spend some time, like half an hour or an hour on a question, but I was able to think it logically through and arrive at the answer, or what I thought was the answer. (M1, p. 7)

By engaging with the mathematics problems, students learned "how to go about solving [problems] and after awhile . . . pick[ed] up on it" (F2, p. 2). Novel problems allowed students to experience a new way of thinking about mathematics, which consequently influenced their approach to problem solving.

Modelling Effective Practice

The course instructor modelled, and students experienced, effective teaching methods throughout MATH 010. In this section, the course instructor's descriptions of his teaching practices are reported. This is followed by the participants' descriptions of such methods.

The course instructor's description of his teaching practices. Dr. Jonker explained his view of the teacher's role in the classroom as follows:

> The teacher's role is to design a program that achieves the purposes that the teacher has for the students and to discern how much of that these students are ready to learn in the given time and to . . . invent situations that will enable that learning. So there's an awful lot of direction . . . from the teacher, but . . . it should be behind the scenes . . . as much as possible and the students . . . gain so much more if [they] have a sense [that they] discovered this by [themselves]. (C1, p. 13)

Dr. Jonker acknowledged that he probably teaches "in a way that's different than what they've seen in most cases" (C1, p. 13). For example, each class began with a "spot question" which required students to complete a problem related to content from the previous class. The problem is shorter in length than the problems that might be presented from *StepAhead*, but served as a brief warm-up for students: "I always pick a problem that connects to what we've been doing that I can gauge the response to" (C1, p. 8).

Following the spot question, students were directed to a problem from the *StepAhead* resource. MATH 010 students then worked at the problem, forming small groups if they wished to discuss possible strategies and ideas, which Dr. Jonker described loosely as, "here's this problem . . . make groups and work at it. Sometimes there's some hands-on stuff" (C1, p. 8). Students worked on the problem for approximately twenty minutes, while the instructor circulated to answer any questions: "I . . . walk around, see how people are doing, stop and talk a little bit" (C1, p. 9). Following this, the instructor asked the class as a whole if anyone would like to "report how they're doing" (C1, p. 8) on behalf of their group or individually, with the process of solving the problem. He then encouraged the presentation of several methods to solve the problem, being very careful not to suggest that one process was more correct than another:

> The one thing I definitely try to get away from is any indication in my behaviour or face that says 'this is the right way to do it' because I want them to see, and, of course, they see that already when they start talking to each another, that there's more than one way to approach it; and when they get different groups reporting, they see how people have gone off in different directions. (C1, p. 9)

By seeing various problem-solving methods valued by the instructor, students experience an opportunity to learn that mathematics is about original and creative solution paths. By orchestrating the class in this way, Dr. Jonker hopes that he is "able to show [students] some different habits that [they] might pick up" (C1, p. 14).

Participants' descriptions of effective teaching practices. The participants were positive about the instructor's methods:

> [Dr. Jonker] was very effective in showing that there is more than just one way to teach and that it is important to see how [the] many different styles of teaching our classmates have . . . that can be directly applied in the classroom. [He] showed the class how . . . rewarding it could be. (F2, p. 12)

Participants also enjoyed the structure of the class: "[Dr. Jonker] . . . introduced [to] us at the beginning of the lesson the question and . . . had us work on it and . . . we started to do that in our lessons" (F1, p. 11). The course instructor was also described as having expected "a minimum of, I would say, five answers, on every question" (M1, p. 15).

Participants recalled during individual interviews that there was a multi-solution requirement:

> He'd have different students come up and show us their different ways to solve [problems] before he showed us maybe another way that he had thought of and that . . . really showed me [a] good and interesting way to learn, instead of just having lessons. (F1, p. 6)

Other participants noted how the instructor explained challenging problems that he had presented to his students by asking for many solutions from different groups of students: "Taking [a problem] up was good . . . because he'd have a bunch of people go up and show all the different ways you could solve it" (F2, p. 12).

The instructional methods shown by the course instructor, which included modelling discovery learning, were felt by participants to be novel and effective. His teaching methods were also thought to be effective by members of the teaching and learning community other than the course participants, which is why Dr. Jonker has received a number of Teaching Awards at Queen's University, including the 1997 W.J. Barnes Teaching Excellence Award, the 1999 Alumni Teaching Award, and the Engineering Society's Golden Apple in 2000. In 2004, he was a recipient of the first Ontario Excellence in Teaching Award distributed by the Canadian Mathematical Society (Canadian Mathematical Society, 2011).

The Practicum Experience

MATH 010 students were required to teach, for one hour, once a week for the duration of the course, the *StepAhead* material covered during in-class sessions to Grade

7 and 8 students at a local elementary school. In this section, the practicum is explained by the course instructor, along with participants' descriptions of their experiences in teaching.

A unique aspect of the practicum experience was that it focused solely on teaching the pre-prepared material from *StepAhead* to a group of high-ability students, without the challenge of a wide range of abilities. Each enrichment lesson was team-taught to a small group of between three to eight Grade 7 and 8 students who had been identified by their teacher as being in the upper academic range in their mathematics achievement. Dr. Jonker explained, "What we're doing here is focusing on one thing . . . how do you understand this mathematics, how can you make it interesting and how can you teach it . . . or get kids to learn it" (C1, p. 8). The micro-teaching experience was said by Dr. Jonker to be less stressful for MATH 010 students than teaching curriculum content to an entire class: "There's freedom and so they can . . . discover their own mistakes" (C1, p. 8). The teaching experience gave MATH 010 students the opportunity to engage in meaningful mathematics instruction, without the pressure of having to assess students on specific curriculum expectations. The practicum was also not evaluated by an Associate Teacher.

One participant remarked that the practicum experience gave her "a new way of looking at how to teach math" (F3, p. 11). Another candidate observed that the experience was valuable because she "got to try out different lesson plans [and] . . . different teaching ideas and techniques because . . . there's no pressure" (F2, p. 3). Another participant discussed how "it was really good to get that experience working with the kids one-on-one and actually seeing how your teaching affected them" (F3, p. 11).

The experience of implementing the teaching methods that were discussed and demonstrated in MATH 010 encouraged participants to think about how to teach mathematics. The fact that MATH 010 combined a practicum with in-class sessions in the same week was seen to be very effective, especially by one participant who had a practicum experience previously as part of her Concurrent Education program:

> MATH 010 allowed you to . . . build off of what you were learning in the classroom, rather than waiting . . . a few months before you go and do a practicum in Con-Ed. . . . So, it's more right there in front so you could actually do it. (F2, p. 8)

The field experience presented a unique, scaffolded, and risk-free opportunity for connecting theory to practice, making MATH 010 a rich context for the development of ideas about learning and teaching mathematics.

Collaboration about teaching and learning. Each week, course candidates developed an enrichment lesson collaboratively with peers and with input from the course instructor. In groups of two or three, MATH 010 students then team-taught during their weekly teaching placement with Grade 7 and 8 students. Students noted that they "worked together to come up with creative solutions to the problems" (F2, p. 10) in order to better prepare for their practicum. Dr. Jonker explained his positive attitude for generating collaboration: "I've become much more aware of the importance of students working together and how effective that is" (C1, p. 4).

The team teaching was explained to be effective in generating reflection about teaching: "They're there in twos or threes and they look at each and they say 'Oh, you didn't do that very well'" (C1, p. 20). For example, Dr. Jonker referred to one particular situation during which "three of them had been there and one of them had explained something quite wrong and got quite stuck and . . . the others pointed it out" (C1, p. 20). One participant explained that he was "able to generate a healthy discussion based on questioning each others' answers in the teaching part of the program" (M1, p. 15). Through collaboration, students were able to challenge ideas and generate new ones.

A Comfortable Classroom Setting

In this section, the instructor's description of his intended classroom setting is presented, followed by participants' descriptions of the comfortable atmosphere that was created during the MATH 010 experience.

The intended goals for the classroom. While the intended goals have been revised over time, Dr. Jonker maintains that his emphasis in the course has always been for his students to develop a positive attitude towards mathematics:

> The intended goal . . . shifts a little bit. . . . [W]hen I first started [MATH 010], I wanted to get students to be . . . excited about mathematics . . . interested in mathematics, to enjoy problem solving and that still is very much my goal. (C1, p. 2)

Dr. Jonker said that he wanted to make mathematics "challenging and fun as opposed to routine and must be done" (C1, p. 5). Working with students throughout the 10 years of teaching MATH 010 also solidified for Dr. Jonker the importance of diminishing many students' fear of mathematics:

> The university students tend to be students, not without exception . . . who are very afraid of mathematics. So, it became clear that the thing that was most important for the university students is that they get rid of some of their fear, that they begin to gain some confidence in their ability to do some mathematics and at the same time to change their belief [about mathematics]. (C1, p. 5)

Participants' descriptions of the classroom. Participants described the MATH 010 classroom setting, which supported their positive attitude towards mathematics:

> I really liked how [Dr. Jonker] structured our class: . . . it was . . . a very comfortable atmosphere and everyone felt okay to share their ideas, and, in learning, that's really important because if the students don't feel comfortable to share their ideas [then] they're not going to learn anything. (F3, p. 5)

The impact that the instructor had on his students became clear as participants discussed his teaching: "What [Dr. Jonker] showed . . . was . . . the idea [to] get everyone involved, make . . . the classroom not as intimidating and . . . no one's answer is wrong" (F3, p. 3). One participant elaborated on the instructor's supportive manner in the classroom: "He was definitely encouraging . . . and he always said there's nothing wrong with asking questions" (M1, p. 16). By creating a classroom in which students felt comfortable to explore their ideas, participants described an increased sense of confidence in performing mathematics tasks.

The Development of Reflective Practice

Students were encouraged to reflect critically about their teaching experiences in classroom discussions, as well as in weekly reflection journals. In this section, the instructor's description of how he encouraged reflection in order to support competence in teaching is presented, followed by participants' descriptions of the reflective practices.

Each Friday, students discussed various aspects of teaching, for example, behavioural problems that they encountered during their practicum that week: "Students get up in front and say this is what we did, this is . . . how we handled this problem and so on. So there's reflection in that" (C1, p. 22). Feedback from the MATH 010 students was encouraged by Dr. Jonker so that students had the opportunity to hear about possible teaching strategies to use in their own classrooms. The instructor showed teaching videos and promoted discussion of educational research about mathematics: "This year I also did some videos . . . Deborah Ball teaching Grade 3 and then I had to show them a video of a U.S. . . . Grade 8 geometry class and one of a [Grade 8] Japanese geometry class and we discussed" (C1, p. 22). Dr. Jonker noted that the purpose of such videos was "to examine just what's going on in these classes and how people do things differently" (C1, p. 22).

Weekly journals also allowed students the opportunity to examine their own teaching and learning experiences within meaningful contexts; namely, the practicum experience. Journals were assessed based on student responses to questions about the practicum experience, including: (a) How did you handle behavioural problems? and (b) What techniques did you use to engage students? (Appendix B). Writing in the weekly journals immediately following the experience provided a vehicle for reflection about teaching practices. One participant noted that without having to write in the journals, "we probably wouldn't have really reflected on our lesson that much" (F1, p. 13). The participant went on to say that the journals made her "think about how [she] could have got more kids to participate or more kids to discuss things” (F1, p. 14). Being required to reflect about what went well during the lesson and what could have been improved was an important aspect of the course that allowed students the opportunity to become aware of their teaching methods.

Summary

The previous section of this chapter described how the MATH 010 course provided a rich context for studying prospective teacher beliefs about the nature of mathematics and mathematics teaching and learning. Given the challenging nature of the *StepAhead* material, prospective teachers were required to consider many solutions in order to solve enrichment questions during weekly in-class sessions and when completing homework problems. Candidates were required to anticipate how Grade 7 and 8 students would respond to and solve such problems. The weekly in-class reflections completed concurrently with the practicum experience presented a unique opportunity for connecting theory to practice. Thus MATH 010 seemed to provide authentic opportunities for participants to articulate their beliefs about teaching and learning.

After the Course Experience

In this section, participants’ descriptions detail their beliefs about the nature of mathematics and mathematics teaching at the end of the MATH 010 course. Their descriptions of their competence in their teaching abilities are also presented, as well as descriptions about their mathematical and pedagogical content knowledge at the end of the course. Data were drawn from responses to items contained in Questionnaire 2 and from individual interviews, both of which were completed during the last week of the course.

Beliefs about the Nature of Mathematics

By the end of the course, many MATH 010 candidates expressed transformed ideas about what defines mathematics as a subject. For example, during an interview, one participant stated: "The course has sort of opened my eyes to [the fact] that math isn't just numbers. It's theories, it's ideas, it's proofs . . . it actually makes sense once you actually look at it further" (F3, p. 10). Another participant described trying to "make sense of" (F1, p. 9) challenging problems. The earlier experience of solving routine mathematics problems was compared to thinking "outside of the box" (F3, p. 1) and using "knowledge of other things" (F3, p. 1) throughout their opportunities to learn in MATH 010.

Problem solving process. By the end of the course, participants placed great value on the process of solving a problem and the many ways of examining a concept through multiple lenses and connections among content strands. Participants discussed changes to the way in which they thought about the problem-solving process: "Now I can . . . see that there's lots of different ways" (F1, p. 5). The course allowed one participant the opportunity to learn about "different ways of explaining things that will make sense to other people" (F1, p. 8). One participant expressed his thoughts about the important aspects of problem solving: "Looking at questions and knowing that there [are] different ways to do it and . . . you just have to think about what they're asking you. That's the key to solving it" (F2, p. 8). In response to Questionnaire 2, one participant noted that during problem solving, “we wanted to know why we were doing what we were doing, rather than just finding the answer” (p. 12).

Beliefs about the Nature of Mathematics Teaching as a Developing Teacher

Beliefs about the nature of mathematics teaching relate to course candidates’ perceptions about how mathematics is taught, which includes effective instructional methods. Participants’ ideas about such strategies at the conclusion of MATH 010 are presented in this section.

Descriptions of hands-on activities. During individual interviews, participants discussed hands-on learning activities as being integral to their transformed beliefs about the nature of teaching mathematics. The interview participants described how the course had provided examples of hands-on activities that allowed them to better understand the complexity of mathematics, while being better able to “visualize the question more” (F2, p. 25). One student elaborated on how the course instructor showed a hands-on learning activity by bringing in rolls of toilet paper to explain perimeter and area:

> [Dr. Jonker] always brought in . . . props. . . . He could have just . . . drawn it on the board, but actually feeling it and being able to . . . measure it in person . . . and then . . . prove it by measuring it with a ruler is . . . good for students to be able to . . . trust the answer that they’ve arrived at. (M1, p. 4)

Another participant discussed a similar experience: “It was great to learn that note-taking did not have to exist and that students could learn and retain the knowledge just as well by working hands-on with the problems” (F2, p. 15). The use of such techniques was described by participants as being very important to their views about teaching: “I've come to realize how important visual aids are for some students to grasp what is being taught and that hands-on activities are important to get certain students to be engaged" (F2, p. 15). By the end of the course, participants believed that teaching mathematics involved instructional approaches to facilitate students' exploration of mathematics creatively, which included hands-on learning activities.

Beliefs about the Nature of Mathematics Teaching as a Student Learner

At the end of the course, candidates had transformed their ideas about how to learn mathematics. In response to items on Questionnaire 2 and during individual interviews at the conclusion of the course, participants described methods that facilitate student learning, which are presented next.

The implications of group work for student learning were evident as participants described their practicum experiences with Grade 7 and 8 students at the end of the course: "Definitely now I see how important it is for [Grade 7 and 8 students] to really work together with other students . . . [and] try and figure things out together" (F1, p. 5). Another participant had a similar realization about collaboration: "I . . . recognized that [collaboration] was a really important part of learning" (F3, p. 2).

Group discussions led students to re-consider their initial ideas about how to learn mathematics. One participant described that, through collaboration, "everyone's sharing their ideas and they feel comfortable and it creates that dynamic . . . learning environment that's really important for kids to learn in" (F3, p. 13). The use of discussion as a strategy for students to share their ideas about how to solve a problem allowed participants to be able to recognize the importance of collaboration: "I . . . recognized that [discussion] was a really important part of learning" (F3, p. 2). By the end of the course, participants valued learning mathematics through collaboration.

Beliefs about Competency as a Student Performing Mathematical Tasks

Participants discussed self-assessments of their competence to problem-solve at the end of the course. Course candidates also described increased confidence as a student performing mathematical tasks, which is described in this section.

Participants' described mathematics as an enjoyable subject: "You can actually have fun with [mathematics]. That's what I was surprised about because . . . you can relate it to any other subject really" (F2, p. 3). Another participant commented, "We found that math can relate to . . . anything, . . . all sorts of real life experiences you can have calculations for" (F2, p. 17).

Participants were also able to express confidence in completing mathematics tasks by the end of MATH 010: "I think . . . this course . . . opened my eyes [and gave] me more confidence . . . to look at a problem and understand it and . . . combine everything that I've learned" (F3, p. 8). One participant stated, "I feel more confident now" (F2, p. 10). Another participant discussed his view of mathematics at the end of the course: "Now I think of math as . . . not that bad. There's different focuses within, so . . . it's become more friendly with a semester under my belt" (M1, p. 9). After completing the course, finding a solution to a challenging mathematics problem also did not seem quite as intimidating as it once did for participants: "I don't jump to thinking I'm not going to be able to find [a solution]" (F3, p. 10). Seeing mathematics as an approachable subject allowed participants to feel both competent and confident about their problem-solving abilities.

Beliefs about Competency as a Developing Teacher

Participants discussed self-assessments of their competence in teaching at the end of the course. During an interview, one participant noted how the course "really prepared us for going into the [intermediate division] classes" (F1, p. 2). The participant said that MATH 010 helped her "learn how to really make sure these kids are understanding math" (F1, p. 3). Another participant commented that the course helped her prepare for teaching mathematics by "learning from the curriculum" (F2, p. 7), while a third participant commented that the course provided her with "a lot of different techniques to use" (F3, p. 13) in the classroom.

One participant commented, however, that she still needed to learn more in order to be an effective mathematics teacher. Specifically, she described that MATH 010 "was just geometry, so [she was] going to have to look at all the different parts [of mathematics]" in order to prepare for her teaching career (F2, p. 14). Another participant commented that he still needed to learn about "teaching and education in general" (M1, p. 19) in order to be a fully competent mathematics teacher. Three participants described increased confidence in their abilities to teach mathematics at the end of the course.

Knowledge of Mathematical and Pedagogical Content

On Questionnaire 2 and during individual interviews, participants were asked about their perception of their mathematics content knowledge as well as their pedagogical content knowledge. For example, participants were asked to review mathematics content knowledge questions during Questionnaire 2 that they had previously answered for Questionnaire 1. One such question related to prime numbers and gave several examples of possible solutions to a problem, which participants were expected to review and assess. Prime numbers was not a topic covered in MATH 010, however, since the 2010 fall semester focused exclusively on geometry. Six participants' answers did not change because they hadn't acquired any further knowledge about prime numbers. Participants elaborated that any change to solving these kinds of content knowledge questions would be in the problem solving process, which would involve generating various approaches to solving the problem. Participants were also asked in Questionnaire 2 if the course had contributed to their understanding of mathematical content questions.

Since the 2010 MATH 010 course focused on geometry, it is not possible to report on change to participants' mathematics content knowledge at the end of the course. Participants' self-assessments of their mathematics content knowledge are the only reference to change, and these descriptions seem to be related to participants' ideas about problem solving and participants' self-assessed competence in performing mathematics tasks, both of which were discussed in previous sections of this chapter.

Summary

The 10 participants in this study appear to have undergone a significant transformation throughout their experiences in MATH 010. Participants described several changes in their beliefs about the nature of mathematics and mathematics teaching as a result of completing the course. By the end of the course, candidates' view of mathematics had changed to a belief that mathematics is a subject that is creative and goes well beyond simple rules and procedures. They described that doing mathematics involves considering a variety of methods to solve a problem and that rules and procedures are just one way of approaching the problem-solving process, which is multi-dimensional. By the end of the course, prospective teachers viewed exploration and collaboration as instructional approaches to facilitate learning and described competence in performing mathematical tasks. Participants also thought mathematics was a more enjoyable subject. In the final section of this chapter, the analysis of the data is presented.

Analysis of Data

In this chapter, the data was presented in order to answer the research questions underpinning this study. It was then important to examine the data for recurring themes that pertained to all data that were collected. For this reason, an analysis of the data was conducted, which first involved the development of a coding scheme (Table 5). Following this, the emergent themes and patterns, which corresponded to specific codes, were identified and summarized to describe trends within the data (Table 6). Specific patterns within the data corresponded to themes about (a) transformative learning and (b) the key elements of the course that contributed to the transformation to beliefs and knowledge. To highlight the transformative experience undergone by the MATH 010 course candidates, the data was presented chronologically to provide a rich narrative of participants' experiences in the order that they happened.

Table 5

Coding Scheme Used in the Analysis of Data

Code	Definition	Example
Algorithmic	Using predetermined formulae to solve problems	"you were given a formula, plug in numbers"
Anxious	Feeling anxious about performing mathematics tasks	"I dreaded solving it"
Collaboration	Learning by collaborating with others	"we worked together for creative solutions"
Confidence	Gaining confidence in teaching abilities	"I feel more confident now"
Connected Topics	Making connections among topics within mathematics	"there's different focuses within"
Contextual Applications	Linking concepts to other applications	"Math can relate to . . . anything"
Critical Thinking	Learning by asking "how" and "why" concepts and solutions come about	"we wanted to know why we were doing it"
Disengaged	Being disconnected from the subject	"why do I need to know this?"
Engaging Tasks	Using activities to engage learners	"we got to try out different lesson plans"
Individual	Learning individually	"Individual classrooms"
Intimidation	Feeling intimidated by a lack of teaching abilities	"I was always worried about teaching math"
Joint Enterprise	Having a common goal of understanding problem solving	"you have to think about what they're asking"
Lecture	Presenting information orally	"the teacher tells you what to do"
Mechanistic Answer-Finding	Using repetitious methods to solve problems	"you knew the steps of how to . . . solve the problem"
Memorization	Learning through memorizing rules and procedures	"you memorize what the teacher did"
Multiple Perspectives	Having many ways to explain and solve a concept or problem	"different ways to solve it"
Multiple Representations	Presenting information multiple ways (visually, physically)	"more than just one way to teach"
Note-Taking	Supplementing oral information with written notes	"note-taking was kind of the typical thing"
One Solution Path	Considering only one way to solve a problem	"there was one way to solve it"
Participation	Taking part in the learning process	"you can actually have fun with it"
Reification	Understanding abstract concepts	"learning different proofs"
Reflection	Being reflective to improve practice	"really reflected on our lesson"
Rich Resources	Using rich tasks for critical thinking	"[StepAhead problems] made you think outside the box"

Table 6

Emergent Themes and Patterns from the Data

	Themes	Patterns	Codes
Pre-Course Profile	Traditional Math Teaching	-ineffective ways to facilitate student learning	Lecture, Intimidation, Note-Taking
	Procedural Learning and Procedural Knowledge	-learning as a passive transmission of knowledge	Anxious, Individual, Memorization
	Instrumental View of the Utility of Math	-not focused on connecting ideas -math as an isolated body of concepts	Algorithmic, Disengaged, Mechanistic Answer-Finding, One Solution Path
The Course Experience	Building Community	-social construction of knowledge	Participation
	Inquiry-Based Approach to Math	-experienced discovery learning -opportunities to consider other explanations	Rich Resources
	Mentoring by an Expert Instructor	-gained understanding of effective teaching strategies through engaging activities	Engaging Tasks
	Safe Opportunities for Practice	-developing, practicing, and applying skills for teaching and learning math	Joint Enterprise
	Meaningful Reflection	-reflect critically about experiences	Reflection
Post-Course Profile	Reform-Based Math Teaching	-effective ways to facilitate student learning	Collaboration, Multiple Representations
	Conceptual Learning and Conceptual Knowledge	-learning by understanding concepts -transformation of knowledge	Confidence, Contextual Applications, Critical Thinking, Reification
	Dynamic View of the Utility of Math	-math as a web of connected topics	Connected Topics, Multiple Perspectives

Results showed that the participants underwent a transformation throughout their journey in MATH 010. The pre-course profile revealed participants' views about the course experience include: (a) the importance of community, (b) an inquiry-based approach to mathematics, (c) mentoring by an instructor who embodies the principles of the contemporary mathematics curriculum, (d) safe opportunities to attempt to emulate the instructor's strategies and behaviours, and (e) meaningful reflection about these experiences. Participants' transformed views about mathematics and its teaching at the conclusion of the course highlight a post-course profile that reflects participants' views about reform-based mathematics teaching, which led to views about conceptual learning methods and a dynamic view of the utility of mathematics.

Summary

In this chapter, data related to each of the three research questions driving this study have been presented to formulate empirical answers to these questions. The data were represented chronologically to describe participants' beliefs about the nature of mathematics and its teaching before and after the course experience. In analyzing the data, it was clear that between the beginning and the end of the course, participants underwent a transformation to their beliefs about the nature of mathematics and the nature of mathematics teaching. By experiencing a well-designed course in which practical experience and discovery learning were foci, participants were able to transform their traditional beliefs about mathematics and teaching, adopting more research-based views that aligned with the vision articulated in the National Council of Teachers of

Mathematics' *Standards*. In the next chapter, the emergent themes from the data, along with their corresponding patterns, are discussed in connection to the literature.

CHAPTER 5

DISCUSSION

The purpose of this study was to examine the ways in which prospective teachers perceived a change to their personal and professional beliefs about mathematics and mathematics teaching, as well as to their content knowledge after experiencing a university course that purported to focus on both. The study was guided by three research questions: (a) What are prospective teachers' beliefs about the nature of mathematics and teaching before the course experience? (b) How do prospective teachers perceive that their beliefs about the nature of mathematics and teaching have changed in response to the course experience? and (c) To what do prospective teachers attribute any changes to their beliefs about mathematics and its instruction?

The previous chapter presented the data for this study and detailed the experiences of 10 prospective teachers enrolled in a university mathematics course, MATH 010, entitled *Fundamental Concepts in Elementary Mathematics for Teachers*. Chapter 5 discusses the data and begins with the 10 study participants' prior experiences in mathematics. As the course candidates entered MATH 010, it was evident that their secondary mathematics education had been very traditional in spite of the fact that they were secondary school students nearly 20 years after the introduction of the National Council for Teachers of Mathematics' *Standards* (1989). They described mathematics teachers as lecturers who transmitted information and themselves as "sponges" who were expected to absorb information passively. This influenced the way in which participants experienced learning; that is, they recalled it as being procedural, a set of steps requiring the substitution of numbers into a formula. This experience, in turn, underpinned their view that mathematics was an instrumental subject, the formulaic use of numbers was simply a means to a correct solution. These views reflected participants' beliefs about doing mathematics and teaching mathematics and were tightly linked to their prior experiences. Relevant educational research suggests that the participants' past experiences could create deeply ingrained beliefs that would be unlikely to change over the course of a 12-week MATH 010 semester (Bransford, Brown, & Cocking, 2000; Kagan, 1992; Vygotsky, 1978).

Darling-Hammond (2006) reviewed seven highly successful, long-standing teacher education programs to explicate the kinds of educational experiences that had been shown to enable powerful learning. Her review suggested that learners are able to achieve high levels of understanding when teachers skillfully design curriculum materials and artfully manage classroom discourse and student behaviours. Darling-Hammond argues that teachers need a solid grounding in mathematics content to be able to lead conceptually-driven mathematics discussions and guide instruction so that students engage with and learn fundamentally important mathematics. She further argues that this professional growth takes time, energy, and consistency. By the end of the 12-week journey through MATH 010, participants experienced a transformation to their beliefs and knowledge because of the skillfully designed course that Dr. Jonker created. By the conclusion of the course, candidates described teaching and learning as a process of collaboration and discovery; they valued hands-on activities and novel tasks that challenged their thinking; they spoke of developing a conceptual understanding of content; and, they learned to make connections among concepts, principles, and procedures. The prospective teachers were able to articulate an explicit purpose for knowing and doing mathematics and recognize that multiple representations of a concept empowered them to better instruct and engage students in mathematics. They also held a concrete vision of how to implement their beliefs in the classroom because they experienced an instructor who embodied these qualities and practices.

This chapter is organized to highlight the significant transformation of participants' beliefs about mathematics and its teaching relative to existing literature. The first section provides a discussion of the 10 participants' experiences prior to MATH 010. To draw attention to the contrast between beliefs held at the beginning and end of the course, the second section describes participants' views about mathematics and its teaching at the conclusion of the course. The course experience is discussed in the third section in order to suggest how and why the participants' beliefs about mathematics and teaching underwent a significant transformation.

It is important to consider what we can learn from the experiences in MATH 010 that led participants to re-consider their initial beliefs about mathematics and its teaching in the 12-week semester. This study supports the contention that one teacher can make a difference in a short period of time and highlights the notion of a transformative curriculum. These findings emerged from the analysis of data and were not discussed in the Literature Review. Thus the focus of this chapter is the transformation to beliefs and knowledge experienced by the candidates in response to the course which was facilitated by an expert instructor. The chapter concludes with overall implications about preservice teacher mathematics education, limitations of this study, and suggestions for further research.

Pre-Course Profile

Teacher candidates typically arrive at Faculties of Education with a similar set of experiences which influence their beliefs about the nature of mathematics and its teaching (Richardson, 1996). These past experiences in mathematics classrooms often lead to beliefs that (a) curricular topics represent isolated bodies of concepts, and (b) learning is achieved through a passive transmission of knowledge from teacher to student (Ambrose, 2004; Ambrose, Bridges, DiPietro, Lovett, & Norman, 2010; Kajander, 2007). The 10 participants in this study were no exception, being a group of prospective teachers who shared similar mathematics classroom learning experiences prior to the start of MATH 010 and, as a result, expressed similar beliefs about doing mathematics and learning mathematics. The participants expressed beliefs that reflected a traditional mathematics learning experience, describing secondary school mathematics instruction as being focused on repetitive computations and having minimal problem-solving opportunities. Mathematics learning experiences were described as lectures during which teachers demonstrated how to use rules and procedures to complete a set of tasks, while students recorded notes. Memorizing and then repeating these rules and procedures (focused primarily on one way to solve problems in order to achieve a correct answer) appeared to be the common experience of the participants.

Research suggests that students' school experiences will influence and inform teachers' beliefs about teaching and, in turn, their beliefs will predict their behaviours while teaching (Bransford, Brown, & Cocking, 2000; Kagan, 1992; Vygotsky, 1978). The pre-course profile of the 10 participants in this study mirrors the findings of these studies, since the candidates' beliefs at the start of MATH 010 seemed to be a mirror image of their experiences in traditional classrooms as learners of mathematics. For example, the 10 participants viewed the use of lecture as the only teaching method because of their almost exclusive experience of this strategy as learners. This background also appears to have led to the view that mathematics is strictly procedural: the mapping of symbols and application of rules to find one correct solution. In other words, these participants held an instrumental view of mathematics (Ernest, 1989), learning mathematics as a recipe without a conception of why particular algorithms were being used for specific purposes. These beliefs, ingrained as a result of the approximately 2000 hours (Ball, 1988) spent as a student apprentice (Lortie, 1975) observing and learning in traditional mathematics classrooms, characterized the profile of the participants in September 2010.

The remainder of this section in Chapter 5 describes how participants' beliefs reflected (a) traditional mathematics teaching, which led to (b) procedural instructional methods and procedural knowledge development, and (c) an instrumental view of the utility of mathematics. These beliefs will be examined through the lens of the body of literature that examines the ways in which traditional teaching experiences affect prospective teachers' views of mathematics and its teaching.

Traditional Mathematics Teaching

The 10 participants in this study had very little experience teaching mathematics prior to the start of MATH 010. Four candidates had one-on-one tutoring experience, while one had completed two weeks and another had completed three weeks of practice teaching as part of a concurrent education course requirement. Four participants had no teaching experience at all. This lack of teaching experience caused them to be anxious about going into classrooms to teach (a required course component for MATH 010). They said that, prior to MATH 010, their experiences in mathematics classrooms consisted of being learners, taking notes while listening to and recording a teacher's explanations in order to replicate the procedures that were described. This method of teaching is characterized by a focus on the teacher's voice and chalkboard notes. Participants' descriptions of the teacher's role as a transmitter of information, characterized by lecture and note-taking, reflect reliance on the teacher for exact procedures to solve mathematics problems (NCTM, 2000).

Participants shared the belief that teaching mathematics through lecture and note-taking was boring and uninteresting. Although teaching methods such as the use of lecture were recognized as being ineffective by the participants, they stated that they would adopt such methods in their own classrooms. Due to their lack of teaching experience and the strong influence of their own teachers' modelling, participants had no other repertoire of strategies from which to choose. These teaching strategies were familiar and represented for them a viable vision for and model of instructional practice.

The National Council of Teachers of Mathematics described what a traditional mathematics classroom looked like:

> In all the math classes that I [*sic*] visited, the sequence of activities was the same. First, answers were given for the previous day's assignment. The more difficult problems were worked on by the teacher or the students at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and the problems assigned for the next day. The remainder of the class was devoted to working on homework while the teacher moved around the room answering questions. The most noticeable thing about math classes was the repetition in this routine. (National Council of Teachers of Mathematics, 1991, p. 1)

Within the mathematics education community, this model of standard, or traditional, instruction was thought to include teaching strategies that were less effective than more research-informed, reform-based, mathematics education practices, in which students are guided to understand concepts and procedures through exploration, discovery, and application to real-world contexts (NCTM, 1989).

Miller (2007), a holistic education scholar, has studied the three stage model of teaching and learning: (a) transmission of knowledge, (b) transaction of knowledge, and (c) transformation of the learner. According to Miller, traditional, teacher-centered classrooms put emphasis on the transmission model of teaching which limits the transfer of knowledge to the learner. The transaction, or exchange, of knowledge from the learner to another person is also absent, thereby limiting the transformation of knowledge to other contexts. According to Swann (1998), transmission involves the learner in a passive state "in which he or she is not particularly responsible for what happens, apart from a willingness to be alert to the input of information" (p. 211). Focusing solely on the transmission model of teaching thus constrains the learner's application of knowledge (Miller, 2007). The 10 prospective teachers who were participants in this study appeared to experience, as learners of mathematics instruction, a passive transmission of knowledge in that their visions of the classroom were restricted to outmoded models of teaching and learning when they began MATH 010.

Procedural Instructional Methods and Procedural Knowledge Development

Hiebert and Lefevre (1986) characterized procedural knowledge as having a sequential nature. Procedural knowledge includes (a) knowing the formal language, or symbols, represented by mathematics, (b) knowing algorithms and rules for completing tasks, and (c) knowing strategies for solving problems. Procedural fluency is required to solve problems but an understanding of the concepts embedded in the procedures enables students to use different methods, remember the procedures more easily, and know whether the procedure elicited the correct response (Kilpatrick, Swafford, & Findell, 2001).

Memorizing procedures in order to solve problems was common for participants prior to the start of MATH 010. Data from Questionnaire 1 showed that, in response to the statement “exact procedures are needed for solving mathematics problems,” all 10 participants agreed. Participants described being able to apply rules and procedures to find the solution to a mathematics problem but admitted that they did not understand why the rules or procedures were used. Participants memorized algorithms without engaging in the use of mathematical reasoning, a strategy thought not to be effective for understanding how to learn mathematics (NCTM, 1991). Memorizing procedures left students feeling unengaged by mathematics. This was shown when participants discussed mathematics as a dull subject prior to the start of MATH 010.

At the start of the study, the 10 course candidates focused primarily on “how to” without clear conceptual links to the procedural knowledge. To achieve high scores in mathematics, as students, the 10 prospective teachers had carried out procedures and used algorithms, formulae, and mathematical rules, mastered through memorization and repetition. The results of this study suggest that participants had a novice understanding of mathematics (Ernest, 1989) prior to MATH 010, which included procedural knowledge that lacked connections between and among pieces of knowledge.

An Instrumental View of the Utility of Mathematics

Participants had experienced traditional teaching strategies and, in turn, adopted procedural learning methods. Their first-hand experiences as learners (over a significant time span) contributed to their view, as adults, that mathematics was a subject that lacked purpose. Participants recalled not knowing why they were completing various tasks in mathematics classes, for example, mathematics involved substituting numbers arbitrarily into a formula to obtain a correct answer. Participants described how simply substituting numbers into a formula had not provided them with a solid foundation for understanding mathematics, even though they had all answered content questions successfully as part of the pre-course questionnaire. Participants described the nature of mathematics as a process of obtaining a correct answer because, for them, that was how mathematics was applied prior to MATH 010. Participants’ beliefs about the nature of mathematics related to their previous experiences which were very procedural. Ainley, Pratt, and Hansen (2006) discussed how “the learning of mathematics encompasses not just the ability to carry out procedures, but the construction of meaning for the ways in which those mathematical ideas are useful" (p. 30). Participants' self-assessments reflected the fact that they believed their expertise was shallow, not deep or conceptual. Course candidates in this study appeared to hold beliefs that align with an instrumental view of mathematics, i.e., one in which facts, rules, and procedures are used as a means to an end (Ernest, 1989). As seen through this lens, topics are unrelated and limited to a set of rules and facts. An instrumental view of mathematics is considered to be a novice understanding of the nature of mathematics (Ernest, 1989).

Conflicting beliefs. At the start of the MATH 010 course, participants sometimes expressed conflicting views about the nature of mathematics. For example, one participant agreed with the statement that "mathematics involved creativity and new ideas" while simultaneously agreeing that "mathematics is only about the substitution of numbers into a formula." Before the course, participants described the discovery of new mathematics ideas by themselves, conflicting with their statements about the repeated use of procedures given by a teacher in order to learn mathematics. Conflicting beliefs have been previously shown to be held by prospective teachers with a novice conception of mathematics (Roth-McDuffie, McGinnis, & Graeber, 2000). Roth-McDuffie et al. used interviews, surveys, and observations to investigate the beliefs held by 21 undergraduate students enrolled in a semester-long mathematics course for candidates preparing to be middle school mathematics teachers. The teacher candidates ascribed to the belief that mathematics involves step-by-step procedures to solve a problem and, at the same time, described mathematics as a dynamic discipline that involves a search for patterns. The 10 prospective teachers in MATH 010 recognized that mathematics was more than an unrelated set of rules, but they expressed a lack of confidence about the connections within mathematics due to the fact that they had never experienced mathematics through discovery learning or as an interconnected field.

Summary

Prior to MATH 010, the 10 participants in this study had experienced a traditional mathematics education, despite having undergone their elementary and secondary education at a time when mathematics teaching and learning was supposed to be reform-based. Their experiences in traditional secondary school classrooms meant their images were of stereotypical mathematics teaching methods. Their experiences as learners, to date, also meant that they held traditional ideas about learning, which they believed involved memorization of procedures, which, in turn, underpinned their views about the lack of utility or application for mathematics. In the next section of this chapter, participants' beliefs following the completion of MATH 010 are discussed. These new beliefs highlight the significant shift undergone in participants' beliefs about mathematics and its teaching as a result of the transformative power of MATH 010.

Post-Course Profile

Widespread evidence has suggested that teacher education courses have little impact on the development of teacher candidates' teaching abilities (Hiebert, Morris, Berk, & Jansen, 2007; Russell, McPherson, & Martin, 2001). Research has shown that prospective teachers often leave university with the same ingrained beliefs about teaching with which they entered (Ernest, 1989; Kane, 2007; Mezirow, 1997; Thompson, 1984). According to Kane (2007), one reason for this is that the structure of teacher education programs provides prospective teachers with general information about teaching and learning rather than specifically linking course work to practicum experiences. Even if a prospective teacher's beliefs about teaching do undergo a transformation, it is thought to take several years to occur (Mezirow, 1997).

The results of the current research suggest that the 10 participants experienced a university mathematics course focused on education that provided an exception to this literature. During the course, all 10 participants were part of a classroom that supported the discovery of multiple solution paths and encouraged and enabled inquiry and collaboration. The prospective teachers' journey was facilitated by an expert mathematics instructor whose teaching strategies placed great emphasis on discovery learning, communication, and conceptual development. In combination with supportive course work and mentoring, the 10 participants also completed a practicum assignment and were encouraged to reflect about this experience. By the end of the course, all 10 participants' beliefs about mathematics and its teaching reflected their new experiences. These results contrast with the literature described previously, which suggested that the deep-seated beliefs held by the MATH 010 participants would persist despite their course work (Ernest, 1989; Hiebert, Morris, Berk, & Jansen, 2007; Russell, McPherson, & Martin, 2001). The data in this study, however, suggest that the participants' beliefs underwent a significant transformation as a result of a 12-week semester.

In analyzing the data, the *Teacher Education and Development Study in Mathematics* (TEDS-M) (Tatto et al., 2008) was the primary source used to assess (a) perceived areas of change in prospective teachers' beliefs about what mathematics is and how it should be taught; and (b) knowledge of mathematics and strategies for and approaches to classroom instruction. The remainder of this section describes how participants' beliefs were transformed to reflect (a) reform-based mathematics teaching, which led to (b) concept-focused instructional methods and conceptual knowledge development, and (c) a dynamic view of the utility of mathematics. These beliefs will be examined through the lens of the body of literature that examines how reform teaching experiences affect prospective teachers' views of mathematics and its teaching.

Reform-Based Mathematics Teaching

Throughout MATH 010, participants experienced and observed the teaching methods of Dr. Jonker, an expert mathematician and mathematics educator. They were given challenging mathematics problems that required a sustained effort and were encouraged to discover their own methods for solving these problems. They were required to reflect on the task and its multiple solution paths to assess their own conceptual understanding. Lastly, they were challenged to facilitate parallel micro-teaching situations with Grades 7 and 8 students. Having been given the opportunity to participate in such an innovative (to them) mathematics class, the participants, by the conclusion of the course, described the value of collaborating with their peers to generate ideas and strategies in order to better understand concepts, practices that had been modelled by Dr. Jonker. Their instructor guided them through opportunities for reasoning based on evidence.

During post-course interviews, participants described how they had used Dr. Jonker's strategies in their own teaching situations during their practicum placements. The 10 participants recognized the effectiveness of Dr. Jonker's teaching strategies and, as a result, had enacted his repertoire of strategies in their own practicum settings. They described the implementation of teaching methods that included collaboration and discovery learning in their own micro-teaching situations with Grade 7 and 8 students.

The National Council of Teachers for Mathematics described a vision of a reform-based mathematics classroom:

> Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. . . . Knowledgeable teachers have adequate resources to support their work and they are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. . . . Students confidently engage in mathematical tasks carefully chosen by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find the methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups . . . they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it. (NCTM, 2000, p. 3)

In mathematics education research, reform-teaching was thought to be necessary in order for students to understand mathematics (NCTM, 2000). Dr. Jonker's course exemplifies the vision of a reform-based mathematics classroom, given participants' descriptions that (a) valued problem solving, (b) required discovery learning, (c) provided opportunities for collaboration, (d) utilized effective resources, and (e) provided the skilled guidance of Dr. Jonker, an expert mathematician, who has an ability to generate high-quality novel problems.

Swann (1999) posits that "learning takes place when a human . . . has a problem, attempts to solve it and survives, creating changes in the world and in the learner" (p.

260). Miller (2007) describes this kind of learning as transformative. Some researchers consider transformative experiences as the only means for learning to occur (Kolb, 1984; Sfard, 1998; Woolfolk, Winne, & Perry, 2003). According to Miller, transformative learning is more likely to take place in student-centered classrooms, characterized by an emphasis on the use of reasoning and critical thinking. The 10 participants in this study described learning as a transformative experience because of the requirement and opportunities for critical thinking in MATH 010.

Transformative learning was evidenced when the participants discussed their opportunity to apply their new understanding of mathematics and mathematics teaching to another context, which, in this study, was a Grade 7 and 8 teaching placement. After a semester in MATH 010, the 10 participants were surprised by how well they were able to teach mathematics to the Grade 7 and 8 students. While they had recognized that they acquired a rich repertoire of strategies, by the end of the course, they realized how much more they needed to know. Students appeared to have acquired subject matter knowledge (Hill & Ball, 2009) about geometry content; however, they did not have the opportunity to demonstrate growth in other mathematics strands. For example, on the post-course questionnaire, one participant described that she could not solve the content question related to prime numbers with confidence because she had not learned anything about prime numbers during MATH 010. Similarly, participants had learned about pedagogical content (Hill & Ball, 2009) topics. They discussed learning about the Ontario curriculum and effective strategies to teach mathematics, such as discovery learning. One participant recognized, however, that he still needed to become familiar with general pedagogy to teach mathematics effectively in his future career.

Concept-Focused Instructional Methods and Conceptual Knowledge Development

According to researchers Hiebert and Lefevre (1986), conceptual knowledge is acquired in two ways: (a) “[by] the construction of relationships between pieces of information” or (b) “[by the] creation of relationships among existing knowledge and new information that is just entering the system” (p. 4). Long (2005) argued that conceptual knowledge is intricately linked with procedures and algorithms. Understanding the concepts embedded in procedures enables the student to (a) use different methods, (b) remember the procedures more easily, and (c) know whether the procedure elicited a correct response (Kilpatrick, Swafford, & Findell, 2001). Conceptual understanding has value and often elicits varied, insightful responses from learners who have grasped the concept (Ball, Lubienski, & Mewborn, 2001; Lampert, 2001).

Based on their self-assessments, the 10 participants in this study described a new sense of conceptual understanding about mathematics that coincided with their experiences in MATH 010. Reform-based mathematics experiences during the course appeared to make a significant contribution to the participants’ views about teaching, which emphasized their new ideas about learning and the meaning of “understanding.”

One participant was able to describe the ways in which she was able to apply prior knowledge to solve a new problem. At the end of the course, she could see how algorithms fit into problem solving procedures and she had developed a new appreciation for the fact that mathematics process rules were not limited to procedural use in a pre-determined process. Skemp (1976) described this kind of understanding as "relational" (p. 21). Relational understanding is the ability to deduce specific rules and procedures from more general mathematics relations. In mathematics, experts are more likely than novices to demonstrate elements of relational understanding, such as understanding processes, rather than simply to attempt the substitution of numbers into a formula (Bransford, Brown, & Cocking, 2000). This was reflected in the participants' descriptions of their approach to problem solving when they described an awareness of the fact that one concept could be applied in multiple ways.

Establishing connections among various topics and algorithms appeared to allow the participants in this study to see the value of nurturing in their students the development of conceptual understanding. Throughout the semester, the prospective teachers built upon their procedural knowledge to form connections among topics in mathematics. Building connections among strands in a mathematics education course has been previously shown to be effective in developing a deeper understanding of problem solving in preservice teachers (Liljedahl, 2005). Figure 1 (Van de Walle & Folk, 2005) is a metaphor for the construction of ideas. When a new idea (the center dot) is introduced, the teacher attempts to facilitate the students' generation of connections to existing knowledge (the outer dots) by activating prior knowledge through purposeful questioning. The more connections a student makes, the richer the learning becomes. This is the essence of using or activating prior knowledge, a key in teaching and learning conceptually and

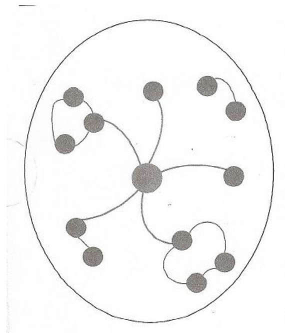

Figure 1. The Construction of Ideas. Adapted from *Elementary and Middle School Mathematics: Teaching Developmentally* (p. 29), by J.A Van de Walle and S. Folk, 2005, Toronto, Canada: Pearson Education. Copyright 2004 by Pearson Education.

considered to be commonly demonstrated by experts (Ross, 2006, Van de Walle & Folk, 2005).

A Dynamic View of the Utility of Mathematics

Thompson (1992) argued that dynamic beliefs undergo change as one evaluates one's beliefs against experiences. A person who holds dynamic beliefs is likely to allow their practice to inform their beliefs (Stipek et al., 2001; Kagan, 1992; Wood, Cobb, & Yackel, 1991), for example, experiencing learning through discovery could lead to reform-based teaching beliefs. The data suggest that, as a result of experiencing MATH 010, the 10 participants appeared to develop common beliefs that were dynamic and open to change as they evaluated their experiences as students in MATH 010. Participants allowed their MATH 010 experiences to inform their beliefs about teaching, learning, and doing mathematics and, as a result, their beliefs appeared to change. This finding contrasts with the literature that supports the position that beliefs are ingrained and unlikely to change (Ernest, 1989; Kane, 2007; Mezirow, 1997; Thompson, 1984).

Ernest (1988), a philosopher of mathematics, described a dynamic view of mathematics as “invention . . . [and] a process of enquiry” (p. 251). Ernest elaborated, saying that mathematics is an infinite field that expands based on human creation. Furthermore, a person having a dynamic view of mathematics embraces a conceptual understanding of mathematics as a tool for problem solving (NCTM, 1991; Thompson, 1992). A dynamic view of mathematics is considered by Ernest (1989) to be an expert understanding of the nature of mathematics.

The 10 participants’ responses suggest that they developed a dynamic view of the nature of mathematics over the semester. Participants’ MATH 010 experiences expanded their views about teaching and learning to include more contemporary methods, including discovery learning, collaboration, and hands-on activities. Participants appeared to see the connections among topics within mathematics. For example, one participant described that problem solving was the basis of mathematics and that problem-solving encompassed many different topics in geometry. As a result, they experienced success in solving challenging problems and they expressed more confidence when completing such problems. They expressed pride in their ability to discover their own methods for solving problems, which was a new experience for them, and felt that they finally understood how and why various algorithms could be applied to solve a variety of problems. They described the nature of mathematics as less formulaic than they previously had, referring to the subject as an intellectual challenge. This new belief arose because mathematics had a new purpose: the 10 participants wanted to be challenged so that they could increase their knowledge of the mathematics necessary for teaching. By seeing multiple representations of a topic, they hoped to be able to see the nature of mathematics through relationships and patterns so that they would be better able to teach mathematics. One participant noted that, through solving challenging problems herself, she had learned how to facilitate Grade 7 and 8 students' conceptual understanding. This dynamic view is aligned with constructivist tenets on which MATH 010 was based; that mathematics is continually growing, changing, and being revised as new problems are explored (Suurtamm & Roulet, 2007).

Summary

Upon completion of MATH 010, the 10 participants in this study had experienced, for the first time, a 12-week, reform-based mathematics course with a focus on teaching and learning. This new experience appeared to transform their beliefs as they began to acquire and use reform-based teaching methods, including collaboration and discovery learning. Their experiences also meant that they described reform ideas about learning, which involve understanding concepts. This, in turn, supported their new views that understanding mathematics would help them to be effective teachers. These differences highlight the significant shift in participants' beliefs about mathematics and its teaching as a result of the transformative power of the course. In the next section of Chapter 5, elements of the course experience that led to this transformation will be discussed.

The Course Experience

Recent literature describes several aspects of teacher education programs that play a role in the transformation of prospective teachers' beliefs and knowledge about mathematics and its teaching (Koleza & Iatridou, 2006; Leonard, Newton, & Evans, 2009). For example, Koleza and Iatridou studied 106 prospective teachers enrolled in a mathematics problem-solving course. Data from pre- and post-course surveys suggest that collaboration and a focus on pragmatic problems were key factors in changing students' beliefs about mathematics. Similarly, Leonard, Netwon, and Evans investigated the change in 25 prospective teachers' mathematical and pedagogical content knowledge after they were enrolled in a semester-long, reform-based mathematics methods course. Data from quantitative questionnaires administered at the beginning and end of the course showed that the reform-based methods were influential in improving prospective teachers' content knowledge and pedagogical content knowledge. For example, facilitating discovery learning allowed participants to explore problem-solving, and subsequently, they described an increased understanding of geometry.

MATH 010 brought together a combination of many of the key elements necessary to expand participants' views about mathematics teaching. Specifically, MATH 010 combined rich and authentic course work with a practicum experience. Students prepared for the practicum through in-class sessions, during which they considered many aspects of explaining and solving a variety of geometry problems. Concurrently, they team-taught this material to a small group of Grade 7 and 8 students. Participants were given opportunities to reflect on their teaching and learning experiences, both individually and in collaboration with their classmates and their instructor. The course experience appears to have been very influential in changing participants' beliefs about the nature of mathematics and its teaching.

For the purposes of this study, data was collected using pre- and post-course questionnaires and individual interviews. Thematic analysis of the data revealed five main themes: (a) building community, (b) an inquiry-approach to mathematics, (c) mentoring by an expert instructor, (d) safe opportunities for practice, and (e) meaningful reflection. In the next section of this chapter, these themes will be examined through the lens of the body of literature that examines the transformative power of various elements of teacher education programs.

Building Community

Wenger (1998) posited that communal interactions have three main components: (a) Mutual engagement: building collaborative relationships (b) Joint enterprise: the shared domain of the community, and (c) Shared repertoire: a set of resources developed by the community. Wenger described this kind of learning as the social construction of knowledge. He argued that these characteristics best foster a community of learners and that "the learning that is most transformative turns out to be the learning that involves memberships in . . . communities" (p. 6). These characteristics of Wenger's definition of a community of learners were mirrored in the MATH 010 experience.

Dr. Jonker created multiple opportunities for mutual engagement. For example, students completed challenging *StepAhead* problems in teams. Participants suggested that working through these problems together allowed the presentation of alternative ideas and allowed formulations to be challenged. John-Steiner (2000) suggested that "generative ideas emerge from joint thinking, from significant conversations, and from sustained, shared struggles to achieve new insights by partners in thought" (p. 3). It appeared that, for most MATH 010 candidates, solving novel problems in cooperation with peers was a new and challenging, but positive, experience. Participants' common struggles and shared successes appear to have been key factors in the formation of collaborative relationships.

Students also completed their practicum in teams of three or four. This created a shared domain, or joint enterprise, within the MATH 010 community. The data support the suggestion that participants engaged in the common goal of improving their instructional skills, in light of their statements about increased confidence in their teaching abilities. Participants described many learning experiences during their classroom practice, including how best to explain concepts and ask probing questions. The data suggest that the participants seemed to develop a set of tools for effectively teaching and learning mathematics, a shared repertoire of strategies created and shared by the MATH 010 community. This set of tools was described by participants as including: (a) discussion to challenge ideas about teaching and learning, and (b) collaboration to generate new ideas within a community of learners. Working collaboratively allowed the participants to transform their vision of the classroom from an image of students working in isolation to solve routine problems to a more reform-based view of teaching and learning that included teams of students working as a community of learners to solve rich and appropriately demanding problems. Preservice teachers who were members of a community of learners as students have been shown to develop instructional strategies to promote communal interactions in their inservice teaching years (Dinsmore & Wenger, 2006). This suggests that the effective learning strategies experienced and practiced as preservice teachers can be implemented successfully as instructional strategies during their inservice teaching years.

An Inquiry-Approach to Mathematics

Inquiry-oriented mathematics educators engage students actively in classroom practices in order to facilitate students' conceptual development (Thompson, 1992; Wood, Cobb, & Yackel, 1991). The teacher supports and guides the learning process, rather than transmitting knowledge, and acts as a facilitator (NCTM, 2000). The structure of MATH 010 reflects an inquiry-based approach to mathematics education. For example, one problem that was posed to participants from the *StepAhead* curriculum described a room of known dimensions that contained a spider and a fly. The problem can be solved using multiple methods and was designed to ensure that students who used such problems from the *StepAhead* resource would have the opportunity to put in a sustained effort at solving a problem because solutions were neither obvious nor formulaic.

StepAhead problems required a conceptual understanding of mathematics but also required students to use procedural knowledge when necessary. For example, in the problem of the spider and the fly, participants needed procedural knowledge to compute area, distance, or volume, given the dimensions of the room. Participants also had to consider various possible solutions by understanding the relationship between area or volume and maximum distance (conceptual understanding). Throughout this process, participants contended that Dr. Jonker never revealed the correct answers but provided support through probing questions and other facilitative strategies such as discussion and collaboration.

Miller (2007) described this kind of curriculum as transformative. A transformative curriculum teaches learners how to apply their knowledge in real-life contexts. Through exploration and by making connections among various domains of knowledge, the student is thought to "experience" the curriculum. By making connections among topics in geometry, participants were able to re-learn some of the mathematics content that they had previously been taught. They experienced familiar content in a new context through problems that had inherent interest because they were unique, novel, and engaging. Participants were able to apply their knowledge in a real-life teaching context, which forced them to consider the fact that their algorithmic mastery was not enough. This kind of curriculum allows knowledge to be created from interpretations in light of new experiences, for example, solving challenging mathematics problems (Mezirow, 1996). Participants experienced the *StepAhead* curriculum as a learner and then applied their knowledge to the practicum as a teacher, an opportunity to learn that had the potential for being transformative (Miller, 2007).

Mentoring by an Expert Instructor

The data from the current study substantiates the research that one teacher can have a powerful influence on student learning (Darling-Hammond, 2006). During the 12-week course, Dr. Jonker was able to create a set of conditions that participants described as key factors in transforming their ideas about doing and teaching mathematics. Such a transformation is thought to take years to occur (Mezirow, 1997), yet Dr. Jonker had developed such expert teaching strategies that he was able to facilitate this infrequent transformation over a short period of time. Berliner (1986), an educational psychologist, examined empirical studies and scholarly literature in order to argue that a teacher's experience alone is not enough to develop instructional expertise. While in search of a definition of the expert pedagogue, Berliner suggested that certain characteristics are associated with such expertise: (a) a range of instructional methods from which to choose, and (b) domain-specific knowledge of teaching. Dr. Jonker had many years of experience as a mathematics educator, but, more importantly, he exemplifies Berliner's characteristics of the expert pedagogue, in his (a) teaching methods, (b) expectations for his students, (c) use of rich resources, and (d) knowledge of students and content.

Teaching methods. Dr. Jonker modelled effective mathematics teaching strategies for his students, which were aligned directly with the National Council of Teachers of Mathematics' vision of an ideal mathematics classroom (NCTM, 2000). Participants suggested that an inquiry-based classroom was created during MATH 010, which included multiple opportunities for discovery and collaboration. Dr. Jonker gave several opportunities to learn concepts from different perspectives. The Pythagorean theorem, for example, was described by Dr. Jonker as embodied knowledge because of the depth at which the theorem had been explored in the course. Students with embodied knowledge have an understanding of the theorem without conscious thought and are able to examine a concept from multiple perspectives. His teaching methods were described as a key factor in transforming participants' beliefs about the nature of mathematics from a subject that involved algorithmic solutions to a discipline that links patterns and relationships.

Participants suggested that this transformation occurred because Dr. Jonker presented them with opportunities to become familiar with key mathematics ideas in geometry, including the theory of Pythagoras, in combination with developing their problem-solving skills.

The effectiveness of Dr. Jonker’s instructional methods went well beyond MATH 010. His teaching was thought to be so effective that he was awarded the first Ontario Excellence in Teaching Award distributed by the Canadian Mathematical Society. He won several teaching awards from Queen’s University, including the 1999 Alumni Teaching Award, and the Engineering Society's Golden Apple in 2000. He also won the 3M National Teaching Award which resulted in his appointment as a Teaching Chair at the Queen’s Center for Teaching and Learning (Canadian Mathematical Society, 2011).

Expectations of Students. Berliner (1986) stated that expert teachers “know their new class even before they [get] to meet them” (p. 10). Dr. Jonker had ambitious expectations for his students before he met them. He created *StepAhead* with the idea to challenge, yet support students, regardless of their background in mathematics. He expected that participants would be able to teach enriched-level problems to Grade 7 and 8 students, despite a lack of teaching experience. He knew what his students would need to know in order to be successful teachers: they need to be able to do the mathematics, reflect on the efficacy of multiple approaches and solutions, and, in turn, facilitate student engagement with the problem; all of which were abilities that participants described being capable of doing, by the end of the course. Teacher expectations have been shown to correlate with student success. In a review of expectations research, Good (1981) notes that research has consistently demonstrated positive relationships between teacher expectations and student achievement. Dr. Jonker had ambitious expectations for his MATH 010 students and, in turn, participants described competence in their ability to (a) problem solve and (b) teach mathematics.

Use of rich resources. Dr. Jonker created a mathematically rich curriculum which provided students with opportunities to learn or re-learn mathematics concepts. Given that there is no standard curriculum for mathematics teacher education in Ontario, nor is there a textbook for teaching intermediate mathematics, Dr. Jonker had limited access to appropriate, high-quality, post-secondary teaching resources. He had researched and reflected over the years about the kinds of problems that he needed to develop in order to challenge his students' thinking and beliefs about mathematics and teaching. From his knowledge, experience, and research he developed *StepAhead* to be used as a class resource for MATH 010 as well as an enrichment guide for Grade 7 and 8 students. For example, for the problem of the spider and the fly, participants were required to find the maximum distance that a fly could separate from a spider in the room. From a post-secondary perspective, an analysis of the problem might involve deriving an algorithm for maximum distance. From a Grade 7 perspective, however, the solution could involve the use of a net, or model of the three-dimensional room, in order to determine the maximum distance using a concrete approach. The fact that *StepAhead* problems are accessible and challenging to both intermediate and university students suggests that the course curriculum provided rich learning tasks for students. Participants described connecting formulae to bigger ideas, for example, connecting the theorem of Pythagoras to an understanding of maximum distance, which suggests rich learning. The *StepAhead* curriculum is based on a problem-solving, inquiry-approach to mathematics, thus is in line with the curriculum expectations for reform-based, transformative learning (Miller, 2007; NCTM, 2000).

Knowledge of students and content. Berliner (1986) stated that expert educators possess domain-specific knowledge and that knowledge is established only over long periods of time: hundreds, sometimes even thousands of hours of learning, experience, and reflection on experience are required for expertise to be developed. By the tenth year of teaching MATH 010, Dr. Jonker had an image of the general profile of students entering his course. He knew how the course needed to be structured in order to facilitate student learning. Participants' learning was reflected in the data. At the end of MATH 010, participants expressed confidence in their problem solving and teaching abilities.

Dr. Jonker had acquired domain-specific knowledge in mathematics throughout the many years he spent as a research mathematician and professor of mathematics and physics. He had also acquired expert knowledge about mathematics pedagogy. A strong focus on pedagogy would be expected of a course within a Faculty of Education, but MATH 010 was a mathematics course. This suggests that Dr. Jonker not only had a profound understanding of mathematics content but also understood the complexity of preparing undergraduate students for teaching mathematics. Dr. Jonker used his domain-specific knowledge of both students and content to implement expert teaching strategies and design a mathematics curriculum to facilitate student learning (Ball, Hill, & Bass, 2005). Berliner (1986) argued that domain-specific knowledge is the basis for transforming subject matter.

Safe Opportunities for Practice

Mezirow (1997) stated that experience transforms learning. He further suggested that "learners need practice in recognizing frames of reference and using their imaginations to redefine problems from a different perspective" (p. 10). Effective practice depends on how well the educator can create a situation that allows the learner to (a) be well informed, (b) be free of coercion, (c) be open to other perspectives, and (d) have an opportunity to assume various roles.

Dr. Jonker designed the course so that students gained first-hand experience in class that could be modified for lessons with small groups of Grade 7 and 8 students. After students had become familiar with the topics during in-class sessions, they led problem solving enrichment lessons with a small group (three to eight) of able Grade 7 and 8 students. The Grade 7 and 8 students were in the higher range of mathematical ability for their class, which meant that they were expected to do well with the new topics.

During the practicum, MATH 010 students stated that they were free to experiment with teaching approaches. This was because there was little risk of being evaluated by an associate teacher because they had no formal assessment during their practicum experience. The only form of assessment for the practicum component of MATH 010 was self-assessment, which was recorded in journals. In that sense, Dr. Jonker appeared to create an opportunity for his students to gain experience teaching mathematics in which they were relatively safe from failure. A traditional teaching practicum within a

Faculty of Education would be evaluated by an experienced teacher and lessons would be conducted within a regular classroom of 25–30 students, who have a wide-range of mathematical abilities, attitudes, and dispositions. A teacher candidate completing such a practicum runs the risk of disapproval by an experienced teacher who may be unsupportive of the non-standard teaching methods modelled and emulated throughout MATH 010. The MATH 010 practicum was free from assessment by an associate teacher which allowed students to experiment with various strategies. Students were able to assume the role of learner and teacher throughout the course which allowed them to experience different perspectives of mathematics teaching. Mezirow (1997) argued that this kind of experience is central for making meaning and "necessary to validate what and how one understands, or to arrive at a best judgment regarding a belief" (p. 10).

Meaningful Reflection

Mezirow (1997) stated that in order to "facilitate transformative learning, educators must help learners become aware and critical of their own and others' assumptions" (p. 10). Transformation takes place through critical reflection upon beliefs and points of view. Cranton (2002) further described that critical reflection involves questioning and examining assumptions "in terms of where they came from, the consequences of holding them, and why they are important" (p. 66). One can learn to become critically reflective when involved in communicative learning. Self-reflection can also lead to significant personal transformations (Mezirow, 1997). One of the major challenges for teacher education is to provide opportunities for the development of the skills for critical reflection on practicum experiences and linking those experiences to what was learned in education courses (Mezirow, 1995).

Schön (1987), a philosopher, wrote that reflection is intimately bound with action and involves simultaneous reflecting and doing. Schön argued that reflection can happen i*n* action or *on* action. Reflection-in-action means that an individual incorporates lessons learned at the time of an experience, building a new understanding to inform his actions. Reflection-on-action happens after the experience when an individual incorporates lessons learned.

Dr. Jonker created many opportunities for reflection-on-action. For example, during in-class sessions, MATH 010 students were able to discuss their practicum teaching each week. The practicum experience allowed participants to reflect based on their interpretation of the Grade 7 and 8 students' engagement in class and grasp of the concepts that were introduced. They were also required to reflect on their practicum experience in weekly journals, which were read by Dr. Jonker and assessed only on completion of written responses to assigned questions (Appendix A). Although the journals were not used as data for this study, the data collected suggest that participants valued the reflective component of the journals. One participant noted that she likely would not have reflected on her teaching if not for the journals.

Participants' transformed beliefs about the nature of teaching mathematics also suggest that MATH 010 created opportunities for reflection-in-action. Schön described that reflection-in-action is a response to an unexpected outcome when an individual is posed with a challenging problem. For example, participants described that, through reflection, they were able to reconsider their views about traditional teaching methods because they saw the effectiveness of reform-based instructional strategies. Through the process of reflection, they described new ideas about teaching mathematics because they had experienced challenges while teaching mathematics. The development of critical reflection was possible because Dr. Jonker created an opportunity for participants to experience teaching and reconsider their beliefs about mathematics teaching.

Summary

MATH 010 created numerous opportunities for students to transform their beliefs about mathematics and its teaching during the 12-week course. The course experience described by the 10 participants in this study suggests that one teacher can make a difference in a relatively short period of time. The course was designed by an expert, award-winning instructor whose inquiry-approach to mathematics provided students with a profound example of exemplary reform-based mathematics teaching. The instructor created important opportunities for practice and collaboration. He was then able to support the development of the skills necessary for critical reflection by closely linking the practicum experience to course work. In the next section of Chapter 5, the closing remarks are presented.

Closing Remarks

This study investigated the ways in which prospective teachers perceived a change to their personal and professional beliefs about the nature of mathematics and what it means to learn and teach mathematics after the experience of a university mathematics course. This study substantiates research that one teacher can make a difference to the transformation of student learning. In a short period of time, the participants in this study had a meaningful mathematics education experience because of the specifically designed elements of MATH 010 created by an expert mathematics instructor.

The discussion of the results revealed significant contributions to preservice teacher mathematics education. It is possible for preservice teachers to experience a change to their beliefs about mathematics and its teaching. A transformative experience allowed the 10 participants in this study to undergo a change to their beliefs in response to a 12-week course. It is possible for students to acknowledge changes to their thinking about instruction and to the strategies they employ for instruction over a relatively short period of time; however, specific conditions must be in place to effect a transformation. The key elements of the transformative learning experience in this study included: (a) an expert instructor, (b) creating a community of learners, (c) active and engaging instruction, and (d) a curriculum composed of carefully constructed problems. The demonstration of reform-based mathematics teaching in MATH 010 allowed participants to learn new and more effective instructional methods.

Limitations

This study took the form of a case study: a group of 10 students enrolled in a course for future elementary mathematics educators and the course instructor. The results of this study can be used to inform teacher education programs and to direct future research, but cannot be used to draw general conclusions about cause and effect of various course elements on student learning in intermediate mathematics courses for prospective teachers. The researcher did not observe the 10 participants during university classes or during the practicum and did not have access to participants' journals. The impact that the 10 participants had on Grade 7 and 8 students was also not measured. Participant descriptions about their experiences, however, were provided as a rich narrative.

Other possible limitations of the study include the duration of the study. The study took place over a four-month semester, which was a short period of time to examine a change in content knowledge about multiple topics in mathematics. Had it been known to the researcher that geometry would be the focus of MATH 010 for the 2010 semester, content questions on Questionnaire 1 could have been focused solely on geometry in order to better examine changes to participants' content knowledge of geometry over the semester.

Finally, Questionnaire 2 was anonymous, which did not allow for individual correlations to be drawn about participants' beliefs or content knowledge from the beginning of the semester to the end. Data was collected about the 10 participants as an illuminative group instead.

Implications for Future Research

The findings of this study suggest several avenues for future research. First, it would be fruitful to conduct research that follows a class of MATH 010 students through several years of professional practice. The purpose of such a longitudinal study would be to investigate whether or not students who adopted the new teaching methods learned in a course, such as MATH 010, would continue to use these strategies over time in their teaching career.

Second, further research to examine the effects of prospective teacher instructional methods on students' learning is needed. For example, research conducted in the Grade 7 and 8 classrooms of MATH 010 prospective teachers would be beneficial in order to examine whether prospective teachers' beliefs about teaching mathematics are evident in their practice.

Third, research is needed to examine whether or not the successful elements of MATH 010 could be replicated with as much effect if a different instructor were to conduct the class. This research would illuminate specific course elements needed in order to recreate the transformative learning experience.

Further research is also needed about the characteristics of an expert instructor. This research would highlight various instructor qualities needed to create a transformative experience for prospective teachers.

Final Thoughts

The findings of the thesis and the implications for further research show that the present study makes a clear contribution to emerging work on preservice mathematics teacher education. This research has had a profound impact on my teaching philosophies. I embarked on this journey in order to explore my own beliefs and knowledge about mathematics teaching. Throughout this journey, I have gained practical knowledge about research-based strategies that correlate with successful classroom practice. MATH 010 has shown me how effective it can be to guide student learning, rather than transmit information, and how important collaboration is for the generation of ideas. This experience has encouraged me to implement rich and novel problems in my classroom and challenge students to have an in-depth understanding of mathematics. MATH 010 will not soon be forgotten and I will strive to create a community of learners in my own classroom throughout the remainder of my career.

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APPENDIX A: *STEPAHEAD* ENRICHMENT LESSON

Enrichment Activity: Measuring the Thickness of Toilet Paper

Lesson Goals

• To present students with a very nice and fun problem that involves the calculation of areas and circumferences of circles

• To challenge students to think hard about how they can apply what they have learned to this non-standard problem

Materials

• copies of Spot Question in Appendix C.15 for each student

• rulers and calculators for each student

• rolls of toilet paper for each group of 3-5 students

Problem Statement

Given a roll of toilet paper with 200 sheets, measure the thickness of the paper. [Note: The rolls you bring may have more or less than 200 sheets - this information is usually given on the plastic wrap in which the toilet paper is sold. Use the actual number to make the problem real for the students.]

Lesson Sequence

1. A nice way to begin this session is with a Spot Question that gets students thinking about the area and/or circumference of a circle. A good example is the Spot Question in Appendix C.15, which asks students to calculate the amount of rope required to wrap around a cylinder a given number of times.

2. After the Spot Question (or other opening activity), divide the class into groups of 3-5 students and give each group a roll of toilet paper. State the problem above, being sure to give them the number of sheets. Here, we will suppose that the toilet paper rolls have 200 sheets. The students will need rulers and calculators. Allow the students to unroll a bit of the paper so that they can measure the length of one sheet of toilet paper. Suppose it comes to 11.1 centimeters.

3. Allow the students to struggle with the problem for as long as possible. If they get frustrated, give them some hints. There are at least two ways to solve the problem: one using area and the other, circumference. Some students may even choose to look at volume (this method is similar to that of area). You will have to decide which method will work best for each group, depending on the work they have done already. Some groups may be completely stuck, in which case, you might suggest that they get started by computing some areas, as outlined in the area method. Try not to give away the solution all at once though. The students need to feel as if they are the ones solving the problem.

4. Two possible solutions are given next. They are presented in sequential steps, which can be used to guide students along. The calculations are based on an actual roll of toilet paper.

APPENDIX B: MATH 010 COURSE OUTLINE

Marking Scheme

Class participation	15%
Teaching Journals	15%
Homework assignments	55%
Final Exam	15%

Expectations on students

- attend classes and participate
- do weekly homework question
- provide a report on every enrichment class (may be done jointly with partner) - this is the teaching journal. This report should indicate what you covered, how you approached it, and how it went. It should also relate the class (briefly) to the Ontario Mathematics Curriculum for grades 7 and 8
- At the end of the term prepare a final summary of the material covered in the enrichment classes, to go to the elementary school.
- write final exam

Teaching Journals

- What topics did you discuss in the class?
- How did you present the topics?
- What techniques did you use to engage the students?
- What are the strands that are reinforced by this material (see Ontario Math Curriculum[1])?
- What sort of discussions and questions took place?
- Did the students understand the material?
- How was the class participation?
- Did you have discipline problems, and, if so, how did you handle them?
- What would you do differently if you could do the session over?

APPENDIX C: LETTER OF INFORMATION FOR COURSE INSTRUCTOR

Faculty of Education, 511 Union Street, Kingston, Ontario, Canada, K7M 5R7

Examining a Preservice Mathematics Education Course and its Effects on Candidates' Beliefs About Mathematics and Teaching and Knowledge of Mathematics Content and Pedagogy

LETTER OF INFORMATION (for course instructor)

2010 09 01

Dear Dr. Jonker,

I am inviting you to allow a research study, entitled *Examining a Preservice Mathematics Education Course and its Effects on Candidates Mathematical Beliefs and Knowledge*, to be conducted in your class, Math 010. The study was granted clearance by the General Research Ethics Board for compliance with the Tri-Council Policy Statement: Ethical Conduct of Research Involving Humans, and Queen's policies.

The aim of this letter is twofold. First, it will describe the purpose and method of the research study. Second, it will request that you agree, in writing, to allow research to be conducted in your class.

This study is investigating the beliefs and knowledge of university students throughout a semester in Math 010. More specifically, the research will investigate (a) perservice teachers' beliefs about the nature of mathematics and mathematics teaching before the course experience (b) prospective teachers' perceptions about their beliefs in regards to the nature of mathematics and mathematics teaching in response to the course experience (c) what aspects of the course that prospective teachers attribute change to their beliefs about mathematics and mathematics instruction to.

The research method in this study requires potential participants in your class to complete two online questionnaires about mathematics beliefs and knowledge: one at the beginning of the course and one at the end of the semester. As well, weekly teaching journals will be photocopied. At the end of the course, 4-6 students will be approached to participate in a one-on-one interview. All data will be collected by the researcher.

Your permission involves the following:

1. Allowing access to students enrolled in Math 010, through email.
2. Providing access to weekly teaching journals. Weekly teaching journals that are required for course completion will be photocopied upon each weekly submission and returned to the course instructor.

Granting access to your students in Math 010 involves no more risk than your normal activities. There are no known physical, psychological, economic or social risks to you associated with agreeing to this research. Agreement on your part in no way obligates you to remain a part of the study. Agreement is voluntary, and you may choose to withdraw your class from the study at any time. Should you choose to withdraw you may request that all or part of the data be destroyed. I intend to publish the findings of the study in professional journals and report them at conferences. Confidentiality will be maintained to the extent possible. At no time will your actual identity be disclosed. You will be assigned a code name; any reference in publications to you or to what you say at any time during the study will be to the code name only. Note that data will not be used for secondary analysis.

Any questions about study participation may be directed to Gina Kerr at gina.kerr@queensu.ca or my supervisor, Dr. Lynda Colgan, at 613.533.2000 ext 75553 or lynda.colgan@queensu.ca. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at 613-533-6081 or chair.GREB@queensu.ca

Please indicate your decision to participate in the study by signing one copy of the *Consent Form* and returning it to me in the enclosed stamped, self-addressed envelope. Retain the second copy for your records.

Yours sincerely,

Gina Kerr

Master of Education Candidate

Attachments: Two (2) copies of the *Consent Form*

APPENDIX D: CONSENT FORM FOR COURSE INSTRUCTOR

CONSENT FORM (for course instructor)

Examining a Prospective Mathematics Education Course and its Effects on Candidates' Beliefs About Mathematics and Teaching and Knowledge of Mathematics Content and Pedagogy

- I agree to allow Gina Kerr access to Math 010 in order to conduct the study entitled *Examining a Prospective Mathematics Education Course and its Effects on Candidates Mathematical Beliefs and Knowledge,* conducted through the Faculty of Education at Queen's University.
- I have read and retained the *Letter of Information* and the purpose of the study is explained to my satisfaction.
- I have had any questions answered to my satisfaction.
- I understand my giving my permission means that class participants may participate in: (a) completion of two online questionnaires that will require approximately 60 minutes each; (b) a semi-structured interview which will be audio-recorded and will last approximately 60 minutes and be held at a time and in a place convenient to individual participants (c) submission of weekly teaching journals that will be photocopied and returned weekly.
- I understand that my participation is voluntary and that I am free to withdraw my class from the study at any time without negative consequences. Should I decide to withdraw my class from the study I may request that all data associated with my participation is destroyed.
- I understand that the researcher intends to publish the findings of this study.
- I understand that a copy of each publication resulting from the research will be mailed to me.
- I have read and signed one copy of the attached *Consent Form*. I have returned the signed copy and retained one copy for my records.
- I am aware that any questions about study participation may be directed to Gina Kerr at gina.kerr@queensu.ca or my supervisor, Dr. Lynda Colgan, at 613.533.2000 ext. 75553 or Lynda.colgan@queensu.ca. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at 613-533-6081 or chair.GREB@queensu.ca

Please sign one copy of this Letter of Consent and return to Gina Kerr prior to the first week of class. Retain the second copy for your records.

I HAVE READ AND UNDERSTOOD THIS *CONSENT FORM* AND I AGREE TO PARTICIPATE IN THE STUDY.

Full name (Please Print): ______________________________

Signature: ______________________________ Date: ______________________

APPENDIX E: RECRUITMENT EMAIL FOR PROSPECTIVE TEACHERS

My name is Gina Kerr and I am a Master's student at the Faculty of Education at Queen's University. I am writing to invite you to participate in a research study that I am conducting. This research will contribute to my Master's thesis, under the supervision of Dr. Lynda Colgan. The research you are invited to participate in will be an idividual interview and two questionnaires on the topic of prospective teacher knowledge and beliefs in the field of mathematics, as well as access to weekly self-reflection journals. The study was granted clearance by the General Research Ethics Board for compliance with the "Tri-Council Policy Statement: Ethical Conduct of Research Involving Humans," and Queen's policies.

The purpose of this study is to examine the effect of Math 010 on prospective teachers' beliefs and knowledge. To do this, I am planning to conduct a questionnaire at the beginning and end of the semester. In addition, I will also be examining your teaching journals, as required by the course instructor. Individual interviews with 4-6 participants will be conducted, following the completion of the final questionnaire. The interview will allow you to discuss in detail questions from the questionnaire.

The questionnaire will be completed online in the first and last week of the semester and will take approximately 60 minutes to complete. The interview will be conducted at a time, and in a private location on the school campus that is convenient for individual participants. The interview will be approximately 45 minutes in length and will be audio recorded.

I intend to publish the findings of the study in professional journals and report them at conferences. At no time will the actual identity of the participants be disclosed, and confidentiality is guaranteed to the extent possible. If at this point, or any point in the future, you have any questions about this research study, you should feel free to contact me, Gina Kerr, M.Ed. student, Faculty of Education, Queen's University (gina.kerr@queensu.ca, 613-583-4462) or my supervisor, Dr. Lynda Colgan (Lynda.colgan@queensu.ca 613-533-2000 ext 75553).

If you would like to volunteer to participate in an individual interview for this study, please respond by e-mail to Gina Kerr at gina.kerr@queensu.ca at your earliest convenience. If you decide to volunteer and respond by e-mail, you will then receive an additional e-mail with further details about the research as well as a Consent form.

Sincerely,   Gina Kerr

APPENDIX F: LETTER OF INFORMATION FOR PROSPECTIVE TEACHERS

Faculty of Education, 511 Union Street, Kingston, Ontario, Canada, K7M 5R7

Examining a Preservice Mathematics Education Course and its Effects on Candidates' Beliefs About Mathematics and Teaching and Knowledge of Mathematics Content and Pedagogy

LETTER OF INFORMATION (for prospective teacher)

2010 09 01

Dear Student,

I am inviting you to participate in a research study, entitled *Examining a Preservice Mathematics Education Course and its Effects on Candidates Mathematical Beliefs and Knowledge***,** being conducted at Queen's University. The research has the support of Dr. Leo Jonker, Math 010 course instructor and member of the Mathematics Department at Queen's University. Also, the study was granted clearance by the General Research Ethics Board for compliance with the Tri-Council Policy Statement: Ethical Conduct of Research Involving Humans, and Queen's policies.

The aim of this letter is twofold. First, it will describe the purpose and method of the research study. Second, it will request that you agree, in writing, to participate in the study.

This study is investigating the beliefs and knowledge of university students throughout a semester in Math 010. More specifically, the research will investigate (a) perservice teachers' beliefs about the nature of mathematics and mathematics teaching before the course experience (b) prospective teachers' perceptions about their beliefs in regards to the nature of mathematics and mathematics teaching in response to the course experience (c) what aspects of the course that prospective teachers attribute change to their beliefs about mathematics and mathematics instruction to.

Your participation would involve the following:

1. Completion of two questionnaires that explore beliefs and knowledge in the field of mathematics. These questionnaires will require approximately 60

minutes to complete online, once in the first week of the semester and once in the last week of the semester.

2. Access to weekly teaching journals. Weekly teaching journals that are required for course completion will be photocopied upon each weekly submission and returned to the course instructor.
3. Four to six participants will be invited to take part in one-on-one semi-structured interviews. The interview, which will be audio-recorded, will last approximately 60 minutes and be held at a time and in a place convenient to you.

Participating in the study involves no more risk than your normal activities. There are no known physical, psychological, economic or social risks to you associated with participation in this research. Agreement on your part in no way obligates you to remain a part of the study. Participation is voluntary, and you may choose to withdraw from the study at any time. Should you choose to withdraw you may request that all or part of your data be destroyed. Please note that if you chose to decline participation in this study this will not affect your Math 010 coursework or participation in classroom activities outlined in the course outline. Students who withdraw from the study will also not be penalized in the Math 010 course.

I intend to publish the findings of the study in professional journals and report them at conferences. Confidentiality will be maintained to the extent possible. At no time will your actual identity be disclosed. You will be assigned a code name; any reference in publications to you or to what you say at any time during the study will be to the code name only. Note that data will not be used for secondary analysis.

Any questions about study participation may be directed to Gina Kerr at gina.kerr@queensu.ca or my supervisor, Dr. Lynda Colgan, at 613.533.2000 ext 75553 or lynda.colgan@queensu.ca. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at 613-533-6081 or chair.GREB@queensu.ca

Please indicate your decision to participate in the study by signing one copy of the *Consent Form* and returning it to me in the enclosed stamped, self-addressed envelope. Retain the second copy for your records.

Yours sincerely,

Gina Kerr

Master of Education Candidate

Attachments: Two (2) copies of the *Consent Form*

APPENDIX G: CONSENT FORM FOR PROSPECTIVE TEACHERS

CONSENT FORM (for prospective teacher)

Examining a Prospective Mathematics Education Course and its Effects on Candidates' Beliefs About Mathematics and Teaching and Knowledge of Mathematics Content and Pedagogy

- I agree to participate in the study entitled *Examining a Prospective Mathematics Education Course and its Effects on Candidates Mathematical Beliefs and Knowledge,* directed by Gina Kerr and conducted through the Faculty of Education at Queen's University.

- I have read and retained the *Letter of Information* and the purpose of the study is explained to my satisfaction.

- I have had any questions answered to my satisfaction.

- I understand my participation involves: (a) completion of two online questionnaires that will require approximately 60 minutes each; (b) a semi-structured interview which will be audio-recorded and will last approximately 60 minutes and be held at a time and in a place convenient to individual participants (c) submission of weekly teaching journals that will be photocopied and returned weekly.

- I give my consent to have the interview audio-recorded.

- I understand that only the researcher and thesis committee will have access to data and that my name will be coded to maintain confidentiality to the extent possible.

- I understand that my participation is voluntary and that I am free to withdraw from the study at any time without negative consequences. Should I decide to withdraw from the study I may request that all data associated with my participation is destroyed.

- I understand that the researcher intends to publish the findings of this study.

- I understand that a copy of each publication resulting from the research will be mailed to me.

- I have read and signed one copy of the attached *Consent Form*. I have returned the signed copy and retained one copy for my records.

- I am aware that any questions about study participation may be directed to Gina Kerr at gina.kerr@queensu.ca or my supervisor, Dr. Lynda Colgan, at 613.533.2000 ext. 75553 or Lynda.colgan@queensu.ca. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at 613-533-6081 or chair.GREB@queensu.ca

Please sign one copy of this Letter of Consent and return to Gina Kerr within the first week of class. Retain the second copy for your records.

I HAVE READ AND UNDERSTOOD THIS *CONSENT FORM* AND I AGREE TO PARTICIPATE IN THE STUDY.

Full name (Please Print): ____

Signature: _____________________________________ Date: ___________________________

APPENDIX H: QUESTIONNAIRE 1 (PRE-COURSE) QUESTIONS

Introduction

Please complete all questions. There are nine sections. I really appreciate your time and effort in completing this questionnaire. It will take approximately one hour. You may complete this questionnaire at different intervals. Please remember to save your sections.

Part 1

Name:

Email:

Phone Number:

1. Degree program in which you are currently enrolled:
2. Course Code/Name of mathematics classes taken at the university or college level (for international students please specify mathematics streams that you have taken):
3. Previous Degrees and/or diplomas earned:
4. Course Code/Name of mathematics classes taken at the high school level (for international students please specify mathematics streams that you have taken):
5. Have you ever been involved in a mathematics club? If so, please describe.
6. Have you ever participated in a mathematics contest? If so, what was it called? (ex. University of Waterloo, Descartes, Pascal, Ontario Association of Mathematics Education)
7. Do you have any experience teaching mathematics? (ex. tutoring, coaching a mathematics team)
8. Why did you take Math 010?
9. What type of employment will you be seeking after graduation?

10. Do you ever do mathematics problems for fun? If so, what kind? (ex. Sudoku, Magic Squares, puzzles)

11. Have you ever attended a mathematics conference? If so, which one(s)?

<u>Part 2</u>

Beliefs about the nature of mathematics

12. To what extent do you agree or disagree with the following beliefs about the nature of mathematics? (strongly agree, agree, slightly agree, slightly disagree, disagree, strongly disagree).

A. Mathematics is a collection of rules and procedures that prescribe how to solve a problem.
B. Mathematics involves remembering and applying definitions, formulas, mathematical facts and procedures.
C. Mathematics involves creativity and new ideas.
D. In mathematics, one can discover and try out many things by oneself.
E. When solving mathematical tasks you need to know the correct procedure or else you will be lost.
F. If you engage in mathematical tasks, you can discover new things.
G. Mathematical problems can be solved in many ways.
H. Mathematics helps to solve everyday problems and tasks.
I. To do mathematics requires much practice, and the correct application of routines and problem solving strategies.
J. Mathematics means learning, remembering and applying.

Beliefs about the nature of teaching mathematics

13. To what extent do you agree or disagree with the following beliefs about teaching mathematics at the **intermediate (Grade 7-9) level**? (strongly agree, agree, slightly agree, slightly disagree, disagree, strongly disagree).

A. In the intermediate division, teachers should spend time in math classes improving students' problem solving skills.
B. Teaching problem solving skills in mathematics classes contributes to developing students' life-long learning.

C. It is difficult to address mathematics curriculum expectations and integrate problem solving components in the regular mathematics program.
D. Teaching mathematics is only about explaining mathematics rules, principles, and procedures.
E. Teaching students to defend their solution process in their own words is an important part of teaching mathematics.
F. Teaching mathematics is about showing the connections between mathematics and other areas of knowledge.
G. Explaining mathematical vocabularies is a crucial element in teaching mathematics.
H. Problem solving should be taught in areas other than mathematics.
I. Focusing on the problem solving process tends to reduce mastery of computations in teaching mathematics.

Beliefs about the nature of learning mathematics

14. To what extent do you agree or disagree with the following beliefs about learning mathematics? (strongly agree, agree, slightly agree, slightly disagree, disagree, strongly disagree).

A. The best way to do well in mathematics is to memorize all the formulas.
B. Students need to be taught exact procedures for solving mathematical problems.
C. To be good in mathematics you must be able to solve problems quickly.
D. Students learn mathematics best by paying attention to the teacher's explanations.
E. When students are solving mathematics problems, more emphasis should be put on getting the correct answer than on the process followed.
F. In addition to getting the right answer in mathematics, it is important to understand why the answer is correct.
G. Teachers should allow students to figure out their own ways to solve mathematical problems.
H. Time used to investigate why a solution to a mathematical problem works is time well spent.
I. It is helpful for students to discuss different ways to solve particular problems.

Beliefs about preparedness to teach mathematics

15. Please indicate the extent to which you think your education thus far has prepared you to do the following for a possible career in teaching: (Not at all, A minor extent, A moderate extent, A major extent)

A. Communicate ideas and information about learning mathematics clearly to students.
B. Set up mathematics learning activities to help students achieve learning goals.
C. Use questions to promote higher order thinking in mathematics.
D. Use computers to aid in teaching mathematics.
E. Establish a supportive environment for learning mathematics.
F. Use assessment to give effective feedback to students about their mathematics learning.
G. Provide parents with useful information about their children's' progress in mathematics.
H. Incorporate effective classroom management strategies into your teaching of mathematics.
I. Have a positive influence on difficult or unmotivated students.
J. Work collaboratively with other teachers.

Beliefs about Self-Efficacy

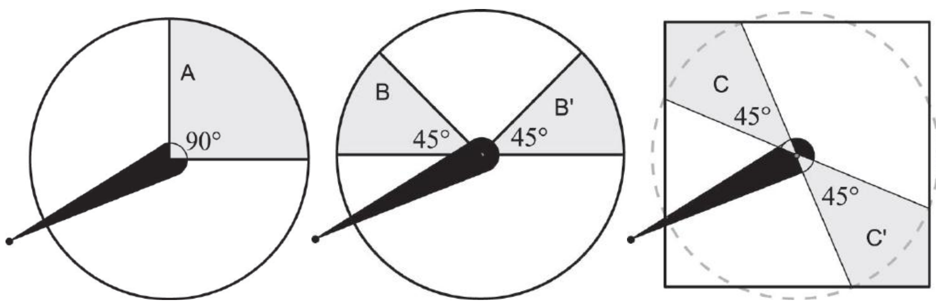
16. To what extent do you agree or disagree with the following beliefs about your self-efficacy? (strongly agree, agree, slightly agree, slightly disagree, disagree, strongly disagree).

A. I have all the types of knowledge needed to be an excellent mathematics teacher.
B. My educational and experiential background has prepared me to be an effective mathematics teacher.
C. I am confident that I can make mathematics interesting and relevant for students.
D. I believe I can gain the respect of my students, their parents, and my colleagues.
E. I believe I am highly employable as a teacher.
F. I believe I have the teaching skills necessary to be an effective mathematics teacher.

G. I believe I can aid in **all** students learning mathematics.
H. I believe I can effectively handle any situation that may arise in the mathematics classroom.

Pedagogical Content Knowledge

17. Your students are examining the three spinners shown below. They are discussing the probability that the spinner stops over a shaded region. Please indicate whether the following statements of four students are Completely True, Partly True or Completely False. If one sentence is true, but the other is false, check Partly True.



Spinner 1 Spinner 2 Spinner 3

a. Sherry says, "The probability is twice as large for Spinners 2 and 3 compared to Spinner 1 because they have two regions to stop and Spinner 1 only has one region."
b. George says, "Spinners 1 and 2 have the same probability since the shaded regions have the same area. Spinner 3, however, has a higher probability than Spinner 2 because the shaded region is a larger area."
c. Paul says, "Spinners 1, 2, and 3 have the same probability because the angles of the shaded regions are the same size."
d. Rainey says, "The probabilities for Spinners 1 and 2 are the same because those areas are the same proportion of the whole circle. For spinners 2 and 3, however, the probabilities are different because the shaded areas for Spinner 3 are a bigger proportion of the whole square than they are of the circle."

18. Mr. Fitzgerald has been helping his students learn how to compare decimals. He is trying to devise an assignment that shows him whether his students know how

to correctly put a list of decimals in order of size. Which of the following sets of numbers will best suit that purpose?

a. .5 7 .01 11.4
b. .60 2.53 3.14 .45
c. .6 4.25 .565 2.5
d. Any of these would work well for this purpose. They all require students to read and interpret decimals.

19. Which of the following problems could be used to illustrate 1 ¼ divided by 1/2? (Mark YES, NO, or I'M NOT SURE for each possibility).

a. You want to split 1 ¼ pies evenly between two families. How much should each family get?
b. You have $1.25 and may soon double your money. How much money would you end up with?
c. You are making some homemade taffy and the recipe calls for 1 ¼ cups of butter. How many sticks of butter (each stick = 1/2 cup) will you need?

20. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in-class. Todd`s work looked like this:

$$\begin{array}{r} 983 \\ \underline{\times\ 6} \\ 488 \\ \underline{+5410} \\ 5898 \end{array}$$

What is Todd doing here? (Mark one answer)

a. Todd is regrouping (carrying) tens and ones, but his work does not record the regrouping.
b. Todd is using the traditional multiplication algorithm but working from left to right.
c. Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.
d. Todd is not doing anything systematic. He just got lucky- what he has done here will not work in most cases.

21. Ms. Abdul is preparing a unit to introduce her students to proportional reasoning. She is considering three versions of a problem that are the same except for the numbers used. Which version of the Mr. Short and Mr. Tall problem below is likely to be the most challenging for students?

a. A picture depicts Mr. Short's height as 4 paper clips and as 6 buttons. The height of Mr. Tall (not shown) is given as 6 paper clips. How many buttons in height is Mr. Tall?

b. A picture depicts Mr. Shorts's height as 4 paper clips and 7 buttons. The height of Mr. Tall (not shown) is given as 5 paper clips. How many buttons in height is Mr. Tall?

c. A picture depicts Mr. Short's height as 2 paper clips and as 9 buttons. The height of Mr. Tall (not shown) is given as 5 paper clips. How many buttons in height is Mr. Tall?

d. All three problems are equally challenging.

22. Ms. Donaldson's class was working on an assignment in which they had to determine the measures of unknown angles in triangles. One student consistently found the measures of unknown angles in right triangles by subtracting the known angle from 90. Ms. Donaldson was concerned that the student might run into difficulty when trying to find the measures of unknown angles in more general triangles. Which of the following questions would be best to ask the student in order to help clarify this issue:

a. What do you get when you add 90 + 62 + 28?

b. Why does subtracting 62 from 90 give you the measure of the unknown angle?

c. How could you find the missing angle in an isosceles triangle?

d. How did you know that this was a right triangle?

e. What if this angle measured 17° instead of 62°?

Mathematics Content Knowledge

23. James wants to run the perimeter of a playing field, the dimensions of which are 50 m by 9000 cm. What distance will James cover by running once around this field?

    a. 280 cm
    b. 280 m
    c. 18 000 cm
    d. 18 100 m

24. The weekly salary for a part-time job selling shoes is calculated using the formula Salary = 5b + y/15, where b represents the number of hours worked and y represents the dollar value of the shoes sold in a week. A salesperson worked 18 hours and sold $885 worth of shoes that week. What was that week`s salary for this salesperson?
    a. $65.00
    b. $90.00
    c. $149.00
    d. $296.20

25. A teacher gave the following problem to her class.
    The numbers in the sequence 7, 11, 15, 19, 23, ... increase by 4. The numbers in the sequence 1, 10, 19, 28, 37, ... increase by 9. The number 19 is in both sequences. If the two sequences were continued, what is the next number that is in BOTH the first and the second sequence?

26. Mr. Allen found himself a bit confused one morning as he prepared to teach. Realizing that ten to the second power equals one hundred ($10^2 = 100$), he puzzled about what power of 10 equals 1. He asked Ms. Berry, next door. What should she tell him?
    a. 0
    b. 1
    c. Ten cannot be raised to any power such that ten to the power equals 1.
    d. -1
    e. I'm not sure

27. Ms. Chambreaux's students are working on the following problem: Is 371 a prime number? As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)
    a. Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.
    b. Break 371 into 3 and 71; they are both prime, so 371 must also be prime.
    c. Check to see whether 371 is divisible by any prime number less than 20.
    d. Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

Pedagogy (Knowledge for teaching)

28. What are some of the challenges that you anticipate facing or have faced as a teacher of adolescent mathematics students?
29. What characteristics of the adolescent learner can you use to your advantage to improve the learning environment in the classroom?
30. How will you structure lessons to take advantage of these characteristics?
31. How will you differentiate instruction for the range of learners in a typical intermediate division classroom?
32. How will you know that students were actively engaged in your lesson?

APPENDIX I: QUESTIONNAIRE 2 (POST-COURSE) QUESTIONS

Please read and consider all questions. Consider your in-class experiences (including problem sets, class discussions and explanations) as well as your practicum experiences (in-school teaching with students) when answering each of the questions. Please explain your answers as thoroughly as possible. Thank you for taking to the time to participate in this study and complete this questionnaire!

Question	Purpose
1. At the beginning of the course the class profile suggested that approximately 70% of study participants held beliefs that mathematics is a collection of rules and procedures that prescribe how to solve a problem. a) In what ways did your beliefs about the nature of mathematics match or differ from this profile at the beginning of the course? b) How would you compare yourself to this aspect of the course profile now?	Explore student perceptions about the nature of mathematics at the beginning of the course (Research Question 1, RQ1). Identify any changes that student might perceive in their beliefs about the nature of mathematics (Research Question 2, RQ2).
2. At the beginning of the course 70% of study participants agreed with the statement "When solving mathematical tasks you need to know the correct procedure or else you will be lost." a) How would you compare yourself to this profile at the beginning of the course? b) How would you compare yourself to this aspect of the course profile now?	Explore student perceptions about the nature of mathematics at the beginning of the course (RQ1). Identify any changes that student might perceive in their beliefs about the nature of mathematics (RQ2).
3. At the beginning of the class, participant responses varied greatly when considering the statement "Mathematics involved creativity and new ideas." a) Where did you stand with regard to this belief at the beginning of the course and why? b) Has this belief changed over the course of the semester? If so, how?	Explore student perceptions about the nature of mathematics at the beginning of the course (RQ1). Identify any changes that student might perceive in their beliefs about the nature of mathematics (RQ2).
4. The previous questions have related to beliefs about the nature of mathematics. To what do	Examine aspects of the course that led to changes (RQ3).

you attribute any change in your beliefs about the nature of what mathematics is, from the beginning of the course to now?	
5. Have your beliefs about the nature of *teaching* mathematics changed during the course? If so, in what ways have your beliefs changed?	Examine aspects of the course that led to changes (RQ3).
6. What factors were most influential in bringing about these changes to your beliefs about the nature of mathematics and its teaching?	Examine the most influential aspects of the course that led to changes (RQ3).
7. Consider the following question from Questionnaire 1: Ms. Chambreaux's students are working on the following problem: Is 371 a prime number? As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.) e. Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9. f. Break 371 into 3 and 71; they are both prime, so 371 must also be prime. g. Check to see whether 371 is divisible by any prime number less than 20. h. Break 371 into 37 and 1; they are both prime, so 371 must also be prime. a) How did you answer this question and what was your justification for doing so? b) Would your answer change if you were to repeat Questionnaire 1? If so, what would your new answer be and why?	Examine math content knowledge at the beginning of the course. Examine how math content knowledge has changed from the beginning of the course to the end.
8. On Questionnaire 1, many participants had varied answers to the following question: Which of the following problems could be used to illustrate 1 ¼ divided by 1/2? (Mark YES, NO, or I'M NOT SURE for each possibility). d. You want to split 1 ¼ pies evenly between two	Examine how math content knowledge has changed from the beginning of the course to the end.

families. How much should each family get? e. You have $1.25 and may soon double your money. How much money would you end up with? f. You are making some homemade taffy and the recipe calls for 1 ¼ cups of butter. How many sticks of butter (each stick = 1/2 cup) will you need? Has your answer changed from the time when you first answered this question to now? If so, how would you compare your initial thinking in answering the problem to your thinking now?	
9. In what ways has the course contributed to your understanding of answering content questions, such as those examples shown in Question 7 and 8?	How do prospective teachers describe changes in their math content knowledge from the beginning of the course to the end?
10. Is there anything else that you would like to share about the course experience that you think is important for me to know?	

APPENDIX J: STUDENT INTERVIEW QUESTIONS

Question	Purpose
Please briefly describe your background in mathematics and in any experience teaching mathematics.	
1. Please describe some of the ways that Math 010 prepared you to teach *StepAhead*.	Identify perceptions about aspects of the course that increased pedagogical content knowledge.
2. What specific course preparations led you to feel confidently about instructing *StepAhead* lessons?	Identify perceptions about the course that lead to self-efficacy and teacher preparedness.
3. Do you agree with the statement: Mathematics involves creativity and new ideas? Why/Why not.	Identify perceptions about the nature of mathematics. (Research Question 2, RQ2)
4. Has your view of the previous statement changed from the beginning to the end of the semester? Why/Why not?	Identify perceptions about the change in beliefs about the nature of mathematics. (RQ3)
5. What experiences did you have that led you to agree/disagree with the statement: Mathematics means learning, remembering and applying?	Identify perceptions about aspects of the course that led to changes in beliefs about the nature of mathematics. (RQ3)
6. Please describe experiences that have changed your thinking about how to teach mathematics?	Identify perceptions about aspects of the course that led to changes in the nature of teaching mathematics. (RQ3)
7. What do you think the purpose of the course was?	
8. Why do you think a teachers' beliefs matter?	Research Question 2
9. What did you learn most? Math? Teaching?	
10. What did you find most challenging about Math 010?	
11. Why is knowledge important for a teacher to know?	
12. Is it good enough to just get high marks in mathematics courses to be an effective math teacher? Why or Why not?	Research Question 2

13. What do you need to be an effective math teacher?	
14. What do you need more experience in to be an effective math teacher?	
15. Do you think that your math courses have and will prepare you to be an effective math teacher? Why or why not?	
16. Has your experience in Math 010 changed the way you approach solving a math problem? If so, in what way?	
17. How would you describe your view on the teacher's role in the classroom?	Research Question 2
18. Has this view changed from what it was before the course experience? If so, why?	Research Question 1
19. Has this experience changed anything else about your views of teaching and learning mathematics, from the beginning of the course to now?	Research Question 2
20. Is there anything else that you think I should know about your experiences in this course?	

APPENDIX K: COURSE INSTRUCTOR INTERVIEW QUESTIONS

Question	Purpose
1. How would you describe your beliefs about the nature of mathematics? (ex. a set of rigid rules and procedures, continually changing)	Identify the course instructor's beliefs about the nature of mathematics.
2. Do you think your beliefs influence the participants of your course? If so, in what way?	Identify how the course instructor views how his beliefs impact his students.
3. In developing the StepAhead program, what resources did you consult and what was your reasoning for those particular resources?	Identify the resources behind the StepAhead curriculum.
4. How does StepAhead relate to the Ontario Mathematics curriculum?	
5. How do you think your students used StepAhead in their practicum experiences to further their students' knowledge?	Identify how StepAhead was viewed to influence knowledge.
6. What is the purpose and main goal of Math 010?	Identify the intended course goals.
7. What do you want students to take away from this course? Math? Resources? Goals?	
8. How has the course changed over time?	Identify course goals.
9. Has the goal of the course always been the same?	
10. What inspires you to teach this course?	
11. Liping Ma describes a "Profound Understanding of Fundamental Mathematics." What does a *profound understanding* mean to you?	
12. How did you develop this course to develop profound understanding?	
13. What does curriculum mean to you and your teaching?	Instructors' view of curriculum.
14. In a perfect world, what would a math curriculum look like?	

15. How has your view of education changed over the course of your career?	
16. What paths lead you to your current position of professor of Math 010?	
17. Is there anything else that you feel is important for me to know regarding your beliefs about mathematics and its teaching, or your development and teaching of Math 010?	

APPENDIX L: EMAIL PERMISSION FROM COURSE INSTRUCTOR

Subject	**RE: A couple Questions**
From	Leo Jonker <leo@mast.queensu.ca>
Date	Monday, February 28, 2011 9:42 am
To	gina.kerr@queensu.ca

Hello Gina,

I am glad to hear things are going well, and I look forward to seeing the final result. I do not mind if you use my real name. According to my records, I first taught math 010 in the fall of 2001, and have taught it annually since then. To say that John Van de Walle was an inspiration would be an overstatement. I did occasionally make use of his book "Elementary and Middle School Mathematics", but less and less as the course developed. So if I called his work an inspiration I exaggerated. I have not read any of his other work.

Leo

From: gina.kerr@queensu.ca [mailto:gina.kerr@queensu.ca]
Sent: Saturday, February 26, 2011 3:14 PM
To: Leo Jonker
Subject: A couple Questions

Hi Leo,

I hope you're doing well. I am working hard to wrap up my thesis and have a couple of questions for you.

1. Could I use your real name in the thesis or would you like me to use a pseudonym?
2. How many years have you been teaching Math 010?
3. You spoke of John Van de Walle as being inspirational to you. Was there any one piece of work that was more influential than another? Or was it a collection of his books?

Thanks so much!

Gina