A New Efficient CSI Signalling Strategy for MMR Systems With Optimal Detection

by

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Abstract

We present a new *Channel State Information* (CSI) signalling strategy for single-branch *Mobile Multihop Relay* (MMR) systems. This novel signalling strategy reduces the signalling overhead at each relay by at least 50%, and eliminates the need for channel estimation at the relays. We prove that this significant overhead reduction comes at the expense of no performance loss at all when hard *Maximum Likelihood* detection is carried out at the destination. Furthermore, we consider the use of our system with concatenated channel codes to carry out soft *Maximum a Posteriori* (MAP) detection, and show that with channel codes employed the optimum detection rule becomes prohibitively complex to implement. We propose two approximate soft MAP detection schemes to make the detection feasible for our system, and demonstrate that the performance is either almost identical or slightly degraded from the ideal case with full CSI at the destination. We demonstrate the validity of our analysis through performance simulation plots.

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Chapter 1

Introduction

1.1 *Mobile Multihop Relaying* (MMR) Systems

With the recent upsurge in demand for wireless communication services, meeting the requirements instituted by today's wireless market is becoming increasingly difficult. Next-generation wireless systems are expected to provide very high data rates and improved quality of service while staying within commercial feasibility by being affordable to deploy and commercialize. To meet this demand, engineers are challenged with design problems requiring solutions that are extremely efficient but tractable to implement. One very promising tool to aid in meeting market expectations is the concept of *Mobile Multihop Relaying* (MMR) [1].

The MMR concept describes wireless transmission systems having an intermediary station(s) between a source and destination to act as a relay point by forwarding data symbols along a specified source-destination path. The relay intermediaries can either be fixed as a part of the infrastructure or mobile depending on the type of application. The fixed-relay usage model of MMR systems makes use of relay stations over places

like building tops, towers, and poles in a way to provide a *Line of Sight* (LOS) channel between the relays and the base station. The mobile-relay usage model employs nomadic relay stations that are deployed temporarily as need arises.

The level of intelligence of relay stations also varies depending on the particular implementation. Relays can either be *regenerative*, following a *decode-and-forward* protocol, or *non-regenerative*, following an *amplify-and-forward* protocol [2]. The former fully decodes the received symbol then forwards the estimate along the path, while the latter simply amplifies the received signal and forwards the amplified version without any intelligent processing. These variations make the use of wireless relaying very broad and implementable in numerous scenarios.

1.2 Advantages of MMR Systems

MMR has proven to be very effective with various wireless systems such as cellular networks [3]–[4], and ad hoc sensor networks [5] among others. For fixed-infrastructure systems (e.g. cellular networks and mobile WiMAX), wireless relaying can be effective for many reasons. One immediate use of relay stations is the ability of extending the coverage area of existing base station terminals and eliminating coverage problems caused by dead spots and deep-shadowed regions. Although these problems can be solved by deploying more base stations, relays remain an equivalent yet cheaper solution. Unlike base stations, relay stations have the flexibility of avoiding a wired connection with the back-bone network and therefore have no geographical location constraints. This fact makes the deployment of relay stations more flexible and cost-effective since no major infrastructure modifications are required.

The deployment of relay stations can also serve to provide shorter hops in each

coverage area, and that can increase the per user throughput of MMR systems over direct link systems. Alternately, relay stations can also serve to reduce the transmission power overhead on base stations while maintaining the same coverage range. This fact can help in reducing amplifier non-linearities and power emission levels. One of the main potential applications for MMR systems is in hand-held services, and this reduction in power transmission can serve to extend the battery life in hand-held devices; a much needed advantage.

Deployment of relay stations can also provide the flexibility of providing coverage in areas where base station coverage is extremely difficult. Examples of these isolated areas are building basements, traffic tunnels, and subway platforms. Due to the fact that relay stations are generally lower in cost than base stations, this application will provide the ability to commercialize the relay terminals such that relay stations in these applications can be owned by the customer. This can be a very convenient option for customers who wish to provide service in such areas.

MMR has also been proposed as a way of integrating cellular and ad hoc networks by using mobile stations in cellular networks as relays [6]–[7]. These systems are very effective in alleviating local traffic congestions experienced at peak-times or peak-locations by effective load balancing. In fact, realizing the gains achieved by utilizing MMR in wireless systems, an official *IEEE* task group was established to introduce *IEEE 802.16j*, which standardizes the use of MMR in the existing *IEEE 802.16e* standard [8]. Fig. 1.1 displays possible uses for fixed relay MMR systems highlighting their advantages.

MMR systems with mobile stations also finds uses in areas where network coverage is needed temporarily for an incidental reason. An example would be an emergency

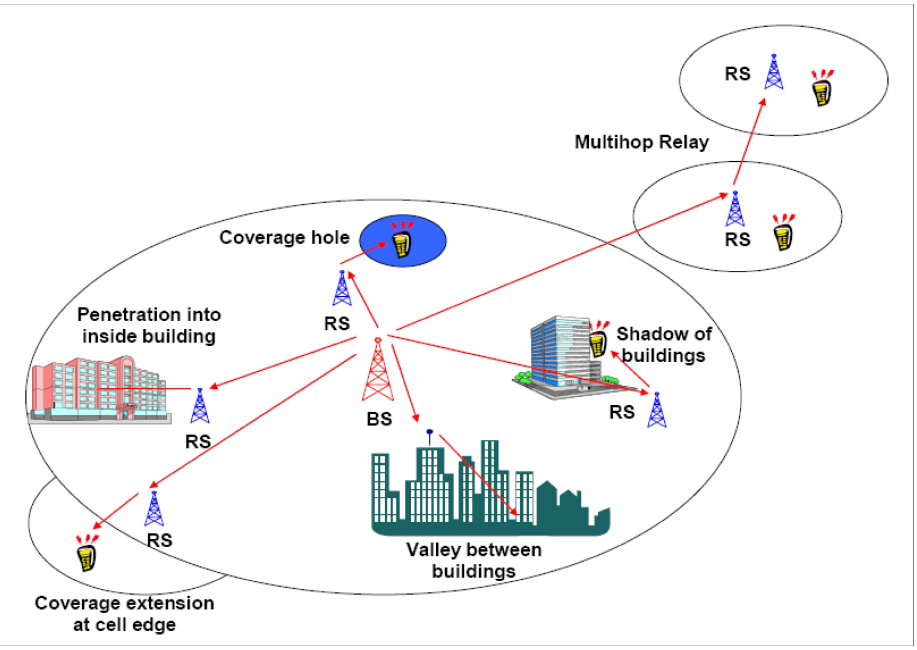

Figure 1.1: Example Uses of fixed-relay MMR Networks [9].

incident or a disaster recovery effort where fixed infrastructure network terminals (base stations or other relays) have been either disabled or even destroyed. A quick and feasible solution would be the deployment of temporary relay stations that can help in alleviating network shortages. Another example would be in events where a heavy load on the network is expected only in certain times. Sporting events, concerts, and other events where a large crowd is expected to gather can find in MMR systems a cheap solution for network shortage.

1.3 Cooperative Diversity Networks

From a theoretical perspective, wireless relay systems have been extensively studied in literature in the context of cooperative diversity [2], [10]–[12]. From a cooperation stand point, relay paths are used to provide either spatial or temporal diversity to improve channel capacity. This concept of cooperation is applicable to MMR networks where redundant paths are used, through advanced routing techniques, to provide the diversity. The performance of cooperative diversity systems has also been extensively analyzed in literature for various cooperative diversity channels [13]–[15]. Other promising studies on cooperative relay systems, like [16]–[18], include the concept of *Coded Cooperation*. Coded cooperation combines the use of channel coding schemes with cooperative relay networks to achieve diversity through channel-coded bit frames.

Relays in cooperative diversity networks could be either *Channel State Information* (CSI)-assisted or *blind* relays, where the former have the ability to estimate the channel of their previous hop while the latter do not carry out any channel estimation. Although CSI-assisted relays can provide some performance improvement, they can

be too costly to implement. There has been encouraging progress in works dealing with systems employing blind relays [14].

1.4 Contributions of The Thesis

Considering signal detection, the destination in almost all works has been assumed to have perfect instantaneous CSI. Although this is a widely accepted assumption, its implications can be very costly because it means an increased relay signalling overhead burden and/or the need for signal estimation at the relays (as in the case of CSI-assisted relays). Few works studied non-coherent detection schemes in the context of cooperative diversity networks [19]–[20], but they do suffer some drawbacks. The optimum *Maximum Likelihood* (ML) detection schemes proposed in those works involve numerical integral evaluations that are very costly to implement at the destination, while the other low-complexity detection schemes proposed are sub-optimum. Furthermore, all the schemes proposed in [19] and [20] are valid only for *Binary Frequency Shift Keying* and *On-Off Keying* schemes. Other than these works, almost all works assume perfect instantaneous CSI at the destination [2], [10]–[15], which requires an increased signalling overhead burden and might entail increased signal estimation complexity at the relays.

In order to address the problems described above, we propose the following:

- A new and efficient CSI signalling strategy for MMR systems. The system enjoys the advantages of reducing the signalling overhead and eliminating the need of channel estimation at each relay. Our system is detected coherently, but does not require the knowledge of instantaneous CSI of each individual hop. This

is done by having the destination know only the end-to-end source-destination channel. We analyze the overhead reduction, and show that as the number of relays increases, the reduction in pilot signalling overhead tends to 100%. Our system is compatible with any two-dimensional M-ary *Quadrature Amplitude Modulation* (M-QAM) or *Phase Shift Keying* (M-PSK) constellation, and can be deployed in any type of channel-fading environment.

- We analyze the *Probability Density Function* (PDF) of the noise component of the system, and consequently derive the likelihood function for the received signal. Through the likelihood function, we derive the optimum hard ML decision rule, and show that it involves complex integral computations. However, we prove that the optimum ML decision rule lends itself to the simple decision metric known as the classical *Minimum Euclidian Distance* metric, avoiding the need to carry out prohibitive integral computations at the destination. Our system therefore achieves the exact same performance as the ideal relay system with full CSI knowledge at a significantly reduced signalling overhead and signal estimation complexity.

- We also consider the use of our system in combination with channel codes. We propose novel approximate *Maximum a posteriori Probability* (MAP) detection schemes to be used with channel codes. Our approximate MAP schemes avoid the need for integrations at the destination. We demonstrate their effectiveness especially for the important dual-hop scenario, and show that they perform either almost identical or very close to MAP detection with full CSI knowledge.

This thesis is organized as follows. In Chapter II we present our system model, and clarify all the assumptions made and their justifications. We derive the optimum hard

ML detector and prove that it can be implemented as a simple minimum Euclidian distance detector in Chapter III. Chapter IV outlines the extension of our system to soft MAP systems by concatenating them with channel codes. We derive the optimum MAP detector and propose two different approximate MAP detection schemes. We also demonstrate their effectiveness through numerical simulations in Chapter V. Chapter VI concludes this thesis and discusses possible future work.

Notation: Bold lower letters denote vectors. $x \sim \mathcal{CN}(\mu, \Omega)$ means that x is a circularly symmetric complex Gaussian random variable with mean μ and variance Ω. $E[\cdot]$ and $Var[\cdot]$ are the expectation and variance operators, and $x|y$ means that the random variable x is conditioned on the knowledge of y.

Chapter 2

System Model

We consider a single-branch multi-hop relay system consisting of a source, a destination, and $(K-1)$ intermediary relays, as shown in Fig. 2.1. Consequently, this systems is called a K-hop system. Each node in this network possesses a single receive and transmit antenna working in a half-duplex mode. At the first time slot, the source transmits a symbol $x \in \mathcal{S}$, where $\mathcal{S}$ is any arbitrary two-dimensional M-QAM or M-PSK constellation, and $E[|x|^2] = E_s$. We consider the amplify-and-forward protocol at each relay and assume that each relay has no instantaneous CSI, but possesses knowledge of channel statistics. In this sense the relays can be considered as blind relays since each relay amplifies the received symbol using a fixed amplification coefficient. We can therefore write the received signal y_K at the destination as

$$y_K = \left(\prod_{i=1}^{K} h_i\right)\left(\prod_{j=1}^{K-1} \alpha_j\right) x + \tilde{n} \tag{2.1}$$

where the noise component $\tilde{n}$ is given by

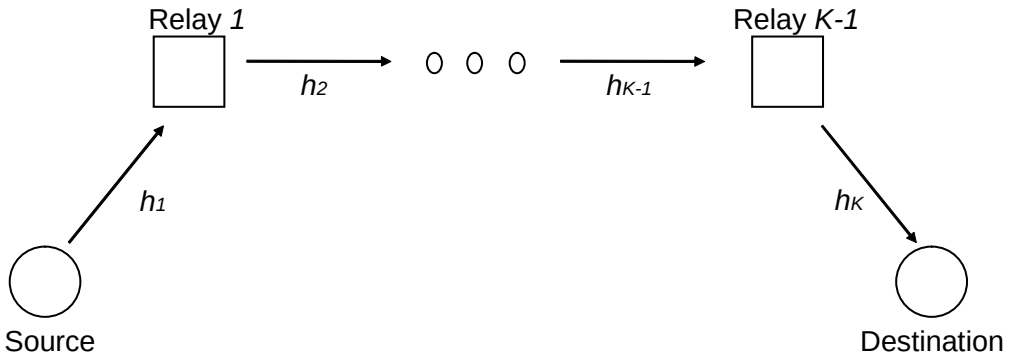

Figure 2.1: Overview of a single-branch MMR relay network

$$\tilde{n} = \sum_{i=1}^{K-1} \left(\prod_{j=i+1}^{K} h_j \right) \left(\prod_{q=i}^{K-1} \alpha_q \right) n_i + n_K. \tag{2.2}$$

Each $h_i, i \in \{1, \cdots, K\}$ represents the channel gain for the i-th hop, and is randomly distributed according to any arbitrary channel model with $E[|h_i|^2] = \Omega_i$. Each value $n_i \sim \mathcal{CN}(0, \sigma_i^2), i \in \{1, \cdots, K\}$ represents the *additive white Gaussian noise* (AWGN) at the i-th hop. Each $\alpha_i, i \in \{1, \cdots, K-1\}$ is the amplification coefficient used at the i-th relay, and is given by

$$\alpha_i = \sqrt{\frac{E_{r,i}}{E_s \Omega_i + \sigma_i^2}}. \tag{2.3}$$

This setting for α_i[1] ensures that each relay satisfies a long term power constraint given by $E_{r,i}$, where $E_{r,i}$ is the average transmission power at the i-th relay. The average *Signal to Noise Ratio* at the i-th hop is therefore given by

$$SNR_i = \begin{cases} \frac{\Omega_1 E_s}{\sigma_1^2}, & \text{for } i = 1 \\ \frac{\alpha_{i-1}^2 \Omega_i E_{r,i-1}}{\sigma_i^2}, & \text{for } i \in \{2, \cdots, K\}. \end{cases} \tag{2.4}$$

2.1 CSI Signalling

2.1.1 Conventional CSI Signalling Strategies

In conventional CSI signalling strategies, the destination is assumed to have full instantaneous CSI for every hop. This may be achieved by using either of two different pilot signalling strategies: the first being a strategy where the source and each relay transmit a pilot signal to the next terminal. Using the pilot signal, the next terminal

[1] The α_i amplification factors are set just as in [14], [21]–[24]

(the next relay or the destination) estimates the channel of its own individual hop. The estimated channel coefficient at each relay is then quantized and transmitted down the path all the way to the destination. In this scenario, each relay must have the ability to estimate its own channel coefficient. This signalling strategy suffers from two major drawbacks. The first drawback is the need of intelligent relays with the ability of estimating the channel. In systems with blind relays, such as the one adopted in this thesis, estimation and quantization of the channel is not possible. Even if this drawback is avoided by using more intelligent relays, this pilot signalling strategy still suffers from another disadvantage that the channel coefficients at each relay will have to be quantized and retransmitted. This digital retransmission of the channel coefficients will incur an increased overhead since the channel coefficient, being a complex number, will need a high bit resolution to quantize accurately. Even if the system uses a quantized codebook for the channel distribution where the codebook index of the estimated channel is transmitted instead of the actual channel coefficient, this codebook index will still need several bits of transmission.

In the other pilot signalling strategy, each terminal, including the source and each relay, generates its own pilot signal, which in turn is relayed all the way to the destination. That is, each relay is responsible for forwarding the pilot signals for all previous terminals, as well as starting the transmission of a pilot signal. The destination eventually will have received K different pilot signals, and will be able to estimate the CSI for each individual hop. This strategy, unlike the previous one, does not suffer from the need of intelligent relays since no channel estimation is carried out at the relays. It also avoids the need for relays to quantize and transmit the channel coefficients of the individual hops. However, the signalling overhead can still be an

issue since each relay is responsible for forwarding all the pilot signals generated by the previous terminals. That means the i-th relay needs to transmit $(i+1)$ pilot signals (one that it generates, and i that it relays from previous terminals). That can be a significant burden especially as K increases.

Since we consider blind relays in the MMR system, the second CSI signalling strategy would be the most suitable one out of the two to obtain full CSI at the destination. For the rest of this paper, we will refer to the second CSI signalling strategy as the *conventional CSI signalling strategy.*

2.1.2 Proposed CSI Signalling Strategy

We propose a new CSI signalling strategy that eliminates all the drawbacks discussed in Section 2.1.1. We will assume that the relays avoid generating their own pilot signals, and that only the source generates one pilot signal that is amplified and forwarded by every relay. The implications of this assumption are as follows:

1. The signalling overhead at the i-th relay is reduced to $1/(i+1)$ of the overhead compared to the conventional signalling strategy.

2. The destination has no knowledge of the instantaneous CSI of each hop. Instead, the destination acquires the overall product of channel gain coefficients only, which is given by $\left(\prod_{i=1}^{K} h_i\right)$ (i.e. the end-to-end source-destination channel).

This reduction in pilot signalling is very significant in terms of transmission power. The ratio between the total number of pilot signals required by the proposed strategy over that required by the conventional strategy is given by

$$\begin{aligned} S &= \frac{K}{\sum_{i=1}^{K} i} \\ &= \frac{K}{K(K+1)/2} \\ &= \frac{2}{K+1}. \end{aligned} \tag{2.5}$$

An attractive feature of this strategy is that its savings improve as K increases. In fact, as K tends to infinity, the ratio in (2.5) tends to zero, i.e. $\lim_{K\to\infty} S = 0$. In what follows, we will see that this overhead reduction comes at absolutely no loss in performance in terms of hard ML detection, and at a small loss in terms of soft MAP detection.

2.2 Likelihood Function

To derive the likelihood function $p(y_K|x)$, we first look at the distribution of $\tilde{n}$. Note that in the conventional CSI signalling strategy, $\tilde{n}$ has a Gaussian PDF as follows

$$\tilde{n}_{|\{h_1,\cdots,h_K\}} \sim \mathcal{CN}\left(0, \sum_{i=1}^{K-1}\Big(\prod_{j=i+1}^{K}|h_j|^2\Big)\Big(\prod_{q=i}^{K-1}\alpha_q^2\Big)\sigma_i^2 + \sigma_K^2\right) \tag{2.6}$$

and therefore the likelihood function $p(y_K|x, \{h_i : i = 1, \cdots, K\})$ is given in closed form as a Gaussian function. With that property, ML detection at the receiver is simply carried out using the classical minimum Euclidian distance rule. Soft MAP detection can also be carried out using any of the existing schemes for the AWGN channel with known CSI.

However, the distribution of the noise component $\tilde{n}$ in the proposed CSI signalling strategy is no longer Gaussian because the individual $h_i, i \in \{1, \cdots, K\}$ are not known

and are still random entities. Expanding (2.2) will give a summation of K terms, each of which is a multiplication of multiple random variables. This makes deriving the PDF of $\tilde{n}$ quite involved. Even if a closed form PDF is derived for $\tilde{n}$, then it would only be valid for a specific PDF model for each $h_i, i \in \{1, \cdots, K\}$.

To approach the problem of finding a general expression for the PDF of $\tilde{n}$, we condition the distribution of $\tilde{n}$ on the knowledge of the channels at each hop just like in (2.6), then we take the expectation over all the channel PDFs. That would give us an integral form for the likelihood function as follows

$$p(y_K|x, \prod_{i=1}^{K} h_i) = \frac{1}{\pi} \int_{\Psi_K} \cdots \int_{\Psi_2} \frac{\exp\left(\frac{-\beta_K}{\sum_{i=1}^{K-1} \left(\prod_{j=i+1}^{K} z_j \right) \left(\prod_{q=i}^{K-1} \alpha_q^2 \right) \sigma_i^2 + \sigma_K^2} \right)}{\sum_{i=1}^{K-1} \left(\prod_{j=i+1}^{K} z_j \right) \left(\prod_{q=i}^{K-1} \alpha_q^2 \right) \sigma_i^2 + \sigma_K^2} \times f_{|h_2|^2, \cdots, |h_K|^2 | \prod_{i=1}^{K} h_i}(z_2, \cdots, z_K) d_{z_2} \cdots d_{z_K} \tag{2.7}$$

where $f_{|h_2|^2, \cdots, |h_K|^2 | \prod_{i=1}^{K} h_i}(z_2, \cdots, z_K)$ represents the joint PDF of $|h_i|^2, i \in \{2, \cdots, K\}$ conditioned on the knowledge of $\prod_{i=1}^{K} h_i$, and

$$\beta_K = \left| y_K - \Big(\prod_{i=1}^{K} h_i \Big) \Big(\prod_{j=1}^{K-1} \alpha_j \Big) x \right|^2 . \tag{2.8}$$

Each range $\Psi_i, i \in \{2, \cdots, K\}$ represent the positive range of each $|h_i|^2$ given that $\prod_{i=1}^{K} h_i$ is known. We have found that it was extremely difficult to solve this integration in closed form (we conjecture that it is impossible), even for the simplest case for $K = 2$ and a Rayleigh fading channel. Interestingly, however, we will show in Section 3.2 that achieving hard ML detection is possible without the need for the integration in (2.7).

Chapter 3

Hard ML Detection

3.1 The Optimum Detector

We have thus far derived the distribution of the noise component of our proposed system. This distribution, also called the likelihood function, will be used to implement the optimum detector for the system. The optimum detector is a detector that makes the best 'guess' at what the transmitted symbol was such that the probability of a correct decision (i.e. error-free communication) is maximized, or equivalently such that the probability of symbol error is minimized. To make that guess, the detector uses whatever information it can acquire from its observation of the channel. In the most general case, the detector will have acquired the received signal vector y_K, and therefore will be required to make a decision on what was the transmitted symbol $x \in \mathcal{S}$ after observing y_K. This type of decision is based on the *a posteriori* probabilities, where the detector makes a decision on the transmitted symbol by computing the probability of each $x \in \mathcal{S}$ given that y_K is known. The optimum decision in that case is to maximize that *a posteriori* probability (i.e. MAP detection) and therefore

the optimal 'guess' is given by:

$$\hat{x}_{MAP} = \arg\max_{x\in\mathcal{S}} p(x|y_K). \tag{3.1}$$

This decision made in (3.1) is most generally the optimum decision rule. If we apply Bayes' rule to (3.1), we can transform it to

$$\begin{aligned}\hat{x}_{MAP} &= \arg\max_{x\in\mathcal{S}} \left[\frac{p(y_K|x)p(x)}{p(y_K)}\right] \\ &= \arg\max_{x\in\mathcal{S}} p(y_K|x)p(x)\end{aligned} \tag{3.2}$$

where $p(x)$ is the *a priori* probability of each $x \in \mathcal{S}$. The probability $p(y_K|x)$ is the likelihood probability of y_K being received given that the particular $x \in \mathcal{S}$ was sent. When all the symbols in $\mathcal{S}$ are equiprobable, (i.e. $p(x) = 1/M, \forall x \in \mathcal{S}$), then the MAP decision rule in (3.2) reduces to the ML decision rule which is given by

$$\hat{x}_{ML} = \arg\max_{x\in\mathcal{S}} p(y_K|x). \tag{3.3}$$

For our proposed system, the assumption of equiprobable symbols is made, and therefore our optimum decision rule is given in (3.3). By substituting (2.7) in (3.3), our optimum ML decision rule is calculated by

$$\begin{aligned}\hat{x}_{ML} = \arg\max_{x\in\mathcal{S}} \int_{\Psi_K} \cdots \int_{\Psi_2} & \frac{\exp\left(\frac{-\beta_K}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} z_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2}\right)}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} z_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2} \\ & \times f_{|h_2|^2,\cdots,|h_K|^2\left|\prod_{i=1}^{K} h_i\right.}(z_2,\cdots,z_K)dz_2\cdots dz_K.\end{aligned} \tag{3.4}$$

To evaluate (3.4), the detector will need to evaluate $K-1$ integrations, which is computationally prohibitive even for the simplest case where $K=2$.

If we assume that the individual hop CSI is given at the destination, the ML decision in (3.3) can be simplified even further. Just as shown in (2.6), the PDF of the noise component conditioned on full CSI knowledge is Gaussian. The likelihood function $p(y_K|x)$ is itself the PDF of the noise component, and therefore using (2.6), the optimum ML decision in (3.3) is given by

$$\hat{x}_{ML} = \arg\max_{x\in\mathcal{S}} \frac{\exp\left(\frac{-\beta_K}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} h_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2}\right)}{\pi\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} h_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2} \tag{3.5}$$

which in turn in equivalent to

$$\hat{x}_{ML} = \arg\min_{x\in\mathcal{S}} \beta_K. \tag{3.6}$$

Equation (3.6) is true since the exponential function is monotonic and strictly decreasing with respect to the argument. This detection rule in (3.6) is called the *Minimum Euclidian Distance* (MED) rule. MED detection is essentially deciding in favor of the transmitted symbol closest to the received symbol in the *Euclidian* sense. MED detection follows from ML detection when the noise distribution is of a Gaussian form.

Unfortunately, the noise component in our system is not Gaussian, however we will show in the next section that ML detection for our system can in fact be reduced to MED detection. This fact will make an ML detector for our system possible since the integration operations in (3.4) will be avoided.

3.2 Detection for Proposed System

Based on the Likelihood function in (2.7), one may obtain the ML solution by numerically integrating (2.7) for all the symbols in the constellation, and then decide in favor of the symbol giving the integration the maximum value. Obviously, the complexity of this detection is prohibitively high. It is easily shown however, that computing the integral of (2.7) is actually unnecessary and that the ML symbol estimate can be simplified to the classical minimum Euclidian distance rule as follows:

$$\begin{aligned}
\hat{x}_{ML} &= \arg\max_{x\in\mathcal{S}} \int_{\Psi_K} \cdots \int_{\Psi_2} \frac{\exp\left(\frac{-\beta_K}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} z_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2}\right)}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} z_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2} \\
&\qquad \times f_{|h_2|^2,\cdots,|h_K|^2|\prod_{i=1}^{K} h_i}(z_2,\cdots,z_K)dz_2\cdots dz_K \\
&= \arg\max_{x\in\mathcal{S}} \frac{\exp\left(\frac{-\beta_K}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} \bar{z}_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2}\right)}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} \bar{z}_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2} f_{|h_2|^2,\cdots,|h_K|^2|\prod_{i=1}^{K} h_i}(\bar{z}_2\cdots\bar{z}_K) \\
&= \arg\min_{x\in\mathcal{S}} \beta_K \qquad (3.7)
\end{aligned}$$

where each $\bar{z}_i, i \in \{2,\cdots,K\}$ is any arbitrary positive constant.

Equation 3.7 is true since the likelihood function is positive and strictly decreasing with respect to β_K, and therefore maximizing the integral will be equivalent to maximizing the integrand. Note that even without the individual hop CSI, the ML solution is still given by the minimum Euclidian distance rule. We are therefore able to reduce the i-th relay's signalling overhead by a factor of $i/(i+1)$ compared to the conventional CSI signalling strategy, and eliminate its need for channel estimation at

no performance loss at all when hard ML detection is carried out.

Chapter 4

Soft MAP Detection

4.1 Soft MAP Detection with Channel Codes

We have shown thus that our system, which enjoys an alleviated burden in terms of relay signalling overhead and channel estimation, will suffer from no performance loss at all for the case of hard ML detection. Unfortunately, that is not the case when soft MAP detection is desired at the receiver. The use of capacity approaching channel codes (e.g. *turbo* [25] and *low density parity check* (LDPC) [26] codes) is in many cases highly desired. These codes are decoded using a *soft-in soft-out* (SISO) channel decoders approaching MAP detection. With these codes, the detector produces soft extrinsic likelihood information that is used by the SISO channel decoder to decode the received frame. These systems can also have the option of iterative joint detection and decoding [27]. In those systems, the channel decoder feeds back information to the detector in an iterative manner to refine the received symbol estimate, and therefore incrementally improve the decoded bits estimate. The information being produced/exchanged by the detector/decoder pair is soft information in the form of

log-ratios. The final objective of the detector/decoder pair is to carry out bit-wise MAP detection, i.e. maximize the *a posteriori* probability of each coded bit. This will be equivalent to obtaining the bit-MAP estimate given by

$$\hat{b}_{m,MAP} = sign\left[L_D(b_m|y_K)\right] \tag{4.1}$$

where

$$L_D(b_m|y_K) = \ln \frac{P\left[b_m = 1|y_K\right]}{P\left[b_m = -1|y_K\right]}. \tag{4.2}$$

Each $b_m, m \in \{1, \cdots, \log_2(M)\}$ represents the coded-modulated bit that is transmitted through each symbol $x \in \mathcal{S}$ such that

$$x = map\left(\left[b_1, \cdots, b_{\log_2(M)}\right]\right). \tag{4.3}$$

The $sign(\cdot)$ function returns $+1$ or -1 depending on the sign of the argument, and each $b_m = +1 (b_m = -1)$ represents a logical one (zero) bit. The $map(\cdot)$ function maps a vector of $\log_2(M)$ bits to a particular M-ary constellation point (e.g. Grey mapping). The $L_D(b_m|y_K)$ value in (4.2) represents the log-*a posteriori* ratio for the m-th bit. The sign of (4.2) represents the final decision on the bit-MAP estimate, while the magnitude represents the reliability of such a decision. This log-ratio form is described as 'soft' information since it holds both the estimate and the reliability of that estimate embedded in it. Using Bayes' rule to (4.2), we can rewrite it as

$$\begin{aligned} L_D(b_m|y_K) &= \ln \frac{P[b_m = +1] \times P[y_K|b_m = +1]}{P[b_m = -1] \times P[y_K|b_m = -1]} \\ &= \ln \frac{P[b_m = +1]}{P[b_m = -1]} + \ln \frac{P[y_K|b_m = +1]}{P[y_K|b_m = -1]} \\ &= L_A(b_m) + L_E(y_K|b_m) \end{aligned} \tag{4.4}$$

where

$$L_A(b_m) = \ln \frac{P[b_m = +1]}{P[b_m = -1]} \tag{4.5}$$

and

$$L_E(y_K|b_m) = \ln \frac{P[y_K|b_m = +1]}{P[y_K|b_m = -1]}. \tag{4.6}$$

The $L_A(b_m)$ value in (4.5) represents the log-*a priori* ratio of the m-th bit. In the very first round of detection/decoding, the bits are assumed to be equiprobable, and therefore $L_A(b_m) = 0$. The $L_E(b_m|y_K)$ value in (4.6) represents the *extrinsic* log-likelihood ratio of the m-th bit. In each detection/decoding step, each element takes in the $L_A(b_m)$ values as input, and then carries out the detection/decoding procedure to calculate $L_E(b_m|y_K)$. The $L_E(b_m|y_K)$ value is then passed on to be used as the $L_A(b_m)$ value for the next detection/decoding step. In the very final detection/decoding round at the channel decoder, the $L_E(b_m|y_K)$ value calculated will be added to the $L_A(b_m)$ producing $L_D(b_m)$ according to (4.4). A decision then will be made to produce $\hat{b}_{m,MAP}$ through (4.1).

Since the main objective of a soft MAP detector is calculating (4.6), we will derive the expression of $L_E(y_K|b_m)$ in the context of our MMR system. First we compute $P[y_K|b_m = +1]$ as follows

$$P[y_K|b_m = +1] = \sum_{x \in \mathcal{S}_{m,+1}} p(y_K|x) \times P[\mathbf{b}_{[m]}] \tag{4.7}$$

where $\mathcal{S}_{m,+1}$ is the subset of $\mathcal{S}$ containing the symbols that have $b_m = +1$ mapped to them. $\mathbf{b}$ represents the vector of coded-modulated bits mapping to the each symbol $x \in \mathcal{S}_{m,+1}$ according to (4.3). $\mathbf{b}_{[m]}$ denotes the $\mathbf{b}$ vector with the m-th element removed. Assuming that the coded bits are independent due to the presence of an interleaver, we can write $P[\mathbf{b}_{[m]}]$ simply as the product of the probabilities of the individual bits given by

$$P[\mathbf{b}_{[m]}] = \prod_{i=1:i\neq m}^{\log_2(M)} P[b_i = (\mathbf{b})_i] \tag{4.8}$$

where $(\mathbf{b})_i$ denotes the actual i-th element of $\mathbf{b}$. Substituting (4.8) into (4.7) will give us

$$P[y_K|b_m = +1] = \sum_{x \in \mathcal{S}_{m,+1}} p(y_K|x) \prod_{i=1:i\neq m}^{\log_2(M)} P[b_i = (\mathbf{b})_i]. \tag{4.9}$$

Similarly, we can also write

$$P[y_K|b_m = -1] = \sum_{x \in \mathcal{S}_{m,-1}} p(y_K|x) \prod_{i=1:i\neq m}^{2^M} P[b_i = (\mathbf{b})_i] \tag{4.10}$$

where $\mathcal{S}_{m,-1}$ is the subset of $\mathcal{S}$ containing the symbols that have $b_m = -1$ mapped to them. Note that $\mathcal{S}_{m,+1}$ and $\mathcal{S}_{m,-1}$ form a partition of the constellation $\mathcal{S}$.

We can write out the individual bit probabilities by modifying the log-*a priori* ratio in (4.5) by writing

$$\begin{aligned}\exp(L_A(b_m)) &= \frac{P[b_m = +1]}{P[b_m = -1]} \\ &= \frac{P[b_m = +1]}{1 - P[b_m = +1]}.\end{aligned} \tag{4.11}$$

A simple manipulation of (4.11) gives

$$P[b_m = +1] = \frac{\exp\left(L_A(b_m)\right)}{1 + \exp(L_A(b_m))} \tag{4.12}$$

and consequently

$$\begin{aligned}P[b_m = -1] &= 1 - P[b_m = +1] \\ &= 1 - \frac{\exp\left(L_A(b_m)\right)}{1 + \exp\left(L_A(b_m)\right)} \\ &= \frac{1}{1 + \exp\left(L_A(b_m)\right)}.\end{aligned} \tag{4.13}$$

By looking at (4.12) and (4.13), we can write a general expression for the probability of each bit location being $(\mathbf{b})_m$ as

$$P[b_m = (\mathbf{b})_m] = \frac{\exp\left(\frac{(1+(\mathbf{b})_m)L_A(b_m)}{2}\right)}{1 + \exp\left(L_A(b_m)\right)}. \tag{4.14}$$

We make the following manipulations to (4.14)

$$\begin{aligned}P[b_m = (\mathbf{b})_m] &= \frac{\exp\left(\frac{L_A(b_m)}{2}\right) \times \exp\left(\frac{(\mathbf{b})_m L_A(b_m)}{2}\right)}{1 + \exp\left(L_A(b_m)\right)} \\ &= \frac{\exp\left(\frac{(\mathbf{b})_m L_A(b_m)}{2}\right)}{\exp\left(\frac{L_A(b_m)}{2}\right) + \exp\left(\frac{-L_A(b_m)}{2}\right)}.\end{aligned} \tag{4.15}$$

Note that the denominator in (4.15) is independent of the elements of $\mathbf{b}$. Now we substitute (4.15) into (4.9) and (4.10) to get

$$P[y_K|b_m=+1] = \sum_{x\in\mathcal{S}_{m,+1}} p(y_K|x) \prod_{i=1:i\neq m}^{\log_2(M)} \frac{\exp\left(\frac{(\mathbf{b})_i L_A(b_i)}{2}\right)}{\exp\left(\frac{L_A(b_i)}{2}\right)+\exp\left(\frac{-L_A(b_i)}{2}\right)} \tag{4.16}$$

$$P[y_K|b_m=-1] = \sum_{x\in\mathcal{S}_{m,-1}} p(y_K|x) \prod_{i=1:i\neq m}^{\log_2(M)} \frac{\exp\left(\frac{(\mathbf{b})_i L_A(b_i)}{2}\right)}{\exp\left(\frac{L_A(b_i)}{2}\right)+\exp\left(\frac{-L_A(b_i)}{2}\right)}. \tag{4.17}$$

We now substitute (4.16) and (4.17) into (4.6) to get

$$\begin{aligned} L_E(y_K|b_m) &= \ln \frac{\sum_{x\in\mathcal{S}_{m,+1}} p(y_K|x) \prod_{i=1:i\neq m}^{\log_2(M)} \frac{\exp\left(\frac{(\mathbf{b})_i L_A(b_i)}{2}\right)}{\exp\left(\frac{L_A(b_i)}{2}\right)+\exp\left(\frac{-L_A(b_i)}{2}\right)}}{\sum_{x\in\mathcal{S}_{m,-1}} p(y_K|x) \prod_{i=1:i\neq m}^{\log_2(M)} \frac{\exp\left(\frac{(\mathbf{b})_i L_A(b_i)}{2}\right)}{\exp\left(\frac{L_A(b_i)}{2}\right)+\exp\left(\frac{-L_A(b_i)}{2}\right)}} \\ &= \ln \frac{\sum_{x\in\mathcal{S}_{m,+1}} p(y_K|x) \prod_{i=1:i\neq m}^{\log_2(M)} \exp\left(\frac{(\mathbf{b})_i L_A(b_i)}{2}\right)}{\sum_{x\in\mathcal{S}_{m,-1}} p(y_K|x) \prod_{i=1:i\neq m}^{\log_2(M)} \exp\left(\frac{(\mathbf{b})_i L_A(b_i)}{2}\right)} \end{aligned} \tag{4.18}$$

which for our system can be written as

$$L_E(y_K|b_m, \prod_{i=1}^{K} h_i) = \ln \frac{\sum_{x\in\mathcal{S}_{m,+1}} p(y_K|x, \prod_{i=1}^{K} h_i) \exp\left(\frac{\mathbf{b}_{[m]}\cdot\mathbf{L}_{A[m]}}{2}\right)}{\sum_{x\in\mathcal{S}_{m,-1}} p(y_K|x, \prod_{i=1}^{K} h_i) \exp\left(\frac{\mathbf{b}_{[m]}\cdot\mathbf{L}_{A[m]}}{2}\right)} \tag{4.19}$$

where $\mathbf{L}_A$ represents the vector of *a priori* bit information that may or may not be available as feedback from the channel decoder, and $\mathbf{b}_{[m]} \cdot \mathbf{L}_{A[m]}$ denotes the inner product of both vectors. The likelihood function in (4.19) for this system is the same one given in (2.7). From (4.19), we see that evaluating the extrinsic information requires evaluating $(K-1)\times M$ numerical integrations per modulated bit, which is intolerable even for the simple dual-hop scenario.

Before we proceed any further, we introduce the following approximation to (2.7) such that $p(y_K|x, \prod_{i=1}^{K} h_i) \approx \tilde{p}(y_K|x, \prod_{i=1}^{K} h_i)$ where

$$\tilde{p}(y_K|x, \prod_{i=1}^{K} h_i) = \frac{1}{\pi} \int_0^\infty \cdots \int_0^\infty \frac{\exp\left(\frac{-\beta_K}{\sum_{i=1}^{K-1} \left(\prod_{j=i+1}^{K} z_j \right) \left(\prod_{q=i}^{K-1} \alpha_q^2 \right) \sigma_i^2 + \sigma_K^2} \right)}{\sum_{i=1}^{K-1} \left(\prod_{j=i+1}^{K} z_j \right) \left(\prod_{q=i}^{K-1} \alpha_q^2 \right) \sigma_i^2 + \sigma_K^2} \times \left(\prod_{i=2}^{K} f_{|h_i|^2}(z_i) \right) d_{z_2} \cdots d_{z_K}. \tag{4.20}$$

Note that the approximation in 4.20 ignores the effect of $\left(\prod_{i=1}^{K} h_i \right)$ on the joint distribution of all the $|h_i|^2$ random variables. With this approximation, the extrinsic information is given by

$$
\begin{aligned}
L_E(y_K|b_m,\prod_{i=1}^{K}h_i) \approx \ln \sum_{x\in\mathcal{S}_{m,+1}} \tilde{p}(y_K|x,\prod_{i=1}^{K}h_i)\exp\left(\frac{\mathbf{b}_{[m]}\cdot\mathbf{L}_{A[m]}}{2}\right) \\
- \ln \sum_{x\in\mathcal{S}_{m,-1}} \tilde{p}(y_K|x,\prod_{i=1}^{K}h_i)\exp\left(\frac{\mathbf{b}_{[m]}\cdot\mathbf{L}_{A[m]}}{2}\right) \\
= \ln \sum_{x\in\mathcal{S}_{m,+1}} \exp\left(\frac{\mathbf{b}_{[m]}\cdot\mathbf{L}_{A[m]}}{2}\right) \\
\times \int_0^\infty \cdots \int_0^\infty \frac{\exp\left(\frac{-\beta_K}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} z_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2}\right)}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} z_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2}\left(\prod_{i=2}^{K} f_{|h_i|^2}(z_i)\right) d_{z_2}\cdots d_{z_K} \\
- \ln \sum_{x\in\mathcal{S}_{m,-1}} \exp\left(\frac{\mathbf{b}_{[m]}\cdot\mathbf{L}_{A[m]}}{2}\right) \\
\times \int_0^\infty \cdots \int_0^\infty \frac{\exp\left(\frac{-\beta_K}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} z_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2}\right)}{\sum_{i=1}^{K-1}\left(\prod_{j=i+1}^{K} z_j\right)\left(\prod_{q=i}^{K-1}\alpha_q^2\right)\sigma_i^2+\sigma_K^2}\left(\prod_{i=2}^{K} f_{|h_i|^2}(z_i)\right) d_{z_2}\cdots d_{z_K}.
\end{aligned}
\tag{4.21}
$$

For more details on soft decoding of concatenated systems, the reader is referred to [27] where the issue is addressed in the context of multi-antenna systems.

4.2 Dual Hop Scenario

In MMR networks it has been argued that the use of a smaller number of hops is more effective [5]. In particular, the special class of dual-hop networks has been shown to be very practical [28]. Dual-hop architectures are particularly popular in the case where the integration between cellular and ad hoc networks is desired. In this section, we propose two approximate MAP detection schemes for dual-hop systems to avoid the

need of evaluating the integral given in (4.21). For the dual-hop case ($K = 2$), (4.21) reduces to

$$L_E(y_2|b_m, \prod_{i=1}^{2} h_i) \approx \ln \sum_{x \in \mathcal{S}_{m,+1}} \exp\left(\frac{\mathbf{b}_{[m]} \cdot \mathbf{L}_{A[m]}}{2}\right) \int_0^\infty \frac{\exp\left(\frac{-\beta_2}{\alpha_1^2 z\sigma_1^2 + \sigma_2^2}\right)}{\alpha_1^2 z\sigma_1^2 + \sigma_2^2} f_{|h_2|^2}(z)dz$$
$$- \ln \sum_{x \in \mathcal{S}_{m,-1}} \exp\left(\frac{\mathbf{b}_{[m]} \cdot \mathbf{L}_{A[m]}}{2}\right) \int_0^\infty \frac{\exp\left(\frac{-\beta_2}{\alpha_1^2 z\sigma_1^2 + \sigma_2^2}\right)}{\alpha_1^2 z\sigma_1^2 + \sigma_2^2} f_{|h_2|^2}(z)dz. \tag{4.22}$$

4.2.1 Gaussian Approximation

Our first approximation for (4.22) is to make the assumption that the noise component $\tilde{n}$ of (2.2) is of a Gaussian distribution. Specifically, we first define a Gaussian random variable $\tilde{n}_G \sim \mathcal{CN}(0, \alpha_1^2\Omega_2\sigma_1^2 + \sigma_2^2)$ such that $E[\tilde{n}_G] = E[\tilde{n}]$ and $Var[\tilde{n}_G] = Var[\tilde{n}]$. We then approximate $\tilde{n} \approx \tilde{n}_G$. Consequently, using the Max-log approximation [27], we can rewrite (4.22) as

$$L_E(y_2|b_m, \prod_{i=1}^{2} h_i) \approx \max_{x \in \mathcal{S}_{m,+1}} \left[\frac{-\beta_2}{\alpha_1^2\Omega_2\sigma_1^2 + \sigma_2^2} + \frac{\mathbf{b}_{[m]} \cdot \mathbf{L}_{A[m]}}{2}\right]$$
$$- \max_{x \in \mathcal{S}_{m,-1}} \left[\frac{-\beta_2}{\alpha_1^2\Omega_2\sigma_1^2 + \sigma_2^2} + \frac{\mathbf{b}_{[m]} \cdot \mathbf{L}_{A[m]}}{2}\right]. \tag{4.23}$$

We can justify this assumption particularly for the cases where the variance of the noise component at the second hop dominates the variance at the first hop, i.e. $\sigma_2^2 \gg \alpha_1^2\Omega_2\sigma_1^2 = (E_{r,1}\Omega_2\sigma_1^2)/(E_s\Omega_1 + \sigma_1^2)$, since in that case $\tilde{n} \approx n_2$ and n_2 has a Gaussian distribution. If we take $\sigma_1^2 = \sigma_2^2$, then this would mean that the Gaussian approximation is very accurate when $E_s\Omega_1 \gg E_{r,1}\Omega_2$, or equivalently when SNR_1 is much higher than SNR_2. This is a typical scenario in the downlink of a relay network

where the channel between the base-station and the relay (which can be an analog repeater) is a Ricean channel with a strong LOS component.

$$\sigma_2^2 \gg \alpha_1^2 \Omega_2 \sigma_1^2 = \frac{E_{r,1}\Omega_2\sigma_1^2}{E_s\Omega_1 + \sigma_1^2} \tag{4.24}$$

4.2.2 Mean Value Theorem Based Approximation

Although the Gaussian approximation performs extremely well when SNR_1 is larger than SNR_2, it degrades in performance when the SNR_2 is higher than SNR_1. This motivates devising another scheme that better evaluates (4.22). We propose our second MAP approximating scheme for (4.22) based on the following theorem.

Theorem 2 (The first Mean Value Theorem for integration (MVT)) [29]: Suppose that $f(x)$ is continuous and bounded, that $g(x)$ is integrable over the interval (p, q) where $p < q$, and that $g(x)$ does not change sign anywhere in (p, q). Then, there exists at least one point γ with $p \leq \gamma \leq q$ such that

$$\int_p^q f(x)g(x)dx = f(\gamma)\int_p^q g(x)dx. \tag{4.25}$$

□

By the MVT, we can write $p(y_2|x)_{app}$ as

$$\int_0^\infty \frac{\exp\left(\frac{-\beta_2}{\alpha_1^2 z\sigma_1^2+\sigma_2^2}\right)}{\alpha_1^2 z\sigma_1^2 + \sigma_2^2} f_{|h_2|^2}(z)dz = \frac{1}{\alpha_1^2\gamma\sigma_1^2 + \sigma_2^2}\exp\left(\frac{-\beta_2}{\alpha_1^2\gamma\sigma_1^2 + \sigma_2^2}\right) \tag{4.26}$$

where $\gamma \in [0, \infty)$. We dropped the constant $1/\pi$ since it cancels out in the calculation of (4.21). Worthy of note here is our justification for choosing the appropriate functions to separate the integrand in (4.26) since the MVT allows some flexibility

in choosing the functions, $f(x)$ and $g(x)$, that form the integrand. Our solution is to always separate the integrand as in (4.26) into a Gaussian function and the PDF of $|h_2|^2$. First of all, this solution is valid for all channels regardless of their distributions. Our integrand will always be equivalent to the Gaussian function on the right hand side of (4.26) multiplied by the integration of the distribution of $|h_2|^2$ which is equal to 1. The second reason for using the Gaussian function is that it provides us with the convenience of using the Max-log approximation, and that can provide a significant complexity reduction in the soft detection process. Therefore, our problem is now directed towards finding the right γ value to evaluate $\tilde{p}(y_2|x, \prod_{i=1}^{2} h_i)$ in (4.26). Note that in (4.26), γ is a function of $\gamma(\beta_2, \mu, \sigma_2^2)$ where $\mu = \alpha_1^2\sigma_1^2$. If we were able to know γ for a fixed $\{\beta_2, \mu, \sigma_2^2\}$ then we could exactly evaluate $\tilde{p}(y_2|x, \prod_{i=1}^{2} h_i)$ by the right hand side of (4.26). One may solve for γ in (4.26) numerically for different possible values of $\{\beta_2, \mu, \sigma_2^2\}$ and use it to exactly evaluate $\tilde{p}(y_2|x, \prod_{i=1}^{K} h_i)$. That solution however may not be feasible because there are three variables and the range of each variable is also too large to handle. To solve this dependency on many variables, we introduce the following manipulations to (4.26)

$$\frac{1}{\sigma_2^2}\int_0^\infty \frac{\exp\left(\frac{-\beta_2/\sigma_2^2}{\left(\alpha_1^2 z\sigma_1^2\right)/\sigma_2^2+1}\right)}{\frac{\alpha_1^2 z\sigma_1^2}{\sigma_2^2}+1} f_{|h_2|^2}(z)dz = \frac{1}{\sigma_2^2\left[\frac{\alpha_1^2\gamma\sigma_1^2}{\sigma_2^2}+1\right]}\exp\left(\frac{-\beta_2/\sigma_2^2}{\left(\alpha_1^2\gamma\sigma_1^2\right)/\sigma_2^2+1}\right) \tag{4.27}$$

which we rewrite as

$$\int_0^\infty \frac{1}{\epsilon z+1}\exp\left(\frac{-\acute{\beta}_2}{\epsilon z+1}\right) f_{|h_2|^2}(z)dz = \frac{1}{\epsilon\gamma+1}\exp\left(\frac{-\acute{\beta}_2}{\epsilon\gamma+1}\right) \tag{4.28}$$

where $\acute{\beta}_2 = \beta_2/\sigma_2^2$ and $\epsilon = \mu/\sigma_2^2$. Dividing by σ_2^2 will allow us to model γ as a

function of $\gamma(\acute{\beta}_2, \epsilon)$, reducing the dependency of γ to two variables, $\acute{\beta}_2$ and ϵ. When studying the behavior of γ, we noticed that it could be modeled, with a great deal of accuracy, by a simple function with respect to $\acute{\beta}_2$ given that ϵ is fixed. We model the relationship of $\gamma(\acute{\beta})_{|\epsilon}$ as

$$\gamma(\acute{\beta}_2)_{|\epsilon} \approx b\acute{\beta}_2^a \tag{4.29}$$

where $b > 0$ and $a \in [0, 1]$ are found numerically by minimizing the expected *Mean Squared Error* (MSE), $E[|\gamma_{|\epsilon} - b\acute{\beta}_2^a|^2]$, where the expectation is taken with respect to $\acute{\beta}_2$. Fig. 4.1 shows an example $\gamma(\acute{\beta}_2)$ curve, both numerically simulated and modeled according to (4.29), at various values of ϵ for a Rayleigh fading channel. Since the actual range of $\acute{\beta}_2$ is in $[0, \infty)$, we have truncated the range to $\acute{\beta}_2 \in [0, \acute{\beta}_{2,max}]$ such that $\int_0^{\acute{\beta}_{2,max}} f_{\acute{\beta}_2}(\omega)d\omega = 0.99$. Now we can solve for γ numerically in (4.28), and then model it as a function of $\acute{\beta}_2$. This numerical calculation is done off-line, and therefore will not incur any detection complexity. The $\gamma(\acute{\beta}_2) \approx b\acute{\beta}_2^a$ functions are calculated once for each ϵ value, and this one-dimensional tabulation can be stored at the destination. In the detection process, ϵ is first calculated then the appropriate $\gamma(\acute{\beta}_2)$ function is looked up. When β_2 is obtained, it is divided by the current σ_2^2 to obtain $\acute{\beta}_2$. The resulting value is then used in the tabulated function to obtain $\gamma(\acute{\beta}_2)$. Note that in theory the MVT is applicable with any adopted channel model, however this tabulation for the functions that approximate γ will have to be carried out for each different channel distribution. This is because the value of γ changes for every different channel distribution. (i.e. the γ value in (4.25) depends on $g(x)$).

For the case of a dual-hop MMR system with $h_i \sim \mathcal{CN}(0, 1)$ (Rayleigh fading on both hops), we give in Table 4.1 an example of the tabulated $\gamma(\acute{\beta}_2)$ functions

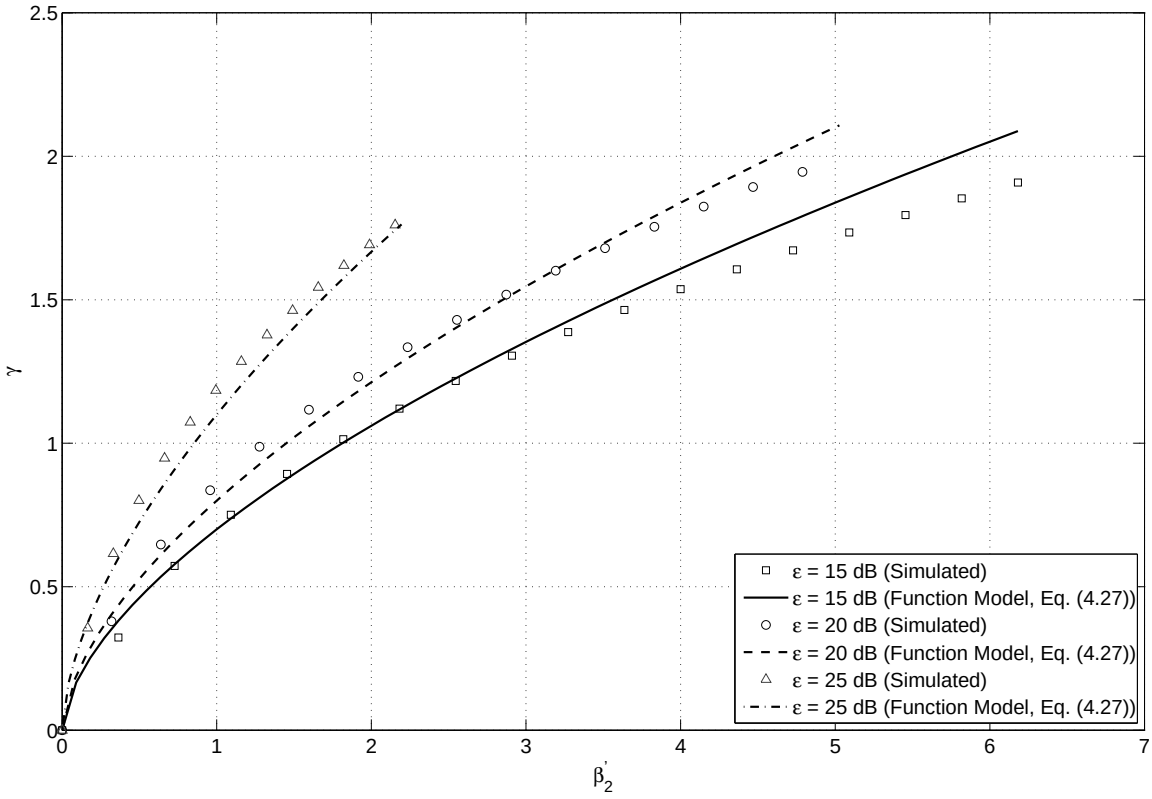

Figure 4.1: γ curve as a function of $\acute{\beta}_2$ at various ϵ values for the dual-hop network in a Rayleigh fading channel.

represented by the coefficients a and b for different values of $\epsilon \in \{0, \cdots, 30\}$ dB. The MSE value, d_{min}, between the function model and the actual γ is also given as a normalized percentage with respect to $E[|\gamma_{|\epsilon}|^2]$. It was observed numerically that the d_{min} values are extremely small, with a value always less than 0.2%.

We will see in Chapter 5 that the Gaussian approximation will perform very close to the ideal case with full CSI at the destination when the ratio $\sigma_1^2/\sigma_2^2 < 0$ dB, and that will roughly correspond to the $\epsilon < 0$ dB range (at high SNR_1, $\epsilon \approx (\sigma_1^2/\sigma_2^2)$ since $\alpha_1^2 \approx 1$). Therefore, there is no incentive to carry out the tabulation of the $\gamma(\acute{\beta}_2)$ functions in that range and that is why we provide Table 4.1 for the range of $\epsilon > 0$.

4.3 Multi-hop Scenario

We discuss in this section the extension of the approximate MAP schemes proposed for the dual-hop systems to systems with general K-hops.

4.3.1 Gaussian Approximation

Similar to Section 4.2.1, the Gaussian approximation can be applied to the multi-hop case by assuming that the distribution of the noise component $\tilde{n}$ is Gaussian. We define $\tilde{n}_G \sim \mathcal{CN}(0, \sum_{i=1}^{K-1} \left(\prod_{q=i}^{K-1} \alpha_q^2\right) \left(\prod_{j=i+1}^{K} \Omega_j\right) \sigma_i^2 + \sigma_K^2)$ and assume that $\tilde{n} \approx \tilde{n_G}$. This assumption is similarly justified on the downlink of the relay network when the SNR at the first hop (and earlier hops) is much better that the SNR at the very last hop. With this assumption, the extrinsic likelihood information can be written similar to (4.23) as

Table 4.1: Coefficients a and b of the function $\gamma(\acute{\beta}_2) = b\acute{\beta}_2^a$ along with the normalized MSE percentage d_{min} between the numerically calculated $\gamma(\acute{\beta}_2)$ and $\gamma(\acute{\beta}_2) = b\acute{\beta}_2^a$ for different ϵ values in a Rayleigh fading channel.

ϵ	a	b	d_{min}
0 dB	0.7	0.5	0.1960 %
1 dB	0.8	0.8	0.0486 %
2 dB	0.6	4.7	0.1406 %
3 dB	0.8	0.4	0.1754 %
4 dB	0.8	0.6	0.1187 %
5 dB	0.7	0.9	0.0547 %
6 dB	0.6	5.1	0.1334 %
7 dB	0.8	0.5	0.1494 %
8 dB	0.7	0.7	0.0628 %
9 dB	0.7	1.1	0.0341 %
10 dB	0.5	4.1	0.0973 %
11 dB	0.7	0.6	0.0587 %
12 dB	0.7	0.7	0.0725 %
13 dB	0.7	1.1	0.0289 %
14 dB	0.5	4.6	0.1168 %
15 dB	0.7	0.6	0.0638 %
16 dB	0.6	0.8	0.0537 %
17 dB	0.7	1.1	0.0325 %
18 dB	0.5	4.7	0.1013 %
19 dB	0.7	0.6	0.0817 %
20 dB	0.6	0.8	0.0475 %
21 dB	0.6	1.1	0.0279 %
22 dB	0.9	3.2	0.0030 %
23 dB	0.6	0.7	0.0855 %
24 dB	0.6	0.8	0.0484 %
25 dB	0.6	1.1	0.0305 %
26 dB	0.9	3.2	0.0033 %
27 dB	0.6	0.7	0.0845 %
28 dB	0.6	0.8	0.0449 %
29 dB	0.6	1.1	0.0257 %
30 dB	0.5	4.7	0.0715 %

$$L_E(y_K|b_m,\prod_{i=1}^{K}h_i)\approx \max_{x\in\mathcal{S}_{m,+1}}\left[\frac{-\beta_K}{\sum_{i=1}^{K-1}\Big(\prod_{q=i}^{K-1}\alpha_q^2\Big)\Big(\prod_{j=i+1}^{K}\Omega_j\Big)\sigma_i^2+\sigma_K^2}+\frac{\mathbf{b}_{[m]}\cdot\mathbf{L}_{A[m]}}{2}\right] - \max_{x\in\mathcal{S}_{m,-1}}\left[\frac{-\beta_K}{\sum_{i=1}^{K-1}\Big(\prod_{q=i}^{K-1}\alpha_q^2\Big)\Big(\prod_{j=i+1}^{K}\Omega_j\Big)\sigma_i^2+\sigma_K^2}+\frac{\mathbf{b}_{[m]}\cdot\mathbf{L}_{A[m]}}{2}\right]. \tag{4.30}$$

4.3.2 On the Mean Value Theorem Based Approximation

By [30, §12.11-Theorem 3] the MVT is applicable to (2.7), which means that the MVT-based MAP approximating scheme for the dual-hop network can be theoretically extended to the multi-hop case. By the MVT, the approximate likelihood function can be rewritten as

$$\tilde{p}(y_K|x,\prod_{i=1}^{K}h_i)=\frac{1}{\pi}\frac{\exp\left(\frac{-\beta_K}{\sum_{i=1}^{K-1}\Big(\prod_{j=i+1}^{K}\gamma_j\Big)\Big(\prod_{q=i}^{K-1}\alpha_q^2\Big)\sigma_i^2+\sigma_K^2}\right)}{\sum_{i=1}^{K-1}\Big(\prod_{j=i+1}^{K}\gamma_j\Big)\Big(\prod_{q=i}^{K-1}\alpha_q^2\Big)\sigma_i^2+\sigma_K^2} \tag{4.31}$$

where each $\gamma_j\in[0,\infty), j\in\{2,\cdots,K\}$. Therefore, knowing the appropriate $\{\gamma_2,\cdots,\gamma_K\}$, the likelihood function for the multi-hop scenario becomes a Gaussian function.

Unlike the dual-hop case, modeling the relationship between $\{\gamma_2,\cdots,\gamma_K\}$ and the variables that influence them is much more difficult because they are interrelated. Each $\gamma_j, j\in\{2,\cdots,K\}$ is actually a function of all others in the set, as well as $\sigma_i^2, i\in\{1,\cdots,K\}$ and β_K. We could introduce a normalization procedure, just as in the dual-hop case, to remove the dependency on $\sigma_i^2, i\in\{1,\cdots,K\}$; but that will not resolve the dependency on $\{\gamma_2,\cdots,\gamma_K\}$. This makes modeling each γ_j value a

multi-variable problem. One possibility is to approximate all $\gamma_i \approx \Omega_i, i \in \{2, \cdots, K : i \neq j\}$, normalize (4.31) with respect to $\sigma_i^2, i \in \{1, \cdots, K\}$, and then model the relationship of $\gamma_j(\beta)$. The issue with this solution is choosing the appropriate γ_j to model using a particular function fitting. In other words which channel do we need to accurately represent and which ones do we assume to be fixed (just like in the Gaussian approximation case), and those assumptions will vary depending on the application. Overall, for more than two hops, the Gaussian approximation is more suitable than the MVT-based approximation.

Chapter 5

Simulation Results

For the performance of hard ML detection, it was clear from Section 3.2 that the performance of the MMR system with our signalling strategy and the conventional strategy is identical. Therefore, we do not provide any simulation results for hard ML detection and only focus on the soft MAP detection case in what follows.

To validate our results, we have conducted numerical simulations for both the dual-hop and three-hop relay systems ($K = 2$ and $K = 3$) employing soft MAP detection at the destination. For all our simulations we have used a rate 1/2 LDPC code of length 204. We have assumed a block Rayleigh-fading channel with each $h_i \sim \mathcal{CN}(0, 1), i \in \{1, \cdots, K\}$, and changes every transmitted frame of 204 bits. We have used a 4-QAM constellation in all of our simulations.

5.1 Dual-Hop Network

For the dual-hop network simulations, we have assumed that the transmitted energy at each relay is normalized such that $E_{r_1} = E_s$. To compare the performance of

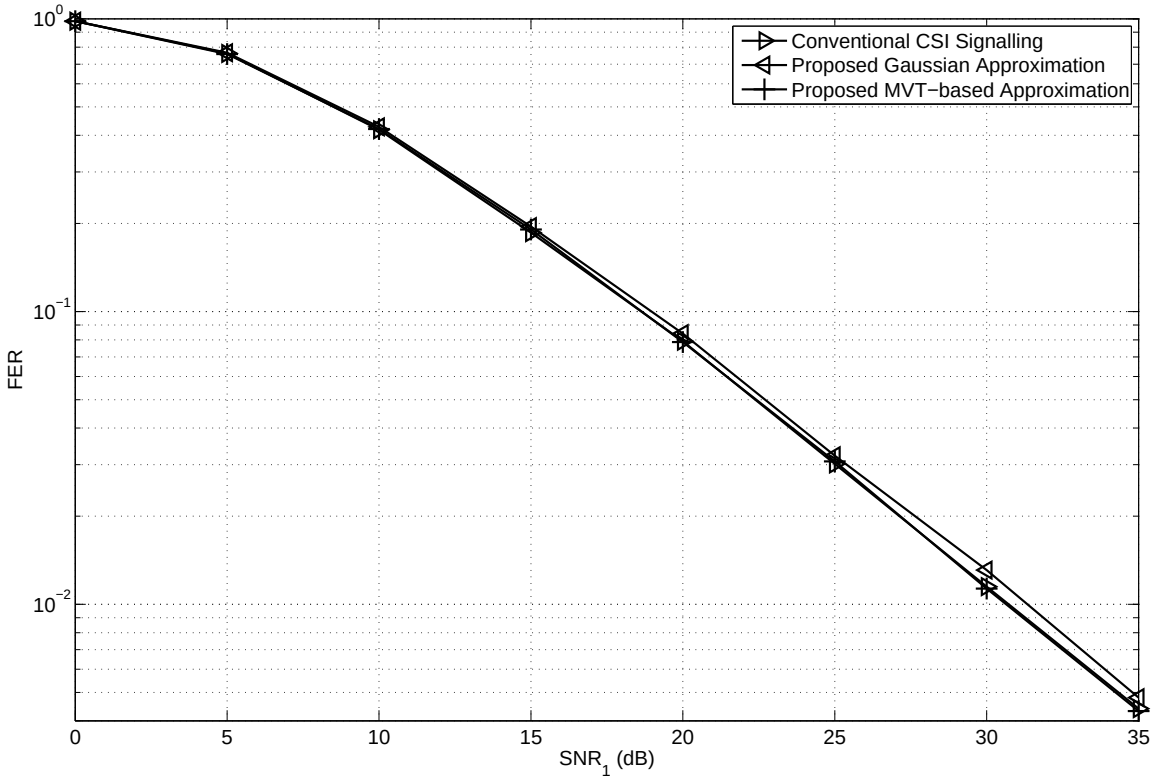

Figure 5.1: FER Performance of dual-hop network at $\sigma_1^2/\sigma_2^2 = 0$ dB.

our proposed schemes as opposed to the case with conventional CSI signalling, we varied the ratio σ_1^2/σ_2^2 for three different cases such that we have $SNR_1 \gg SNR_2$, $SNR_1 = SNR_2$, and $SNR_1 \ll SNR_2$. Fig. 5.1 shows the FER performance as a function of SNR_1 at $\sigma_1^2/\sigma_2^2 = 0$ dB. In it we see that the Gaussian approximation performs about 0.5 dB from conventional CSI signalling. As σ_1^2/σ_2^2 increases the Gaussian approximation starts degrading in performance, reaching about a 2 dB degradation in the extreme case of $\sigma_1^2/\sigma_2^2 = 30$ dB, as shown in Fig. 5.2. For the region of $\sigma_1^2/\sigma_2^2 > 0$ dB, our MVT-based approximation moves closer to the performance of the system with conventional CSI signalling. At $\sigma_1^2/\sigma_2^2 = 0$ dB, is it seen in Fig. 5.1

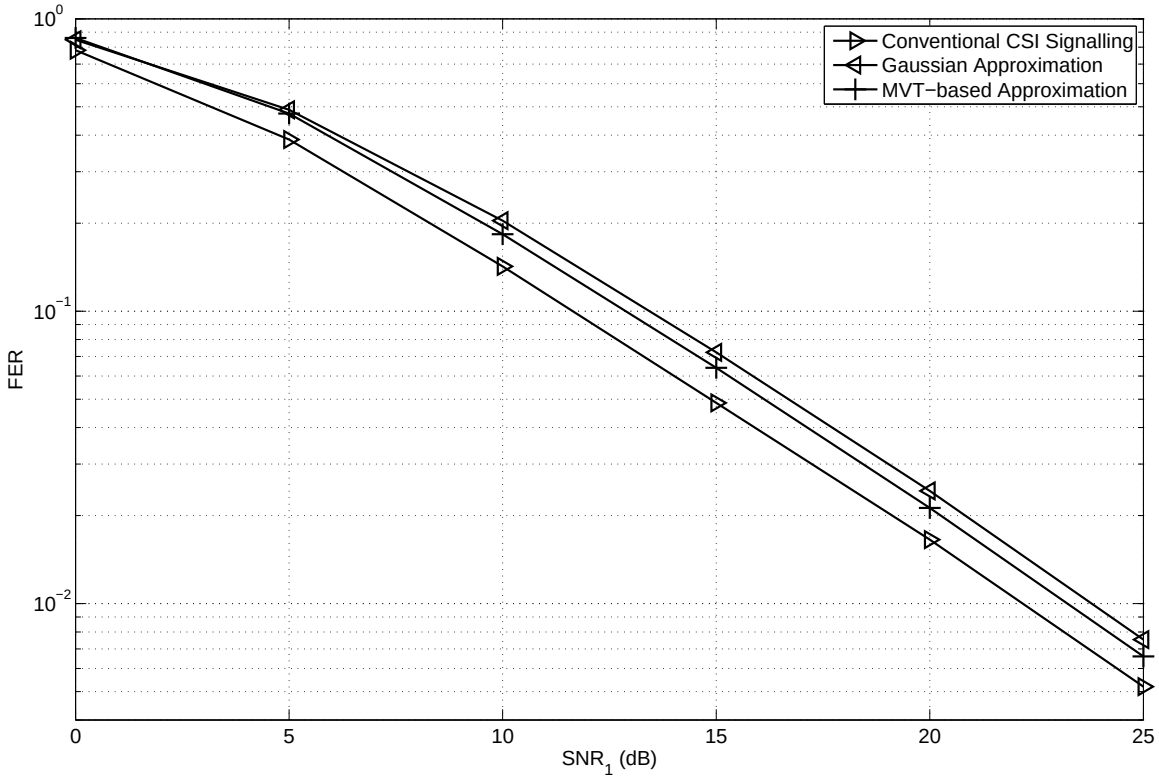

Figure 5.2: FER Performance of dual-hop network at $\sigma_1^2/\sigma_2^2 = 30$ dB.

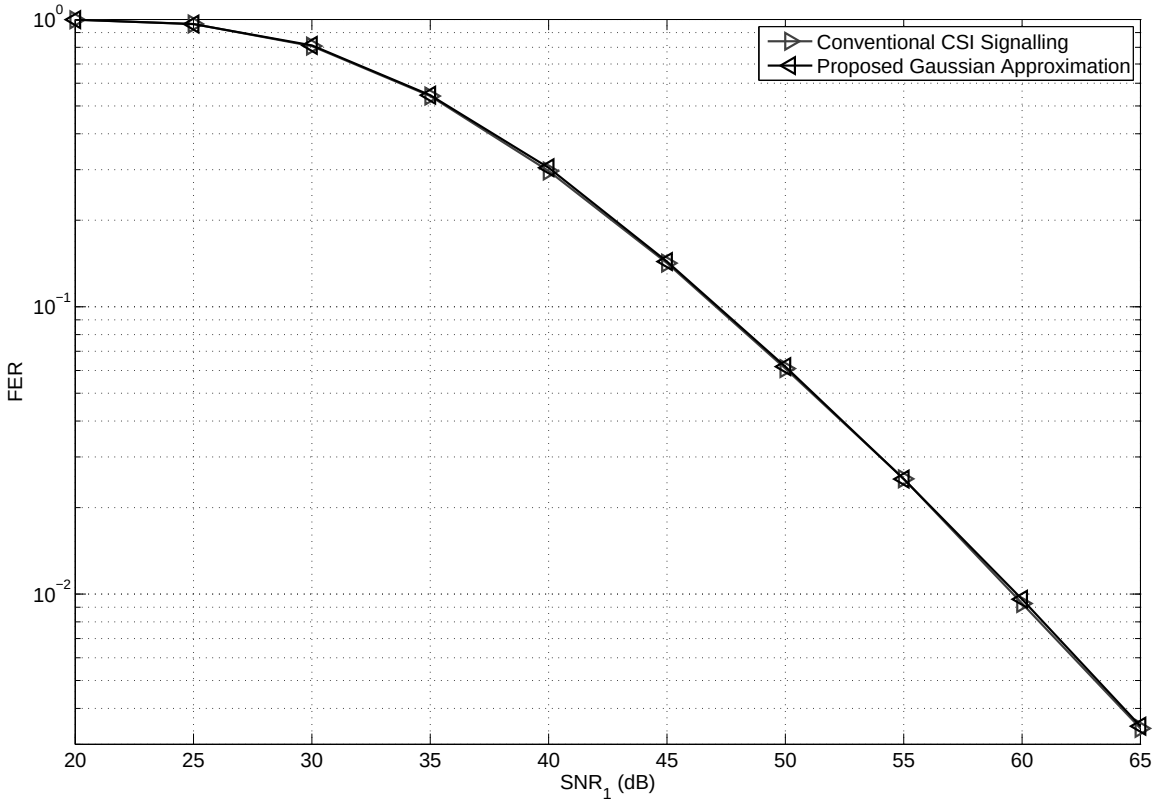

Figure 5.3: FER Performance of dual-hop network at $\sigma_1^2/\sigma_2^2 = -30$ dB.

that the MVT-based approximation performs almost identically to the conventional CSI signalling strategy. In the extreme case of $\sigma_1^2/\sigma_2^2 = 30$ dB, the MVT-based approximation loses about 1.5 dB in performance just as shown in Fig. 5.2. For the $\sigma_1^2/\sigma_2^2 < 0$ dB region, the Gaussian approximation starts approaching the performance of the conventional CSI signalling strategy, and therefore there is no real incentive to use the MVT-based approximation. On the other extreme case of $\sigma_1^2/\sigma_2^2 = -30$ dB, we see in Fig. 5.3 that the Gaussian approximation performs almost identically to the conventional CSI signalling case. Note that this small performance loss in the $\sigma_1^2/\sigma_2^2 = 0$ dB and $\sigma_1^2/\sigma_2^2 = 30$ dB cases comes at a significant 50% reduction in pilot

signalling overhead at the relay.

5.2 Three-Hop Network

Again for the three-hop network we have assumed that the transmitted energy at each relay is normalized such that $E_{r_i} = E_s, i \in \{1, 2\}$. Just as in the case of the dual-hop system, we have conducted performance simulations for three different channel conditions by varying the ratios of σ_1^2/σ_2^2 and σ_1^2/σ_3^2. At $\sigma_1^2/\sigma_2^2 = \sigma_1^2/\sigma_3^2 = 0$ dB, it is observed in Fig. 5.4 that the Gaussian approximation performs less than 1dB away from the conventional CSI signalling case. When $\sigma_1^2/\sigma_3^2 = -30$ dB and $\sigma_1^2/\sigma_2^2 = 0$ dB, we see in Fig. 5.5 that the performance of both the Gaussian approximation and the conventional CSI signalling case is almost identical. That is because when SNR_1 and SNR_2 dominate SNR_3, $\tilde{n} \approx n_3$ and its PDF approaches the Gaussian distribution. In the case of $\sigma_1^2/\sigma_2^2 = \sigma_1^2/\sigma_3^2 = 30$ dB we see in Fig. 5.6 an extreme case of performance degradation for the Gaussian approximation that results in a performance loss of about 2.5 dB. That is due to the fact that when SNR_1 is much worse than the other hops, the PDF of $\tilde{n}$ deviates away from a Gaussian PDF, and thats when the Gaussian approximation loses some of its accuracy. Note here that the performance loss for the $\sigma_1^2/\sigma_2^2 = \sigma_1^2/\sigma_3^2 = 0$ dB case and $\sigma_1^2/\sigma_2^2 = \sigma_1^2/\sigma_3^2 = 30$ dB case comes at a significant 60% reduction in relay pilot signalling overhead.

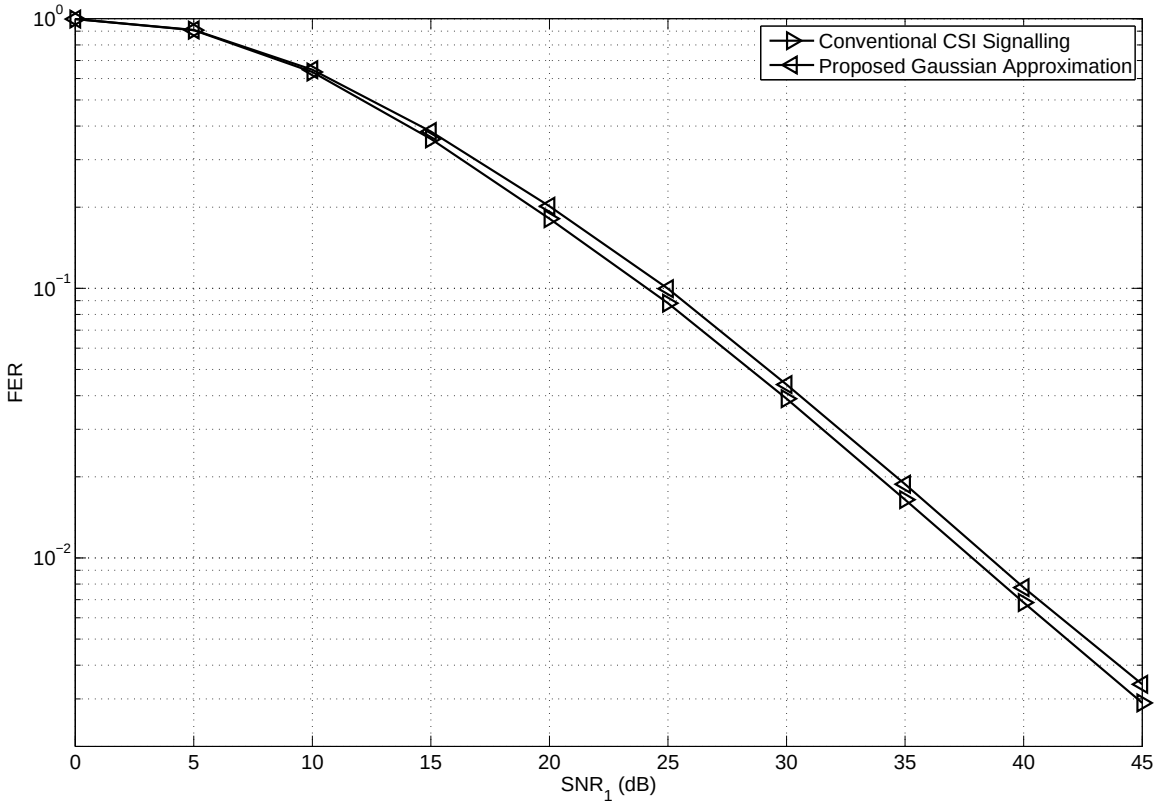

Figure 5.4: FER Performance of three-hop network at $\sigma_1^2/\sigma_2^2 = \sigma_1^2/\sigma_3^2 = 0$ dB.

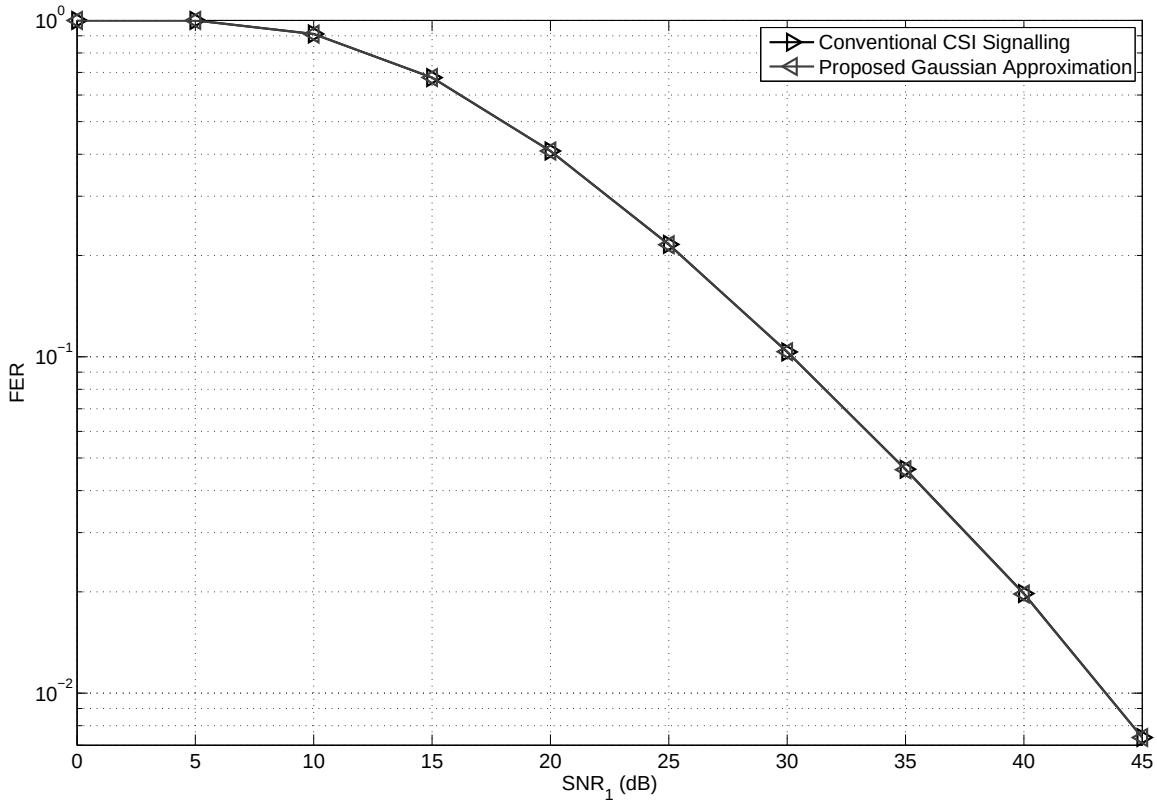

Figure 5.5: FER Performance of three-hop network at $\sigma_1^2/\sigma_2^2 = 0$ dB and $\sigma_1^2/\sigma_3^2 = -30$ dB.

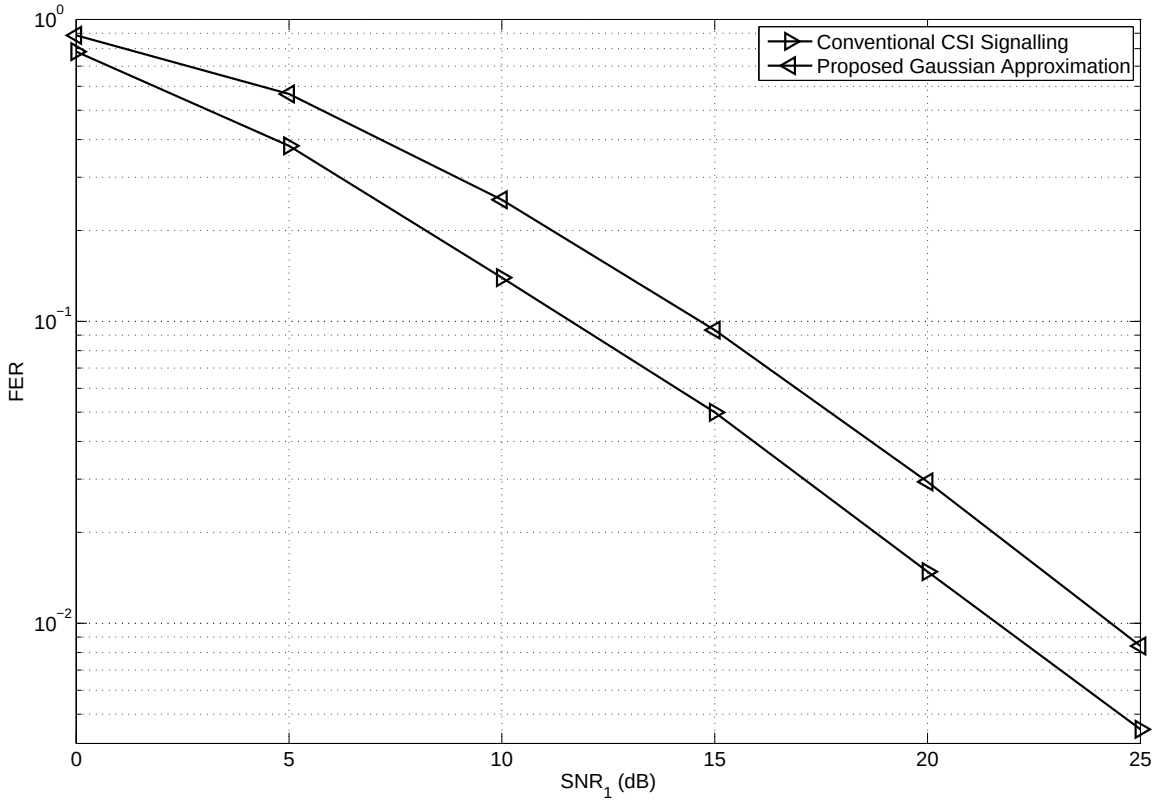

Figure 5.6: FER Performance of three-hop network at $\sigma_1^2/\sigma_2^2 = \sigma_1^2/\sigma_3^2 = 30$ dB.

Chapter 6

Conclusions and Further Work

6.1 Conclusions

This thesis proposed a new CSI signalling strategy for MMR networks. The system does not require CSI knowledge of individual hops at the destination, and consequently enjoys a significant reduction in the signalling overhead at each relay (at least 50%). Furthermore, our system does not require instantaneous CSI knowledge at the relays, and therefore eliminates their channel estimation burden. We have analyzed the signalling overhead ratio between our proposed CSI signalling strategy and the conventional CSI signalling strategy and showed that the ratio tends to zero as the number of relays increases. That means that the overhead savings of our CSI signalling strategy tends to 100% as the number of relays increases.

We have shown that this significant reduction in signalling overhead and channel estimation burden of our proposed CSI signalling strategy does not entail any performance penalty in terms of hard ML detection. We have proven that hard ML detection can still be reduced to the classical minimum Euclidian distance detector

even when the CSI of individual hops is not available at the destination. That is to say that we have proven that CSI of individual hops is unnecessary information for optimal hard ML detection.

When soft MAP detection is desired, we have shown that optimal detection requires the evaluation of multiple integrals at the receiver for each modulated bit, which is not feasible even for the simplest dual-hop systems. For the case of soft MAP detection for dual-hop systems, we have proposed two approximate MAP schemes that avoid the need of necessary integration computations at the destination. The Gaussian approximation performed almost identical to the conventional CSI strategy when $SNR_1 \gg SNR_2$, but suffered from some loss in performance when $SNR_1 \ll SNR_2$. That is where the more accurate MVT-based approximation performed closer to the conventional CSI signalling strategy. For the multi-hop case (i.e. more than one relay), we have shown that our Gaussian approximation scheme extended nicely to the multi-hop case and demonstrate that the performance is comparable to the conventional CSI signalling case except when $SNR_K \ll SNR_1$.

6.2 Further Work

The work presented in this thesis still has the potential for further development. One main issue to study is the extension of our proposed CSI signalling strategy to cooperative diversity networks. For example, the use of multi-branch single-hop networks has been extensively studied in literature as explained in Section 1.3, and finds use in many applications. Proposing such a reduction in signalling overhead for these systems would be extremely promising. The main issues to tackle would be the detection complexity and the possibility of achieving optimum detection in a

simplified manner.

Another important aspect is studying the extension of the MVT-based approximation to the case of more than one relay. In Section 4.3.2, we discussed further considerations for extending the MVT-based approximate MAP scheme to the multi-hop case. Addressing these considerations and devising a MAP scheme that performs closer to the conventional CSI assisted case would be highly desired.

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