The Monetary Transmission Mechanism and Business Cycles: The Role of Multi-stage Production with Inventories

by

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Abstract

This thesis studies the role of multi-stage production for the monetary transmission mechanism. I employ a monetary search model to show how multi-stage production influences both the long run and the short run effects of money growth. Multi-stage production provides an additional channel for money growth having effects through intermediate goods between different production stages. Extending Shi's (1998) model from a single-stage to a multi-stage production model, I show that money growth rate has an unconventional long run effect on quantities per match, and the long run response of input inventory investment is different from that of output inventory investment.

Contrary to classic search models, the steady state effect of money growth on the quantity of finished goods per match is not monotonic and depends on the money growth rate. Furthermore, in steady state the quantities per match first increase with the growth rate of money, before falling for large growth rates. Input inventories arise due to search frictions. Money growth also has hump-shaped real effects on steady state input inventory investment. The intermediate goods build a bridge between the labor market and the finished goods market. Intuitively, households hire more labor with higher future revenue and produce more intermediate goods in order to match the employment level. With more labor and more intermediate goods, finished goods

producers can produce more when matched. As a consequence, they are stuck with more input inventories. Moreover, my model suggests that changes in the money growth rate would be one of the reasons for the decline of the inventory-to-sales ratio since the mid-1980s.

Finally, I calibrate my model to quarterly US data. Contrary to other work, my model is able to replicate the stylized facts on inventory movements over the business cycle by solely relying on monetary shocks. The theoretical impulse response functions can quantitatively reproduce the corresponding empirical ones estimated in a structure autoregressive model. Moreover, the quantitative analysis supports the argument that input inventories amplify aggregate fluctuations over business cycles.

Dedication

I lovingly dedicate this thesis to my parents, who have offered me unconditional love and support throughout my life. Also, this thesis is dedicated to my husband, who encouraged me each step of the way. Finally, this thesis is dedicated to my son, who has helped me develop into a real grown-up.

Acknowledgments

I owe sincere and earnest thankfulness to my Ph.D. supervisors, Prof. Allen Head and Prof. Thorsten Koeppl, for their enormous guidance and encouragement. Prof. Allen Head showed me how to understand the big picture of economics and guided me in the right direction. Under Prof. Thorsten Koeppl's wholehearted guidance, I have been able to become a highly motivated, independent, and well-organized economist.

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Chapter 1

Introduction

In this thesis, I study the role of multi-stage production for the effects of money growth on both the steady state and fluctuations in the economy. To base this study in real situations, I build a multi-stage production model in which intermediate goods connect the labor market and the finished goods market. I show that the multi-stage production model improves the depth of understanding of the monetary transmission mechanism. Intermediate goods play an important role in the monetary transmission mechanism. In particular, they affect both the steady state effects of money growth on real variables and the corresponding short run dynamic responses. By calibrating the model to the U.S. data, I show that a multi-stage production model can reproduce the stylized facts of input inventories by relying solely on monetary shocks. Moreover, the model predicts that input inventories amplify the effects of monetary shocks over the business cycles.

The quantitative analysis shows that input inventories amplify monetary shocks over business cycles. Moreover, input inventories are quantitatively important in terms of matching the data. These findings are consistent with the inventory literature in which input inventories are empirically more important than output inventoreis. Empirical studies show that inventories are procyclical and amplify aggregate fluctuations over business cycles. Such effects are mainly attributed to input instead of output inventories,[1] because input inventories are the biggest and most volatile component of inventories. Humphreys, Maccini and Schuh (2001) show that, on average, manufacturing firms hold more than two times the input inventories than output inventories, and that input inventories are three times more volatile. However, most of the literature either does not distinguish between input and output inventories or only looks at output inventories (e.g. Jung and Yun (2006); Khan and Thomas (2007a); Ramey and West (1999); Shi (1998)).

In my model, I employ a large household model with a monetary propagation mechanism à la Shi (1998). I extend Shi's model from single-stage production to two-stage production by introducing intermediate goods. Households produce intermediate goods in the first stage and finished goods in the second stage. I assume that the intermediate goods are from home production in the benchmark model and generalize this assumption in the full model. Besides material inputs, labor (which can be hired from a frictional labor market) is also required for finished goods production. In the model, intermediate goods facilitate finished goods production and only finished goods yield utility for households. Input inventories arise in my model because of a search friction in the finished goods market. Unmatched finished goods producers hold unused intermediate goods at the end of each period. This is in sharp contrast to the literature, where a fixed delivery cost or a stockout avoidance motive is often

[1] Input inventories are inventories of materials and supplies and inventories of work-in-process, and output inventories are inventories of finished goods.

assumed to rationalize inventories.[2] Moreover, only input inventories are modeled in this paper by assuming that finished goods producers produce in the finished goods market only if they form a match.

In the steady state analysis, I find that the multi-stage production affects the impacts of the money growth rate on real variables. The model predicts that the money growth rate has an unconventional long run effect on quantities per match, and the long run response of input inventory investment is different from that of output inventory investment. In particular, the effects of money growth on the quantity of finished goods per match are not monotonic and depend on the money growth rate. In particular, when the money growth rate is low, finished goods producers trade more goods in each match in face of a higher money growth rate. On the other hand, the quantity of finished goods per match decreases because of the declined real money balances. These results are very different from that of a one sector search model, in which money growth has monotonic real effects on the quantity of goods per match.[3]

The intermediate goods build a bridge between the labor market and the finished goods market. For low levels of the money growth rate, households post more vacancies when the money growth rate increases. In the benchmark model, I assume an extreme case, whereby material inputs and labor are not substitutable, as a result of which households produce more intermediate goods in order to meet the employment level. By having more labor and more intermediate goods, finished goods producers could produce more when matched. In contrast, if the money growth rate is high,

[2]For the (S, s) model, see Fisher and Hornstein (2000); Khan and Thomas (2007a,b) and Scarf (1960). For the stockout avoidance model, see Bils (2004); Bils and Kahn (2000); Coen-Pirani (2004); Wen (2008) and Wen (2011).

[3]See Shi (1997, 1999). The quantity of goods per match in the Lagos and Wright type of monetary search model also decreases with the money growth rate, for example Aruoba, Waller, and Wright (2011); Lagos and Wright (2005); Rocheteau and Wright (2005) and Telyukova and Wright (2008).

households post fewer vacancies because the profitability of hiring decreases. Therefore, households produce fewer intermediate goods and the quantity of finished goods decreases with a higher money growth rate.

This unconventional response of quantities per match implies that, for low levels of the money growth rate, households hold more input inventories when the money growth rate increases. Since input inventories are simply intermediate goods that are unused at the end of each period, they depend on the total number of unmatched finished goods producers and the quantity of intermediate goods held by each finished goods producers. In the benchmark model, the quantity of intermediate goods held by each finished goods producer equals the quantity of goods per match. When the quantity of goods per match increases with the money growth rate, it has a positive effect on input inventories, which enables the input inventory investment (as well as input inventories) to increase with the money growth rate and, hence, with GDP (and final sales) in the long run. Such a response of input inventories is different from that of output inventories which decrease with output (and final sales) in the long run. As a result, input and output inventories play different roles in the monetary transmission mechanism.

The multi-stage production not only connects the labor market and the finished goods market through intermediate goods, it also transfers the effects on final sales from the downstream market back to the upstream market. Since agents are randomly matched in the finished goods market, households do not observe which finished goods producers would get a match before entering the market. Thus households have to adjust intermediate goods levels for all finished goods producers including unmatched producers in response to a money growth shock. As a consequence, unmatched finished goods producers hold more intermediate goods if the money growth rate is low. Therefore, households are stuck with more input inventories even when their final sales increase. A negative effect on final sales will be transferred back to the intermediate goods market when the money growth rate is high.

By calibrating a generalized model to the quarterly U.S. data, I show that monetary shocks are also helpful for explaining inventory behavior. The quantitative analysis shows that without technology shocks, a large household model with multi-stage production can reproduce stylized facts of input inventories by relying solely on monetary shocks. Most importantly, the model predicts procyclical inventory investment, a countercyclical inventory-to-sales ratio, more volatile output relative to final sales, and a positive correlation between inventory investment and final sales. Researchers in the inventory literature also use multi-stage production to model input inventories, although they use different frictions to rationalize inventories. But most papers in the inventory literature rely on real shocks to capture inventory regularities. If there were preference shocks, there would be tradeoffs between inventory investment and final sales. As a result, these models, such as the production smoothing model and stockout avoidance model, usually predict counterfactual results, in particular, countercyclical inventory investment and a negative correlation between final sales and inventory investment. With technology shocks, the model produces qualitatively similar impulse response functions as compared to the empirical ones.

The multi-stage production is the key for being able to match the data. The theoretical impulse response functions show that the quantity of finished goods per match stays above the steady state during the transition following a monetary shock.

Such positive responses of quantities per match are strong enough to cause input inventories to move with final sales in the same direction during the transition, which is essential for reproducing the positive correlation between input inventory investment and final sales.

In a search model, the responses of input inventories are very different from that of output inventories. My model predicts hump-shaped responses of inventories to a positive money growth shock.[4] In contrast, output inventories serve as a buffer stock in a single-stage production model and respond negatively to a positive money growth shock as shown by Menner (2006). Therefore, output inventories decrease whenever sales increase, and it is hard to replicate inventory regularities without technology shocks. The sluggish responses of input inventories shed light on the "slow speed of adjustment" puzzle referred to in the literature.

Since some researchers argue that the reduced inventory level is attributed to the "Great Moderation", I use the multi-stage production model to examine the role of input inventories over business cycles. My results support this argument and predict that input inventories amplify aggregate fluctuations over the business cycle. Moreover, my results suggest that besides the improvement in inventory management, changes in the money growth rate is another possible reason for explaining the declining trend of (input) inventory-to-sales ratio since mid-1980s.

Search frictions are an important feature of this model. Theoretically, they are attributed to non-monotonic steady state responses. Quantitatively speaking, if both monetary shocks and technology shocks are modeled, search frictions in both goods

[4]This result is consistent with the argument made by Jung and Yun (2006) that output inventories and input inventories indeed have different behavior in response to shocks. They show hump-shaped responses of the sales-stock ratio and U-shaped responses of finished goods inventories to an expansionary monetary policy shock.

markets are important for matching the data. If only the monetary shocks are modeled, the intermediate goods market should be more responsive to the shock in order to match the stylized facts of inventories. Nevertheless, in both cases, the intermediate goods market should be more responsive than the finished goods market, otherwise final sales are not volatile enough and the inventory-to-sales ratio becomes more volatile than GDP, which is not consistent with empirical observations.

The remainder of this thesis is organized as follows. In Chapter 2, I discuss related work in both the inventory literature and the search literature. In Chapter 3, I describe the data and document the empirical evidence on input behaviors. I also estimate a structural vector autoregressive model and reports empirical impulse response functions in order to compare with my theoretical impulse response functions. In Chapter 4, I describe a benchmark search model with multi-stage production and study how the multi-stage production affects the long run effects of money growth. In Chapter 5, I investigate the short run dynamics of a richer environment and examine the role of input inventories over the business cycle. I also conduct a sensitivity analysis of some parameters relative to the baseline calibration. Finally, Chapter 6 concludes this thesis.

Chapter 2

Literature Review

My thesis first fits into the inventory literature. Although there are debates within that literature about the role of inventories in the business cycle, a consensus has been reached regarding the importance of studying inventory behavior in order to understand the propagation mechanism of the business cycle. The importance of inventories can be implied by a series of stylized facts. As summarized by Khan and Thomas (2007b), there are five stylized facts associated with inventories and related economic aggregates. First, the volatility of inventory investment is large, accounting for, on average, 29.5% of the volatility of GDP. Second, inventory investment is procyclical. Third, inventory investment is positively correlated with final sales. The corresponding correlation coefficients with GDP and final sales are 0.67 and 0.41 respectively, as reported by Khan and Thomas. The forth stylized fact is a corollary of the third fact according to an accounting identity, namely output is more volatile than sales. This empirical result is often viewed as evidence that inventories play a destabilizing role over business cycles. In particular, inventories amplify aggregate fluctuations over the business cycle. Finally, the inventory-to-sales ratio is countercyclical.

Given the empirical stylized facts of procyclical inventory investment and its positive correlation with production, it is commonly believed in the literature that inventories play a destabilizing role in the economy, and especially that inventories amplify aggregate fluctuations over the business cycle. GDP volatility substantially decreased between 1984 and 2008 which lead to two decades of a "Great Moderation".[5] One explanation for this reduced volatility is the better inventory management that was adopted by firms after the 1980s. Around the same time, the inventory-to-sales ratio started to show a significant downward trend in the durable goods sector. For a further explanation see Kahn, McConnell, and Perez-Quiros (2002), who also show that improved information technology played a significant role in reducing output volatility. Irvine and Schuh (2005a,b) also provide evidence for this argument. On the other hand, Iacoviello, Schiantarelli, and Schuh (2011) argue that although inventories are important for propagating shocks, they do not amplify aggregate fluctuations. See also McConnell and Perez-Quiros (2000) and Cesaroni, Maccini, and Malgarini (2009) for an empirical study on these issues in a European context.

Together with the third regularity, a countercyclical inventory-to-sales ratio implies that inventories move with final sales in the same direction but in lower magnitude. This refers to the "slow speed of adjustment" puzzle in the literature; see Blinder and Maccini (1991). These stylized facts are also important criteria used in the literature to evaluate the model performance.

Beyond the stylized facts discussed at the beginning of this chapter, researchers have tried to highlight the importance of input inventories in business cycle fluctuations, which was first pointed out by Blinder and Maccini (1991). By decomposing

[5] See McConnell and Perez-Quiros (2000) and Ramey and Vine (2004) for identification of the structural break.

the data, they found that, in manufacturing, investment in input inventories is bigger and more volatile than investment in output inventories; and concluded that "most researchers seem to have barked up the wrong tree." This is true for even the narrowest definition of input inventories. This evidence is also confirmed by Humphreys, Maccini, and Schuh (2001), who examined longer time series. They found that manufacturing firms hold an average of more than twice the amount of input inventories than output inventories, and input inventories are three times more volatile than output inventories. Recent literature also shows different cyclical behavior between input and output inventories. As shown by Iacoviello, Schiantarelli, and Schuh (2011), the input inventory-target ratio is countercyclical, and the output inventory target ratio is "mildly procyclical." Jung and Yun (2006) also argue that output inventories and input inventories may have different behavior in response to shocks.

There are three main approaches used to rationalize inventories in the literature. They are the production smoothing model, the stockout avoidance model and the (S, s) model.[6] These models can be used to model either single-stage production or multi-stage production. Blinder and Maccini (1991), Fisher and Hornstein (2000) and Ramey and West (1999) focus on single-stage production and do not distinguish between input and output inventories. Humphreys, Maccini, and Schuh (2001) and Iacoviello, Schiantarelli, and Schuh (2011) use multi-stage production models to introduce intermediate goods and input inventories. But, nevertheless, all of these models have problem in regards to reproducing the stylized facts of inventories under demand shocks.

[6] Fisher and Hornstein (2000), Khan and Thomas (2007a,b) and Scarf (1960) use the (S, s) model; Bils (2004), Bils and Kahn (2000), Coen-Pirani (2004), Wen (2008) and Wen (2011) use the stockout avoidance motive; and Blinder and Maccini (1991), Ramey and West (1999) and Wang and Wen (2009) use the production smoothing motive to rationalized inventories.

As summarized by Blinder and Maccini (1991), the traditional production smoothing model is well known for its failure to reproduce procyclical inventory investment and more cyclical production relative to sales (also see Ramey and West (1999) for a comprehensive review). Firms in this model hold inventories due to a buffer-stock motive, because they face stochastic demand and increasing marginal cost. Since firms use inventories as a buffer to smooth production costs, inventories decrease whenever demand increases and vice versa, regardless of whether the shock is anticipated or unanticipated. As a result, the model implies sales are more volatile than production. The biggest improvement in this regard has been made by Wang and Wen (2009) who try to incorporate a production-cost-smoothing motive into the Dixit-Stiglitz RBC model. Under an aggregate TFP shock and an idiosyncratic cost shock, their model can predict standard inventory regularities, especially procyclical inventory investment and a hump-shaped output response.

The stockout avoidance model is self-explanatory. Firms hold inventories in this model, because they face delivery/production lags and have to commit to production before shocks are realized. Contrary to convex cost in the production smoothing model, firms hold inventories in the (S, s) model due to a fixed delivery cost.

Khan and Thomas (2007b) reviewed and evaluated the stockout avoidance model and the (S, s) model. They conclude that the (S, s) model with capital and technology shocks can reproduce key stylized facts of inventories, such as procyclical inventory investment, more variable production relative to sales, a countercyclical inventory to sales ratio and a positive correlation between inventory investment and sales. On the other hand, the (S, s) model predicts counterfactual results when there are shocks to demand. Specifically, it predicts countercyclical inventory investment, more variable sales relative to production and a negative relationship between final sales and inventory investment. Beyond possessing the ability to match the data, the computational cost of (S, s) model is significant because the (S, s) band varies over time.

In the same paper, Khan and Thomas also incorporate a basic stockout avoidance into a DSGE model. They find that under aggregate shocks, "firms almost never hold any inventories" regardless of whether they are shocks to technology or marginal utility of consumption. After introducing idiosyncratic shocks, the generalized stockout avoidance model without capital is able to reproduce procyclical inventory investment and a positive relationship between inventory investment and sales under preference shocks. But, such an improvement severely sacrifices the ability to match the long run average inventory-to-sales ratio. Moreover, the stockout avoidance model performs even worse under technology shocks. By embedding the stockout avoidance motive into a standard RBC model, Wen (2008) can match the long-run average inventory-to-sales ratio and reproduce inventory regularities under either aggregate demand shocks or labor cost shocks. But, the model performance is very sensitive to parameter values under TFP shocks. Moreover, the model cannot generate a hump-shaped output response.

Besides the importance of inventories, empirical studies also show that monetary aggregates are highly correlated with output. Thus, it would be interesting to explore the effect of monetary shocks on inventory behavior. Kryvtsov and Midrigan (2010a) find that the inventory-to-sales ratio is countercyclical.[7]. They also confirm the previous empirical findings of Blinder and Maccini (1991) that inventory investment is unresponsive to the real interest rate. Jung and Yun (2006) show hump-shaped responses of the sales-stock ratio and U-shaped responses of finished goods inventories

[7]This result is also true if conditional on monetary shocks

to an expansionary monetary policy shock. Both papers use a sticky price approach and require high depreciation rates and a low degree of real rigidities to match empirical findings.

Although models in the inventory literature can match the stylized facts of inventories with technology shocks, the importance of demand shocks for explaining inventory behaviors are far from conclusive.[8] Since inventories are similar to money in the sense that they are dominated by interest bearing assets, some researchers use search models to rationalized inventories.[9] Although they are not focusing on inventory behavior, their works shed light on the role of inventory in the monetary propagation mechanism (see, for example, Li (1994), Menner (2006), Shi (1998) and Wang and Shi (2006)).[10]

The money search model has experienced rapid growth since Kiyotaki and Wright (1991, 1993). It provides a microfoundation for monetary economics by assuming a decentralized trading mechanism. Contrary to the money in the utility model or the cash-in-advance model, money is essential in the money search model, which, in turn, provides confidence to monetary policy analysis. Li (1994) uses a search model to study the relationships among steady state inventory accumulation, the inflation tax and welfare. Shi (1997) generalized the models of Kiyotaki and Wright (1991, 1993) to allow for both divisible goods and divisible money. Building on Shi (1997),

[8]Kahn (1987) shows that the excess velocity of production relative to sales can be reproduced only by demand shock.

[9]Like modeling the money in a cash-in-advance model or in a money-in-utility model (e.g. , Cooley and Hansen (1989) and Sidrauski (1967)), some researchers assume that inventories yield direct utility (see Kahn, McConnell, and Perez-Quiros (2002)), facilitate production (see Kydland and Prescott (1982) and Ramey (1989)), or are required for sales (see Bils and Kahn (2000), Coen-Pirani (2004) and Lubik and Teo (2012)) instead of modeling them endogenously.

[10]Shibayama (2008) models input inventories as unsold goods by assuming price posting and studies inventory cycles. His model is a multi-stage production model and can reproduce the stylized facts of inventories. I will discuss more in Chapter 5.

Shi (1998) proposed a model that combined both labor search and money search to study the monetary propagation mechanism. This model is tractable and can be used to study the short run effects of money growth. These features are very important for studying how multi-stage production model affects the impacts of money growth, especially the short run dynamic effects on input inventories.

The benefits of using a search model are summarized as follows. First, inventories arise naturally in a search model. Firms would hold excess inventories at the end of each period due to search frictions in the goods market. Therefore, inventories in a search model are forced inventories. But households are still able to choose the end of period inventory level by endogenizing buyers' search intensities. Second, the value of money is endogenous in a search model, which is important for monetary policy analysis to be credible. Since multi-stage production takes one more step towards reality and can improve the depth of understanding of the monetary transmission mechanism, the search model is more suitable than alternative models. Finally, as shown in Shi (1998), monetary shocks have persistent effects on real variables in the short run which is a key element for reproducing the stylized facts of input inventories.

In this thesis, I extend Shi's model from single stage production to multi-stage production and model input inventories instead of output inventories.[11] As summarized in Chapter 1, my model predicts that multi-stage production influences the effects of money growth on real variables. Input inventories respond in an opposite way than output inventories to money growth shocks both in the long run and in the short run. Another striking result is that employment responds positively to technology shocks in my model, but responds negatively in a one-sector search model.

[11] So my model is better for analyzing production-to-order industries, for example, Original Equipment Manufacturers, airplane manufacturing and construction.

My model is also related to Bental and Eden (1993), who use sequential markets to model inventories. In their model, since producers produce before buyers show up, inventories arise whenever some markets fail to open, for example when some batches of buyers do not show up for some markets. My model is related to their model in the sense that, while inventories are forced inventories in both models, goods markets are sure to open and only open once (in each period) in my model. Despite the similarity, in my model, inventories arise because of search frictions instead of uncertainty of demand (as is the case in the model of Bental and Eden). Moreover, the pricing mechanisms are different between these two models.

In the next chapter, I describe the data and document the stylized facts of the input inventories. I also estimate a structural vector autoregressive model in order to compare the emprical impulse response functions with these of my model in Chapter 5.

Chapter 3

Data and Empirical Facts

In this chapter, I describe the data and document the empirical evidence on input inventory behaviors. In order to test how monetary shocks affect macroeconomic aggregates, including input inventories, I estimate a structural vector autoregressive model. First, results from the Granger-Causality tests show that the inventory-to-sales ratio is Granger-caused by both the money growth rate and productivity.

Then, results from the forecast error variance decomposition show that the money growth shock help to explain the variances of productivity, the inventory-to-sales ratio and employment to a limited extent. The productivity shock is still the main force driving fluctuations and has more of an impact than monetary shocks.

Finally, for the sake of comparison with the theoretical impulse response functions predicted in Chapter 5, structural impulse response functions with both the money growth shock and the productivity shock are reported.

3.1 Data and Stylized Facts

As mentioned in the previous chapter, the inventory behaviors can be characterized by a series of stylized facts. I also use the stylized facts summarized in Khan and Thomas (2007b) to evaluate my model's performance. In particular, these stylized facts are the following: 1) the correlation between GDP and the inventory-to-sales ratio, 2) the correlation between GDP and inventory investment, 3) the correlation between final sales and the inventory-to-sales ratio, 4) the correlation between final sales and GDP, 5) the correlation between final sales and the inventory-to-sales ratio, 6) the standard deviation of final sales relative to GDP, 7) the standard deviation of inventory investment relative to GDP, 8) and the standard deviation of inventory-to-sales ratio relative to GDP.

To be consistent with the theory presented in the next section, I focus on input inventories instead of a broader concept of inventories that do not distinguish between output and input inventories. Thus, throughout my thesis, the net inventory investment corresponds to the net input inventory investment and the inventory-to-sales ratio corresponds to the input inventory to sales target ratio in my thesis. I will use net inventory investment and inventory to sales ratio for short, if no confusion will arise.

I use quarterly U.S. data from the Bureau of Economic Analysis, the Bureau of Labor Statistics and the Federal Reserve Bank of St. Louis on inventories for the manufacturing sector, final sales, employment, money stock and the velocity of money.[12] The sample period is from 1967:Q1 to 2010:Q4. All of the variables are real variables. The input inventories include inventories of materials and supplies

[12] See Appendix A for the list of data sources.

and inventories of work-in-process. Final sales are manufacturing sales. GDP is calculated according to the accounting identity, which equals final sales plus net inventory investment. I use M2 as the money stock since it is stable within the sample period. Define the gross rate of money growth by $\gamma_t \equiv M_{t+1}/M_t$. Therefore, the velocity of money in my model is the Velocity of M2 Money Stock.

Data on input inventories and manufacturing sales are not consistent across different sample periods in the sense that they are chained with different base years. I adjust all of the data based on year 1996 to year 2005. Since, for the period from 1967 to 1996, the Bureau of Economic Analysis provides data on real manufacturing and trade inventories for both base years, I calculate an adjustment ratio by dividing these two series. The final sales data chained with 1996 dollars are multiplied by this adjustment ratio to match other chained 2005 dollars data.

I use manufacturing sector employment data in the structural VAR estimation. Since the data on labor force and unemployment for the manufacturing sector cannot be collected, I use data on the aggregate labor force, the aggregate employment level and the aggregate unemployment level to calculate the long run labor participation rate and the long run unemployment rate in the calibration. By doing this, I implicitly assume that the average labor participation rate and the average unemployment rate in the manufacturing industry are the same as the corresponding aggregate rates.

Some series are not available at the quarterly frequency, such as manufacturing employment, the total value of shipments and the total cost of materials. I convert my annual data into quarterly data by cubic spline interpolation. The time series of total factor productivity are calculated as the Solow residual. Since the individual production function has to be aggregated, which involves the matching rate in the intermediate goods market, my expression for productivity is more complicated. The details are described in Chapter 5.

Table 1: Stylized Facts

	Data*	KT(2007b)**
corr(GDP, IS)	-0.7561	-0.381
corr(GDP, NII)	0.7891	0.669
corr(FS, IS)	-0.7547	-0.700
corr(FS, GDP)	0.9869	0.943
corr(FS, NII)	0.6800	0.411
$\sigma(FS)/\sigma(GDP)$	0.8357	0.710
$\sigma(NII)/\sigma(GDP)$	0.2195	0.295
$\sigma(IS)/\sigma(GDP)$	0.8410	0.545

* Data sources are listed in Appendix A. Input inventories includes Materials and supplies inventories and work-in-process inventories. GDP: log of real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: log of final sales. To compare the results with that in Khan and Thomas (2007b), NII are detrended as a share of GDP. All data are HP filtered with a weight of 1600.

** corr(FS, IS) is borrowed from Kryvtsov and Midrigan (2010), Table 1, and is estimated conditional on monetary shocks.

Table 1 reports summary statistics for the stylized facts of input inventories which are calculated from my sample. This table also compares those statistics with the ones reported in Khan and Thomas (2007b), who use a broad definition of inventories. As the table shows, the cyclical properties of the input inventory are similar to the one of inventories overall. In particular, the input inventory investment is procyclical, the inventory-to-sales ratio is countercyclical and GDP is more volatile than final sales. On the other hand, by isolating input inventories, I find a much higher negative correlation between GDP and the input inventory-to-sales ratio which is -0.7561, relative to -0.381. The input inventory investment is more correlated with final sales and GDP. Moreover, the input inventory-to-sales ratio is more volatile than the general inventory-to-sales ratio relative to GDP. These features imply that final sales are more volatile than input inventories and that the input inventories are very able to move in the same direction as final sales and GDP during the transition following shocks.

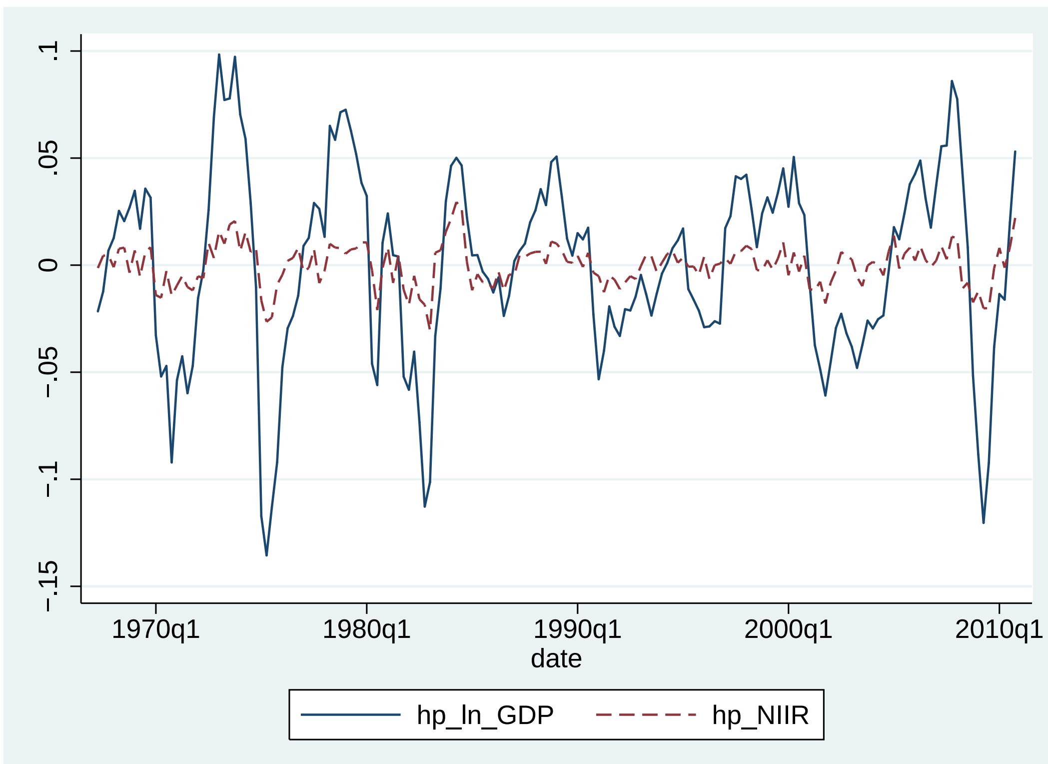

Figure 1: GDP and Inventory Investment

Figure 1 shows the exact evidence. By plotting the detrended series of GDP and inventory investment together,[13] we can see that although input inventories fluctuate within each cycle, they are still very procyclical. This implies that firms hold more input inventories during a boom and cut back input inventories during a recession.

[13]The figure of detrended final sales is very close to that of GDP.

Thus I argue that a lower input inventory level after 1984 induces a reduction in the fluctuations in input inventory stocks, which is one of the reasons for the reduction in the volatility of GDP.

3.2 Structural VAR model

In order to explore the effects of monetary shocks on input inventories, I construct a five-variable structural VAR model in the following way:[14]

$$Y_t = A_0 + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + u_t, \tag{1}$$

where Y is a $K \times 1$ vector of aggregate variables, $A_0, \ldots, A_p$ are $K \times K$ matrices of parameters, u_t is the $K \times 1$ vector of serially uncorrelated structural errors with $u_t \sim N(\mathbf{0}, \Sigma)$ and $E[u_t u_s{}'] = \mathbf{0}_K$ for all $s \neq t$, and Σ is an $K \times K$ diagonal matrix.

To impose short run restrictions, the above equation can be rewritten as:

$$A(I_K - A_1 L - A_2 L^2 - A_p L^p) Y_t = A u_t = B \epsilon_t, \tag{2}$$

where L is the lag operator, A and B are $K \times K$ matrices of parameters, and ϵ_t is a $K \times 1$ vector of orthogonalized shocks; i.e., $\epsilon_t \sim N(0, \mathbf{I}_K)$ and $E[\epsilon_t \epsilon_s{}'] = \mathbf{0}_K$ for all $s \neq t$. The model is identified by placing short run restrictions on contemporaneous correlations and on the covariances of the error, i.e., matrices A and B. Assume A is lower triangular, B is a diagonal matrix, and both matrices are nonsingular.

The vector Y includes the following aggregate variables: the money growth rate

[14] See Christiano, Eichenbaum, and Evans (1999) for details.

Table 2: Unit Root Tests

	t-value	1% cr. value	5% cr. value	p-value
hp_gamma	-6.346	-3.493	-2.887	0.0
hp_ln_A	-4.652	-3.492	-2.886	0.0
hp_ln_empl	-5.682	-3.492	-2.886	0.0
hp_ISR	-4.598	-3.492	-2.886	0.0
hp_NIIR	-5.223	- 3.493	-2.887	0.0

The ADF tests are performed for level specification and differenced specification respectively for the following variables: M2 (money stock), M2V (M2 velocity), IS (inventory-to-sales ratio), FS (final sales). NII are net inventory investment which is calculated as a share of GDP and tested for level specification.

(hp_gamma), productivity (hp_ln_A), employment (hp_ln_empl), the inventory-to-sales ratio (hp_ISR) and inventory investment (hp_NIIR). All of the variables are detrended using a HP filter with $\lambda = 1600$. Since NII could be negative, I follow the work of Khan and Thomas (2007b) and normalize NII as a share of GDP. Table 2 reports the results of the Augmented Dickey-Fuller tests for each series. The ADF tests do reject the unit root null at the 1% significance level for all series.

Since the money growth rate is my policy instrument, a money growth shock is an orthogonalized shock (i.e. shock to ϵ_M) to the money growth rate in this model. To be consistent with the theory predicted in the next chapter, the variables in Y are ordered as the following: (hp_gamma , hp_ln_A, hp_ln_empl, hp_ISR, hp_NIIR). This ordering implies that monetary shocks affect GDP and other aggregate variables contemporaneously. This assumption was first made by Bernanke and Blinder (1992), who argued that the output data is only available to the central bank with a lag. I estimate my structural VAR system with 2 lags, which are suggested by Akaike's Information Criterion (AIC).

Table 3: Granger-Causality Tests

Equation	Excluded				
	hp_gamma	hp_ln_A	hp_ln_empl	hp_ISR	hp_NIIR
hp_gamma	0.000	0.015	0.000	0.344	0.987
hp_ln_A	0.000	0.000	0.000	0.011	0.837
hp_ln_empl	0.375	0.110	0.000	0.000	0.008
hp_ISR	0.001	0.034	0.000	0.000	0.931
hp_NIIR	0.350	0.985	0.599	0.004	0.000

The p-values are reported for Granger causality Wald tests. The null hypothesis is the estimated coefficients on the lagged values of the variable in each row are jointly zero for each equation.

3.2.1 Granger-Causality Tests

Table 3 reports the p-values for the Granger-causality tests for the structural VAR. First, I find a feedback effect between the money growth rate and productivity. These two variables Granger-cause each other at the 5% significance level. The results for the first equation show that net inventory investment and the inventory-to-sales ratio do not Granger-cause the money growth rate at the 10% significance level. The results of employment show that the employment Granger-cause the money growth rate at the 1% significance level, but the reverse causality direction does not hold.

Second, the results for the second equation show that the net inventory investment do not Granger-cause productivity at the 10% significance level. On the other hand, employment and the inventory-to-sales ratio Granger-cause productivity at the 1% and 5% significance levels, respectively.

Third, my tests show no evidence that either the money growth rate or productivity Granger-cause employment. On the other hand, the inventory-to-sales ratio and inventory investment Granger-cause employment at the 1% significance level. This

Table 4: Variance Decomposition of hp_gamma

Steps	hp_gamma	hp_ln_A	hp_ln_empl	hp_ISR	hp_NIIR
1	1.0	0.0	0.0	0.0	0.0
4	78.9	19.2	1.4	0.6	0.0
8	77.4	19.9	1.6	1.0	0.0
12	75.9	19.8	2.8	1.2	0.3

The results are reported in percentage points.

causality direction may be due to manufacturing sector employment. Because manufacturing sector employment is only a proportion of aggregate employment and is affected by the sensitivity of the manufacturing sector to monetary shocks, it may behave differently from aggregate employment.

Finally, the inventory-to-sales ratio is Granger-caused by both the money growth rate and productivity at the 5% significance level. But, the net inventory investment rejects the same null at the 10% significance level. Nevertheless, I maintain the order of my variables because it is the monetary propagation mechanism suggested by my theory that I am interested in testing.

3.2.2 Variance Decomposition

Tables 4-8 report the results of forecast-error variance decompositions for each variable. First, the money growth rate only explains 1.2% of the variance in productivity at the first quarter, yet it reached 17% at the fifth quarter. It can explain 7% of the variances in employment at the twelve quarter, 9% of the variances in inventory-to-sales ratio at the fourth quarter and 5% of the variance in net inventory investment at the twelve quarter. Overall, the money growth helps to explain the real variables, but only to limited extent not much.

Table 5: Variance Decomposition of hp_ln_A

Steps	hp_gamma	hp_ln_A	hp_ln_empl	hp_ISR	hp_NIIR
1	1.2	98.8	0.0	0.0	0.0
4	16.7	58.7	22.1	1.0	1.5
8	12.0	43.1	38.4	1.2	5.3
12	12.4	43.2	37.0	1.7	5.7

The results are reported in percentage points.

Table 6: Variance Decomposition of hp_ln_empl

Steps	hp_gamma	hp_ln_A	hp_ln_empl	hp_ISR	hp_NIIR
1	0.2	4.9	95.0	0.0	0.0
4	1.6	24.6	64.9	6.2	2.8
8	7.4	26.7	51.0	12.2	2.8
12	7.1	24.5	52.4	12.2	3.9

The results are reported in percentage points.

Table 7: Variance Decomposition of hp_ISR

Steps	hp_gamma	hp_ln_A	hp_ln_empl	hp_ISR	hp_NIIR
1	0.0	66.4	11.5	22.1	0.0
4	9.4	35.6	8.7	44.9	1.4
8	7.3	27.4	25.6	34.1	5.6
12	7.8	30.0	25.9	29.1	6.4

The results are reported in percentage points.

Table 8: Variance Decomposition of hp_NIIR

Steps	hp_gamma	hp_ln_A	hp_ln_empl	hp_ISR	hp_NIIR
1	1.3	1.1	4.4	38.3	54.9
4	3.7	21.0	9.9	27.3	38.1
8	4.9	18.5	19.5	25.4	31.7
12	5.0	21.0	22.6	22.4	29.1

The results are reported in percentage points.

The productivity shocks, rather than monetary shocks, are still the main force driving fluctuations. Productivity helps to explain the variance of those three real variables. It can explain 25% of the variance in employment at the fourth quarter, 66% of the variance in inventory-to-sales ratio at the first quarter and maintains at 30% at the twelve quarter and 21% of the variance in net inventory investment at the twelve quarter. Since productivity does not explain the variance of money growth rate at the first quarter, and only to do so with a lag, it is consistent with our identification restrictions that the money growth rate affects productivity contemporarily, but the reverse is not true.

The manufacturing sector employment is quite self-explanatory and is not very sensitive to monetary shocks. The variance in employment is explained by employment itself up to 95% at the first quarter. Moreover, the manufacturing sector employment explains 26% of the variance in inventory-to-sales ratio at the twelve quarter and 21% of the variance in net inventory investment. The inventory-to-sales ratio explains 38% of the variance in net inventory investment at the first quarter.

3.2.3 Impulse Response Function

For the sake of comparison with theoretical impulse response functions in Chapter 5, the impulse response functions are reported for both the money growth shocks and the productivity shock. The responses of real variables are similar, except for the initial responses. As will be discussed in Chapter 5, the theoretical impulse response functions match the empirical impulse response function quantitatively better if aggregate fluctuations originate from productivity shocks.

Figure 2 depicts the impulse response functions for the structural VAR. The shock

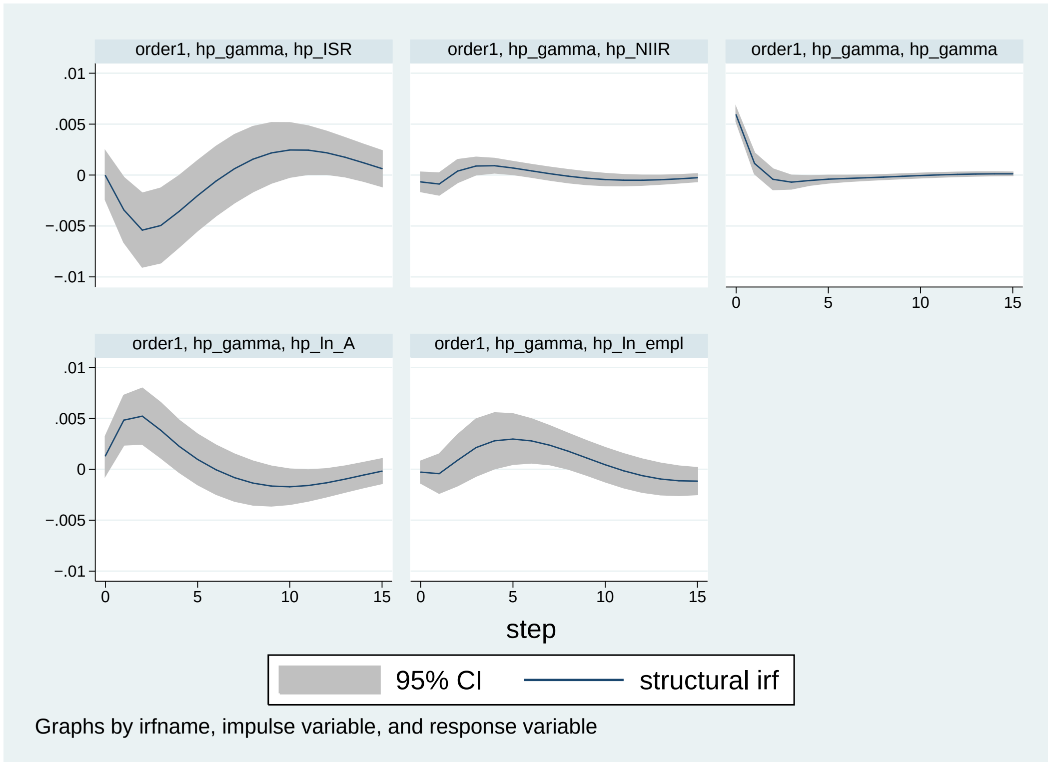

Figure 2: Empirical IRF: Money Growth Shock

is a one standard deviation shock to the money growth rate. The net inventory investment and the employment decrease during the first period then quickly increase above the steady state. The monetary shock drives productivity above the steady state contemporaneously, with productivity increasing for two periods before falling back to the steady state. Finally, the inventory-to-sales ratio drops for two quarters then slowly increases above the steady state.

I also report the responses of my structure VAR system to one standard deviation shock to productivity (Figure 3). First, a productivity shock induces a negative response of the money growth rate. The productivity shock is not very persistent on the money growth rate, since the money growth rate goes back to its steady state level

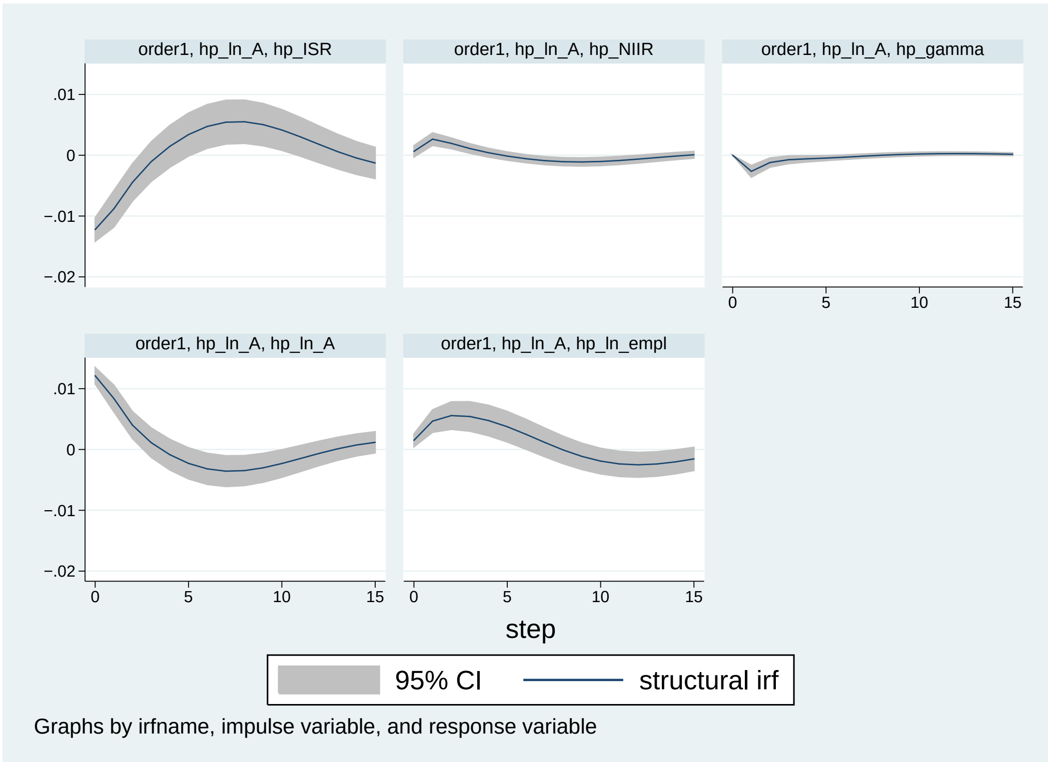

Figure 3: Empirical IRF:Productivity Shock

within 2 quarters. The other responses are similar to the corresponding ones under money growth shocks, except for the initial responses. For example, the inventory-to-sales ratio drops immediately when the shock happens. The inventory investment and the employment increases from the first period instead of decreasing as in the case of monetary shocks.

Chapter 4

A Search Model With Multi-stage Production

4.1 The Environment

In this chapter, I use a large household model to show how multi-stage production affects the long run impacts of money growth. In order to capture the spirit of multi-stage production, I assume there are two subperiods in each period. The first subperiod is for home production of intermediate goods. The finished goods market opens in the second subperiod, in which agents can produce and trade finished goods. Producing finished goods requires both labor and intermediate goods as inputs. Thus, there is also a labor market opened every period.

4.1.1 The Household

The model economy consists of many types of households denoted by set H. The number of households in each type is large and normalized to one. There are also many types of finished goods, which set is denoted by H^f. The measures of H and H^f are the same. Each household has two types of technologies which can be used to produce intermediate goods and household specific finished goods, respectively.

Each household consists of six groups of agents (with the accompanying measure in parentheses): intermediate goods producers (a_p^i), finished goods buyers (a_b^f) and entrepreneurs (a_p^f) who are active in the second subperiod; leisure seekers (n_0), workers ($a_p^f n_t$), and unemployed agents (u). Each entrepreneur consists of a finished goods producer and a finished goods seller. The number of agents (a_p^i, a_b^f, a_p^f, u) remain constant, while the number of effective buyers changes over different periods, because I allow households to choose search intensities every period. The number of leisure seekers and workers (n_0, n_t) varies over time.

4.1.2 The Timing

Time is discrete. Figure 4 depicts the timing of the model. In the first subperiod, households produce intermediate goods for their finished goods production with disutility $\varphi(q_t^i)$, where q_t^i denotes the quantities of goods. The function φ satisfies $\varphi' > 0$, $\varphi'' > 0$ for $q^i > 0$, and $\varphi'(0) = \varphi(0) = 0$. At the end of the first subperiod, newly produced intermediate goods, together with input inventories, are evenly shared among entrepreneurs. Households do not consume intermediate goods. Intermediate goods are storable across periods.

The finished goods market opens in the second subperiod. The finished goods $h^f \in H^f$ produced by household h is desired only by some other types of households. Assume there are search frictions in the finished goods market. Thus, an intrinsically useless object, called fait money, can facilitate trades in the finished goods market. Furthermore, assume there is no double coincidence of wants, so barter trades are excluded in this model. At the beginning of each period, each household divides the

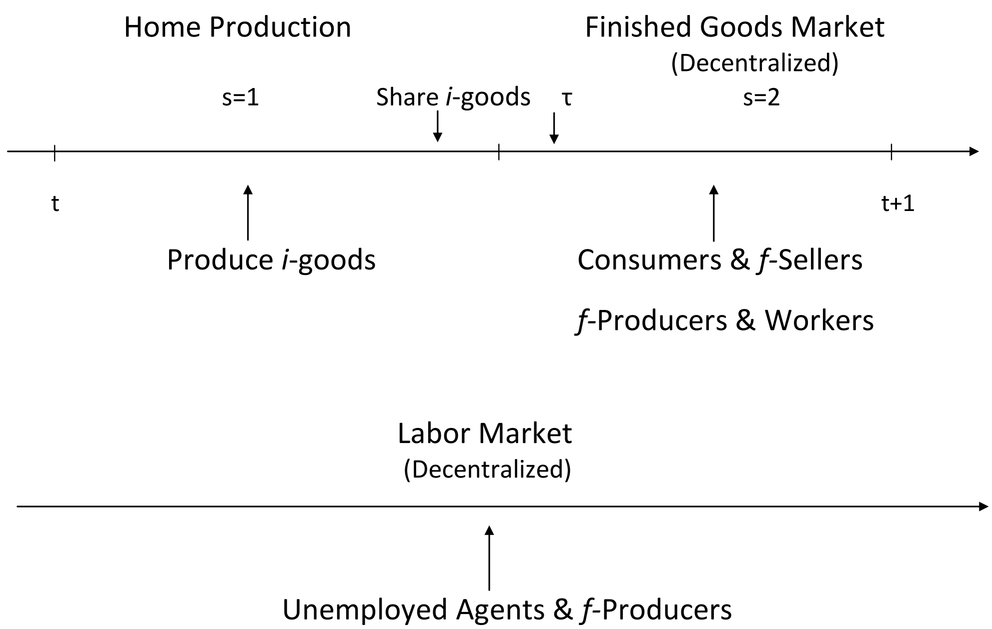

Figure 4: Timeline: Benchmark Model

nominal money balance evenly among its finished goods buyers and chooses buyers' search intensities (s_t^f).

Each finished goods producer carries intermediate goods. Once a buyer and seller have been matched, the seller places the order for its customer. The corresponding finished goods producer then produces the product. As discussed in the introduction, this paper focuses on input inventories instead of output inventories by assuming that finished goods producers produce if and only if they are matched. The terms of trade include the quantity of goods and the quantity of money, denoted by $(\hat{q}_t^f, \hat{m}_t^f)$, respectively, which are determined by Nash bargaining. The price level in the finished goods market is $P_t^f = m_t^f / q_t^f$.

During the transaction, each household receives a lump-sum transfer (τ_t) which will be added to next period's nominal money balance. At the end of the period, finished goods buyers bring trade receipts, and entrepreneurs bring profits and unused intermediate goods back to the household. Workers bring wage income back to the household. At the end, the household and agents share consumption. Since agents regard the household's utility as a common objective and share consumption and inventories, the idiosyncratic risk generated by search friction is smoothed within each household. The household carries the new nominal money balance (M_{t+1}) and input inventories which depreciate at a rate of δ_i over each period. Workers hired in the last period separate from current jobs at an exogenous rate δ_n.

4.1.3 The Matches

Let us describe the matching technologies in the frictional markets. Variables with a hat refer to an arbitrary household. In the finished goods market, the total number of matches is determined by the following Cobb-Douglas matching function:

$$g(\hat{s}^f) = z_1^f (a_b^f \hat{s}^f)^{\xi} {a_p^f}^{1-\xi}, \quad \xi \in (0,1), \tag{3}$$

where $z_1^f > 0$ is a constant. Denote the ratio of buyers to sellers as $B^f = a_b^f / a_p^f$ and $z^f = z_1^f (B^f)^{\xi-1}$. Then the matching rate for each unit of a buyer's search intensity is $g_b^f(\hat{s}^f)$ and the matching rate for each seller is $g_s^f(\hat{s}^f)$, where,

$$g_b^f \equiv z^f (\hat{s}^f)^{\xi-1}, \tag{4}$$

$$g_s^f \equiv z^f B^f (\hat{s}^f)^{\xi}. \tag{5}$$

Thus buyers and sellers get desirable matches at rates $s^f g_b^f$ and g_s^f respectively.

In the labor market, each finished goods producer posts vacancies v_t. Unemployed agents search for jobs. I assume matched workers start to work in the next period and the wage is negotiated according to Nash bargaining. Workers supply one unit of labor inelastically. Wages W_t are paid in nominal terms, regardless of whether or not their employees formed a match. As in the standard labor search model (e.g., Blanchard and Diamond (1989)), the total number of matches between unemployed workers and producers are: $\bar{\mu}(a_p^f \hat{v})^{\phi} u^{1-\phi}$, where $\phi \in (0,1)$ and $\bar{\mu}$ is a constant. The hat on variables refers to an arbitrary producer. The total number of matches for each firm is $\mu(\hat{v})v$, where $\mu(\hat{v}) \equiv \bar{\mu}(a_p^f \hat{v}/u)^{\phi-1}$ is the number of matches per vacancy; and the number of matches per unemployed agent is $\mu(\hat{v})a_p^f \hat{v}/u$.

4.2 The Household's Decision Problem

At the beginning of each period, the household divides the nominal money balance evenly among its finished goods buyers and chooses buyers' search intensities (s_t^f). Assume all sellers' search intensities and unemployed workers' search intensities are inelastic with no cost to households. The household also chooses consumption level (c_t), the number of vacancies for each firm (v_t), the next period's employment level (n_{t+1}), the next period's nominal money balance (M_{t+1}), and the next period's input inventory level (i_{t+1}). The household takes the terms of trade as given when making these decisions. The terms of trade are determined by Nash bargaining and will be described later.

The household's utility function, $U(c)$, is strictly increasing and concave, and satisfies $\lim_{c\to 0} cU'(c) = \infty$ and $\lim_{c\to\infty} cU'(c) = 0$. $\varphi(q_t^i)$ is the disutility of producing intermediate goods. φ^f is the disutility of working in the finished goods market. $\Phi^f(s_t^f)$ is the disutility of searching in the finished goods market. The function Φ^f

satisfies $\Phi' > 0$ and $\Phi'' > 0$ for $s > 0$ and $\Phi(0) = \Phi'(0) = 0$. Finally, $K(v_t)$ is the disutility of posting vacancies, which has the same properties as Φ. Let F^*_{bt} (with measure $s^f_t g^f_{bt} a^f_b$) be the set of matched finished goods buyers in the period t. Similarly, F^*_{pt} (with measure $g^f_{st} a^f_p$) is the set of matched finished goods sellers in the current period.

I assume the production function of the finished good is a Leontief production function:

$$q^f_t = \min\{a_t, n_t\}, \tag{6}$$

where a_t is the quantity of material inputs and n_t is labor inputs hired in the last period. A Leontief production function implies that intermediate goods and labor are not substitutable.[15] Moreover, I make this assumption in order to derive analytical results.

By putting all the necessary ingredients together, the representative household's decision problem can be summarized as follows. The representative household taking the sequence $\{\hat{q}^f_t, \hat{m}^f_t, \hat{W}_t\}_{t\geq 0}$ and initial conditions $\{M_0, i_0, n_0\}$ as given, chooses $\{C_t, q^i_t, s^f_t, v_t, M_{t+1}, i_{t+1}, n_{t+1}\}_{t\geq 0}$ to maximize its expected lifetime utility:

$$\max \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{-1}[U(c_t) - \varphi(q^i_t) - a_p \hat{n}_t \varphi^f - a^f_b \Phi^f(s^f_t) - a^f_p K(v_t)] \tag{7}$$

[15]Because I model input inventory in this model, such simplicity makes sense here. For example, firms cannot produce more cars with fewer windows while using more labor. But for the sake of comparison, I use a Cobb-Douglas production function for my quantitative analysis in Chapter 5.

subject to the following constraints for all $t \geq 0$:

$$\begin{aligned}
c_t &\leq s_t^f g_{bt}^f a_b^f \hat{q}_t^f, & (8)\\
\frac{M_t}{a_b^f} &\geq \hat{m}_t^f, \quad \forall F_{bt}{}^* & (9)\\
q_t^f &= \min\{a_t, n_t\}, & (10)\\
a_t &\leq i_t + q_t^i / a_p^f, \quad \forall F_{pt}{}^* & (11)\\
q_t^f &\geq \hat{q}_t^f, \quad \forall F_{pt}{}^* & (12)\\
M_{t+1} &\leq M_t + \tau_t + a_p^f \hat{n}_t \hat{P}_t^f \hat{W}_t - s_t^f g_{bt}^f a_b^f \hat{m}_t^f + g_{st}^f a_p^f \hat{m}_t^f - \hat{P}_t^f a_p^f \hat{W}_t n_t, & (13)\\
0 &\leq a_p^f [(1-\delta_n) n_t + v_t \mu_t - n_{t+1}], & (14)\\
a_p^f i_{t+1} &\leq (1-\delta_i)[a_p^f i_t + a_p^f q_t^i - g_{st}^f a_p^f a_t]. & (15)
\end{aligned}$$

Constraint (8) and (9) are standard in a large household model. Constraint (8) is a budget constraint, which requires that the household's consumption does not exceed the total amount of finished goods obtained by its buyers. Constraint (9) states that in order to successfully trade with a matched seller, the buyer must have enough money. Constraint (10) is the Leontief production function, which implies that the usage of intermediate goods and labor are equal.

The intuition behind constraint (11) and constraint (12) is similar to the money constraint (9). Constraint (11) states that the usage of intermediate goods is constrained by the finished goods producer's intermediate goods holdings. Similarly, constraint (12) requires that matched finished goods producers should have enough workers and intermediate goods to produce finished goods.

Constraint (13) is the law of motion of money, which states that the nominal money balance at the beginning of next period will be no larger than the nominal money balance carried from last period plus changes in the nominal money balance. The changes in the nominal money balance come from the lump-sum transfer received in the second subperiod, the money spent by finished goods buyers, profits from entrepreneurs and wages earned by workers. Entrepreneurs obtain money if, and only if, their sellers can find desired matches, while wages have to be paid to workers at the end of the period, regardless of whether they matched or not.

Constraint (14) is the law of motion of employment, which states that at the beginning of next period, the number of workers in each firm is no larger than the number of workers who still stay with the current job, plus newly hired workers. The last constraint is the law of motion of inventories, which implies that the household's next period inventory level is no larger than unused intermediate goods depreciated at a rate of $\delta_i \in (0, 1)$.

Denote the multipliers of money constraint (9) by Λ_t^f. Let Ω_{at} be the shadow price of (11) at the beginning of period $t + 1$. I am interested in equilibria with a positive inventory level, which requires that inventories have positive values in each period (ex. $\Omega_{at} > 0$). The multiplier of (12) is denoted by Ω_f. Since entrepreneurs get positive surplus from trading finished goods, it is optimal for them to hire enough workers and have enough intermediate goods in hand. Let the shadow prices of (13), (14), and (15) at the beginning of period $t+1$ be Ω_{mt}, Ω_{nt}, and Ω_{it} respectively, which are measured in terms of the household's period t utility.

Constraint (9) and (11) are restricted to be binding in equilibrium. By plugging c_t into the household's utility function, substituting a_t and q_t^f by n_t, and holding conditions (12), (13), (14) and (15) with equality, we can derive the first-order conditions with respect to $(M_{t+1}, i_{t+1}, n_{t+1}, s_t^f, v_t, q_t^i)$:

$$\Omega_{Mt} = \beta\mathbb{E}[\Omega_{Mt+1} + s_{t+1}^f g_{bt+1}^f \Lambda_{t+1}^f], \quad (16)$$

$$\Omega_{it} = \beta\mathbb{E}[(1-\delta_i)\Omega_{it+1} + g_{st+1}^f \Omega_{at+1}], \quad (17)$$

$$\begin{aligned} \Omega_{nt} &= \beta\mathbb{E}[(1-\delta_n)\Omega_{nt+1} + g_{st+1}^f[\Omega_{ft+1} - \Omega_{at+1} \\ &- (1-\delta_i)\Omega_{it+1}] - \hat{P}_{t+1}^f \hat{W}_{t+1}\Omega_{Mt+1}], \end{aligned} \quad (18)$$

$$\Phi^{f\prime}(s_t^f) = g_{bt}^f[U'(C_t) - \omega_t^f]q_t^f, \quad (19)$$

$$\Omega_{nt} = K'(v_t)/\mu(\hat{v}_t), \quad (20)$$

$$\varphi'(q_t^i) = g_{st}^f \Omega_{at} + (1-\delta_i)\Omega_{it}. \quad (21)$$

Condition (16) equates the opportunity cost of obtaining one more unit of money and the expected benefits of carrying one more unit of money into the next period. Such benefits include the shadow price of money and the shadow value of relaxing the money constraint in the finished goods market. Similarly, condition (17) equates the opportunity cost of obtaining an additional unit of input inventory and the expected benefits of carrying it over to the next period. Such benefits include the shadow value of inventories and the shadow value of relaxing the intermediate goods usage constraint.

Condition (18) equates the opportunity cost of hiring an additional worker and the expected benefits generated by this worker in the next period. This opportunity cost does not only include the shadow value of labor and the wage paid in terms of period $t+1$ utilities $(\beta P_{t+1} W_{t+1} \Omega_{Mt+1})$, but also includes the expected cost of tightening the period $t+1$ intermediate goods usage constraint and the expected shadow value of inventories discounted at the proper rate.

Condition (19) states that the opportunity cost of increasing the search intensity, which involves search costs and the real money balance, equals the marginal utility of consumption. Condition (20) equates the marginal cost of posting a vacancy and the expected benefits. The last condition equates the marginal cost of producing one more unit of intermediate goods and the marginal benefits which include the shadow value of inventories and the cost of relaxing the second subperiod intermediate goods usage constraint.

4.3 Terms of Trade

Let us specify the terms of trade for the finished goods market and the labor market. Because this is a large household model, each agent in the household is negligible and can be viewed as an identity of a small measure (ε). Since each agent's contribution to the household is also negligible, we compute the terms of trade brought by each agent first, then take the limit $\varepsilon \to 0$. Variables with a bar refer to the buyer in the other household and are taken as a given by the representative household.

4.3.1 Goods Markets

The terms of trade in the finished goods market are denoted by $(q_t^f \varepsilon, \bar{m}_t^f \varepsilon)$, where $q_t^f \varepsilon$ is the quantity of finished goods and $\bar{m}_t^f \varepsilon$ is the quantity of money. Thus, the trading surpluses of these two agents to their households are:

$$\text{seller's trade surplus:} \quad \Omega_{Mt}\bar{m}_t^f \varepsilon - \Omega_{ft} q_t^f \varepsilon, \tag{22}$$

$$\text{buyer's trade surplus:} \quad U(\bar{c}_t + q_t^f \varepsilon) - U(\bar{c}_t) - (\bar{\Lambda}_t^f + \bar{\Omega}_{Mt})\bar{m}_t^f \varepsilon. \tag{23}$$

Normalizing surpluses by ε, the terms of trade are determined by Nash bargaining between buyer and seller with equal weights as demonstrated by Shi (1998):

$$\max_{\bar{m}_t^f, q_t^f} \left[\Omega_{Mt}\bar{m}_t^f - \Omega_{ft}q_t^f\right]^{1/2} \times \left[\frac{U(\bar{c}_t + q_t^f\varepsilon) - U(\bar{c}_t)}{\varepsilon} - (\bar{\Lambda}_t^f + \bar{\Omega}_{Mt})\bar{m}_t^f\right]^{1/2}. \qquad (24)$$

By substituting $\bar{m}_t^f = P_t^f q_t^f$, solving for the first-order conditions and taking the limit $\varepsilon \to 0$, I can get the following equations:

$$(\bar{\Omega}_{Mt} + \bar{\Lambda}_t^f)P_t^f = U'(c_t), \qquad (25)$$

$$P_t^f \Omega_{Mt} = \Omega_{ft}. \qquad (26)$$

In the finished goods market, the first condition equates the marginal utility of consumption with the opportunity cost of spending money. The second condition states that the shadow value of real money balances equals the opportunity cost of obtaining money. Denote the shadow value of real money balance in the finished goods market by $\omega_t^f = P_t^f \Omega_{Mt}$.

4.3.2 Labor Market

The terms of trade in the labor market, denoted by $(W_{t+1}\varepsilon)$, are determined by Nash bargaining between the producer and the unemployed worker. Assuming the producer's bargaining weight is σ, where $\sigma \in (0,1)$. Since the producer's surplus of hiring ε more workers is $\{\Omega_{nt} - \beta\mathbb{E}[(1-\delta_n)\Omega_{nt+1}]\}\varepsilon$, by rearranging condition (18), we can reinterpret the producer's surplus in terms of real money balances:

$$\beta\mathbb{E}[\Omega_{ft+1}g_{st+1}^f - \omega_{t+1}^f W_{t+1} - a_p^f g_{st+1}^f \Omega_{at+1} - g_{st+1}^f \Omega_{it+1}(1-\delta_i)]\varepsilon. \qquad (27)$$

The unemployed agent's contribution to his household's utility is $\beta(\bar{\omega}^f_{t+1} W_{t+1} - \varphi^f)\varepsilon$, where $W_{t+1}\varepsilon$ is the expected wage income in terms of the real money balance. Normalizing surpluses by $\beta\varepsilon$, the wage rate maximizes the weighted Nash product of these two agent's surpluses:

$$\max_{W_{t+1}} \mathbb{E}\Big[[\Omega_{ft+1} g^f_{st+1} - \omega^f_{t+1} W_{t+1} - a^f_p g^f_{st+1} \Omega_{at+1} - g^f_{st+1} \Omega_{it+1}(1-\delta_i)]^{\sigma} \times [\bar{\omega}^f_{t+1} W_{t+1} - \varphi^f]^{1-\sigma}\Big]. \tag{28}$$

The wage rate can be obtained after taking the limit $\varepsilon \to 0$ on the first-order condition:

$$\mathbb{E}\Big[\omega^f_{t+1} W_{t+1}\Big] = \mathbb{E}\Big[(1-\sigma) g^f_{st+1}[\Omega_{ft+1} - \Omega_{at+1} - \Omega_{it+1}(1-\delta_i)] + \sigma\varphi^f\Big]. \tag{29}$$

The wage rate equals the weighted sum of the expected future benefit of hiring ε more workers and the opportunity cost of working.

4.4 Equilibrium

4.4.1 Characterization

In this section, I will describe the equilibrium and study how the multi-stage production affects the effects of money growth in the long run. In particular, the multistage production provides an additional channel through intermediate goods for money growth to have effects.

Although households produce and consume different types of goods, they are identical in the sense that they have the same utility function and production technologies. Thus, we can define the symmetric search equilibrium as follows:

Definition. *A symmetric search equilibrium is a sequence of household's choices* $\{\Gamma_{ht}\}_{t\geq 0}$, $\Gamma_{ht} \equiv (c_t, q_t^i, s_t^f, v_t, M_{t+1}, i_{t+1}, n_{t+1}^f)_h$, *expected quantities in trade* $\{\hat{X}_t\}_{t\geq 0}$, $\hat{X}_t \equiv (\hat{m}_t^f, \hat{q}_t^f, \hat{W}_t)$, *and the terms of trade* $\{X_t\}_{t\geq 0}$, *such that*

1. *all of these variables are identical across households and relevant individuals;*
2. *given* $\{\hat{X}_t\}_{t\geq 0}$ *and the initial conditions* (M_0, i_0, n_0), $\{\Gamma_{ht}\}_{t\geq 0}$ *solves the household's maximization problem, with* $(s^f, v) = (\hat{s}^f, \hat{v})$;
3. X_t *satisfies (25), (26) and (29);*
4. $\hat{X}_t = X_t \ \forall t \geq 0$.

As is standard, in order for money to play the role of a medium of exchange, we have to restrict our equilibrium to $\lambda^f > 0$. Similarly, we assume $\Omega_{at} > 0$, which requires that output producers prefer producing to hoarding intermediate goods in the second subperiod. Denote $k(v_t) = K'(v_t)/\mu(\hat{v}_t)$. Condition (11) and the Leontief production function imply that $i_t = q_t^f - q_t^i$. These three restrictions will be verified in the steady state. Then "hat" and "bar" are suppressed for a symmetric equilibrium. Condition (13) is reduced to $M_t + \tau_t = M_{t+1}$ under symmetry. Define the gross rate of money growth by $\gamma_t \equiv M_{t+1}/M_t = (M_t + \tau_t)/M_t$. By substituting conditions (9), (11), (20), (21), (25), (26), (29) and $P_t^f = M_t^f/q_t^f$ into conditions (14) - (19), we can eliminate $(M, i, n, \lambda^f, \Omega_a, \Omega_f, \Omega_n, m^f, W)$, and the dynamic system is characterized in terms of $(s^f, \omega^f, v, q^i, q^f)$ by the following conditions:

$$
\begin{aligned}
\mathbb{E}[\gamma_t \omega_t^f q_t^f / q_{t+1}^f] &= \beta E\{\omega_{t+1}^f + z^f (s_{t+1}^f)^{\xi} [U'(c_{t+1} - \omega_{t+1}^f)]\}, &(30)\\
\Omega_{it} &= \beta \mathbb{E}[\varphi'(q_{t+1}^i)], &(31)\\
k(v_t) &= \beta \mathbb{E}\big[(1-\delta_n) k(v_{t+1}) + \sigma z^f B^f (s_{t+1}^f)^{\xi} \omega_{t+1}^f &(32)\\
&+ (1 - z^f B^f (s_{t+1}^f)^{\xi}) \sigma (1-\delta_i) \Omega_{it+1} - \sigma \varphi'(q_{t+1}^i) - \sigma \varphi^f\big], &\\
\Phi^{f\prime}(s_t^f) &= z^f (s_t^f)^{\xi - 1} [U'(c_t) - w_t^f] q_t^f, &(33)\\
\mathbb{E}[q_{t+1}^i] &= \mathbb{E}\{q_{t+1}^f - (1-\delta_i)[1 - z^f B^f (s_t^f)^{\xi}] q_t^f\}, &(34)\\
\mathbb{E}[q_{t+1}^f] &= (1-\delta_n) q_t^f + v_t \mu(v_t), &(35)\\
c_t &= a_p^f B^f z^f (s_t^f)^{\xi} q_t^f. &(36)
\end{aligned}
$$

Condition (34) of the dynamic system is of particular interest. It implies that if a monetary shock were to hit the economy, the corresponding effect would be propagated through the inventory channel. As shown by the right hand side of this equation, the quantity of next period home production (q_{t+1}^i) not only depends on the next period's quantity of finished goods (q_{t+1}^f), but also depends on the current period's quantities (q_t^f). Since condition (11) is restricted to being binding in equilibrium, the effects of a monetary shock would depend on the inventory level.

By rewriting $(s_t^{f*}, \Omega_i^*, v^*, q^{i*})$ as functions of (ω^{f*}, q^{f*}), the steady state system can be reduced to two equations with two unknowns:

$$z^f[s^f(\omega^{f*}, q^{f*})]^\xi = \frac{\gamma - \beta}{\beta} \frac{\omega^{f*}}{U'(c(\omega^{f*}, q^{f*})) - w^{f*}}, \tag{37}$$

$$\begin{aligned}(1-\beta(1-\delta_n))k(v(q^{f*})) + \beta\sigma\varphi^f &= \beta\{\sigma z^f B^f(s^f(\omega^{f*}, q^{f*}))^\xi \omega^{f*} \\ &+[(1 - z^f B^f(s^f(\omega^{f*}, q^{f*}))^\xi)\sigma(1-\delta_i)\beta - \sigma]\varphi'(q^i(\omega^{f*}, q^{f*}))\}.\end{aligned} \tag{38}$$

It is clear that the money growth rate has real effects on steady state variables. The details will be described in the next section. Moreover, at least one steady state, which satisfies $\lambda^f > 0, \Omega_a > 0$, can be pinned down by these two equations (see Appendix B for a proof). For the sake of simplicity, I assume that a unique steady state exists.

4.4.2 Long Run Effects of Money Growth

In this section, I explore the role of the multi-stage production on the long run effects of money growth. The multi-stage production model affects the monetary transmission mechanism, especially the effects of money growth on the quantity of finished goods per match and on the inventory investment. The multi-stage production model shows that, for low levels of the money growth rate, agents trade more in each match in the finished goods market when the money growth rate increases. This is in sharp contrast to the standard search model which predicts that the quantity of goods per match decreases with the money growth rate monotonically. Such an unconventional effect implies that, for a low level of the money growth rate, increasing the money growth rate has positive effects on input inventory investment, while it has negative effects on the output inventory investment in a standard search model.

The unconventional effects of money growth on the quantity of finished goods per match can be proven by using Proposition 3.5 and Corollary 3.6, as outlined in Shi (1998). These proposition and corollary can be applied here, because $n = q^f$ in my model. The unconventional effects are summarized by the following proposition.[16]

PROPOSITION 1. *For sufficiently low levels of money growth, the steady state quantity of finished goods traded in each match increases with the money growth rate. To the contrary, it decreases with the money growth rate when the level of money growth is sufficiently high.*

In other words, agents tend to bring more real money balances to the market in face of moderate inflation. The intuition behind this proposition is the following. For sufficiently low levels of money growth, households post more vacancies when the money growth rate increases. Steady state employment increases. In an extreme case of no substitutability between labor and intermediate goods, more workers require more intermediate goods as input.[17] Since labor and intermediate goods are shared within each household, agents can produce more in each match with Leontief production function.

As discussed in the introduction, since input inventories are the biggest and most volatile component of inventories, it is worthwhile to explore the effects of money growth on input inventory investment, which can be summarized by the following proposition which is formally proved in Appendix C:

[16]This can be proven by rearranging equation (38) and applying Proposition 3.5 and Corollary 3.6 as outlined in Shi (1998) to the equation. See a sketch of this proof in Appendix C.

[17]In Chapter 5, I use a Cobb-Douglas production function and give a numerical example to illustrate Proposition 1-3.

PROPOSITION 2. *For sufficiently low levels of money growth, the steady state net inventory investment increases with the money growth rate. To the contrary, it decreases with the money growth rate for sufficiently high levels of money growth.*

The non-monotonic q^f is attributed to the positive effects of money growth on input inventory investment. Since the responses of input and output inventory investments are the same in the case of high money growth rates, I focus only on the effects of moderate inflation. The intuition behind Proposition 2 is the following. By plugging the steady state equation of q^i into equation (15), we can see that the inventory investment depends on the quantity of intermediate goods held by each output producer (or output per match) and the number of unmatched producers $(1 - g_s^{f*})$:

$$NII^* = a_p^f \delta_i i^* = (1 - \delta_i)\delta_i a_p^f [1 - g_s^{f*}] q^{f*} \tag{39}$$

where, $q^{f*} = i^* + q^{i*}/a_p^f$. Money growth affects the long run input inventory investment along these two margins. First, increasing the money growth rate has a positive effect on the quantity of finished goods per match provided the money growth rate is sufficiently low. This implies that each unmatched finished goods producer holds more intermediate goods at the end of the period. Second, the money growth rate has a negative effect on the number of unmatched finished goods producers. Since a higher money growth rate decreases the shadow value of money, buyers search more intensively in order to spend money more quickly. As a result, the total number of matched producers increases and the number of unmatched producers decreases. Overall, the positive effect dominates the negative effect if the money growth rate is low and, as a result, input inventory investment increases with moderate inflation.

Multi-stage production transfers the effects on q^{f*} from the finished goods market back to the initial stage of the production chain and affects the decision of material input. Because buyers and sellers are randomly matched in the finished goods market, households do not observe which output producers would get a match. Thus, households have to adjust the input levels for all output producers in response to changes in the money growth rate. As a consequence, unmatched producers hold more unused intermediate goods and households are stuck with more inventories at the end of each period, if the money growth rate is low. The above analysis is illustrated in Figure 5.

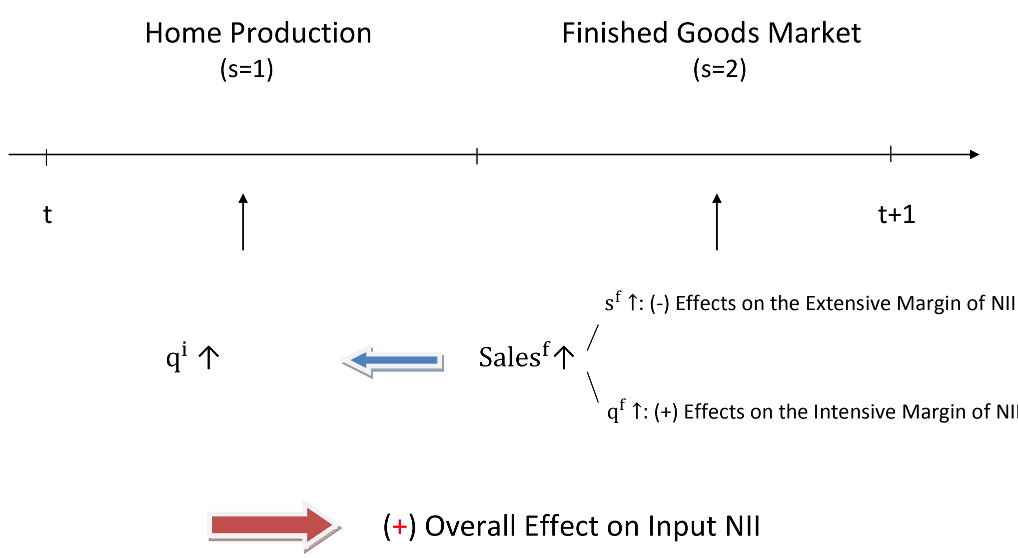

Figure 5: Effects with a Higher Money Growth Rate: Low γ

The long run effects of money growth on final sales and GDP are the same as in the standard search model, namely, for low levels of money growth, they increase with the money growth rate, but decrease for higher money growth rates. This can be easily verified by using the accounting identity GDP = Final Sales + Inventories. Thus, the Friedman rule does not hold in this model. The critical money growth rate is 6.34%, which is higher than the calibrated value of 1.67%, so both GDP and net inventory investment increase with moderate inflation in this model.

Finally, GDP volatility has substantially decreased since 1984, which leads to two decades of "great moderation".[18] At about the same time, the input inventory-to-sales ratio started to show a significant downward trend.[19] Consistent with the empirical evidence found by Iacoviello, Schiantarelli, and Schuh (2011), this model predicts that the long run inventory-to-sales ratio decreases as the money growth rate decreases, because the calibrated money growth rate is below the critical value. So my model suggests that changes in the money growth rate would be one of the reasons for the decline of the inventory-to-sales ratio since the mid-1980. This finding can be summarized in the following proposition (see Appendix C for proof):

PROPOSITION 3. *For sufficiently low levels of money growth, the steady state ratio of input inventory to final sales increases with the money growth rate. To the contrary, it decreases with the money growth rate for sufficiently high levels of money growth.*

4.4.3 Comparisons between models

Multi-stage production with input inventories plays a crucial role for the non-monotonic effects of the quantity of finished goods per match. Multi-stage production extends the standard search model to include the choice of material inputs. Intermediate goods serve as a buffer to meet the labor input requirement. By plugging condition (11) into condition (12), the equation $n^* = i^* + q^{i*}/a_p^f = q^{f*}$ is held in equilibrium.

[18]See McConnell and Perez-Quiros (2000) and Ramey and Vine (2004) for identifying the structural break.

[19]Kahn, McConnell, and Perez-Quiros (2002) show a similar trend for durable goods.

It is clear that q^{f*} is determined directly by n^* in the steady state as long as q^{i*} can be adjusted before the finished goods production. While in a single stage production model with only output inventories as outlined in Shi (1998), labor affects the quantity of goods per match indirectly through output inventories, and the effects of which are offset by the level of inventories. The corresponding condition in Shi (1998) is $i^* + f(n^*) = q^*$, where i^* is the output inventories, $f(\cdot)$ is the firm's production function and q^* is the quantity of goods per match. Since output inventories serve as buffer stocks, they decrease with employment (or output) if the money growth rate is low, as does the quantity of goods per match. Unlike the single-stage production model, input inventories build a bridge between labor and the quantity of finished goods per match by choosing the quantity of material inputs.

Contrary to the response of input inventory investment in a multi-stage production model, output inventory investment in a single-stage production model decreases with the money growth rate monotonically. The different responses of the quantity of goods per match are essential for generating the differences across the two models. Both, input and output inventory investment depend on the difference between the quantity of goods per match and the newly produced goods. As can be seen by rearranging equation (39), equation $NII^* = a_p^f \delta_i [q^{f*} - q^{i*}/a_p^f]$ holds in my model, while equation $NII^* = a_p^f \delta_i [q^* - f(n^*)]$ would hold in a standard search model. In a single-stage production model, the quantity of goods per match decreases with the money growth rate monotonically; in this way, output inventory investment decreases as $q^* - f(n^*)$ decreases. Output inventories serve as buffer stocks and decrease with sales. But, in a multi-stage production model, q^{f*} increases with moderate inflation, which implies that it is possible for the net inventory investment increases as households buy more intermediate goods. Thus, the multi-stage production model with input inventories is like the stockout avoidance model, in which inventories increase with sales. The single-stage production model with output inventories is like the production smoothing model, in which inventories smooth production and decrease with sales.

In the next chapter, I conduct a quantitative analysis to test the performance of my model. I also investigate the short run dynamics of a richer environment and examine the role of input inventories over the business cycle. Finally, I conduct a sensitivity analysis of some parameters relative to the baseline calibration.

Chapter 5

The Role of Multi-stage Production in the Monetary Propagation Mechanism

In this chapter, I take the model to the data and explore the short run dynamic responses to both money growth shocks and productivity shocks. In order to match the data, I generalize the model to incorporate technology shocks. The model is calibrated to match US quarterly data and is able to reproduce the stylized facts of input inventories. Moreover, consistent with the inventory literature, the multi-stage production model predicts that input inventory amplifies aggregate fluctuations over the business cycle.

5.1 Full Model

I use a richer model in the sense that matching frictions also exist in the intermediate goods market and the Cobb-Douglas production function is used instead of the Leontief production function. The detailed differences are the following. First, as the features of the finished goods market, there are also many types of intermediate goods, which are denoted by H^i. The measures of H, H^i and H^f are the same. Each household can produce the household specific intermediate goods. Trade occurs in the intermediate goods market, because the type $h \in H$ household cannot use its own specific intermediate goods as input to produce its specific finished goods.

Second, besides the six groups of agents mentioned in the benchmark model, each household has one more group of agents, which is the intermediate goods buyers (a_b^i). Third, each household has to make two more decisions in each period, one of which is how to divide money between markets, with the proportion Δ_{t+1} for intermediate goods buyers. At the beginning of each period, the household divides each market's nominal money balance evenly among each type of buyers. The other additional decision is the intermediate goods buyers' search intensities, denoted by s_t^i.

Once a buyer and seller are matched, the seller produces intermediate goods for his partner on the spot with disutility, $\varphi(\hat{q}_t^i)$, where $\hat{q}_t^i$ denotes the quantities of goods. The input buyer pays the amount of money, $\hat{m}_t^i$, to the intermediate goods seller. The terms of trade are determined by Nash bargaining. The function φ satisfies $\varphi' > 0$ and $\varphi'' > 0$ for $q > 0$, and $\varphi'(0) = \varphi(0) = 0$. Variables with a hat refer to an arbitrary household. At the end of the first sub period, the intermediate goods market closes. Input buyers bring trade receipts back to the household. The household adds traded intermediate goods to the input inventories, which are carried from the last period, and then divides the intermediate goods evenly among entrepreneurs. It is assumed intermediate goods sellers would not bring their money holdings back to the household until the end of the period. This assumption simplifies the model in the sense that the equilibrium conditions do not involve the intertemporal price ratio of one good market relative to the other. The new timing is summarized in Figure 6.

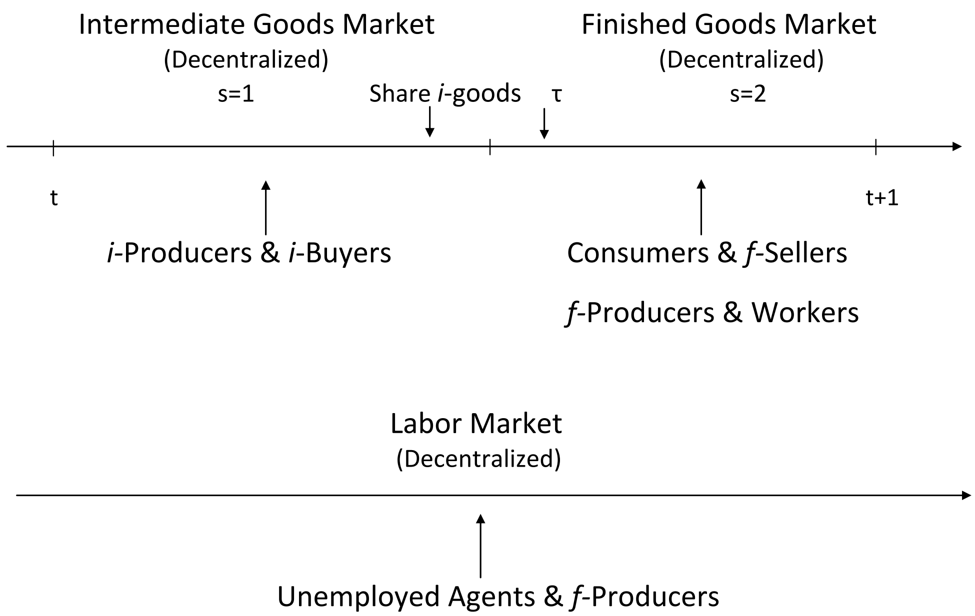

Figure 6: Timeline: Full Model

Similarly to the benchmark model, the total number of matches in the intermediate goods market is determined by the following matching function: $g(\hat{s}^i) \equiv z_1^i (a_b^i \hat{s}^i)^{\xi} {a_p^i}^{1-\xi}$, $\xi \in (0,1)$. As such, the matching rate for each unit of a buyer's search intensity is $g_b^i \equiv z^i (\hat{s}^i)^{\xi-1}$ and the matching rate per seller is $g_s^i \equiv z^i B^i (\hat{s}^i)^{\xi}$, where $B^i = a_b^i / a_p^i$ and $z^i \equiv z_1^i (B^i)^{\xi-1}$. Finally, a buyer and a seller get desirable matches at rates $s^i g_b^i$ and g_s^i, respectively.

The last difference from the benchmark model is the Cobb-Douglas production function used in this model:

$$q_t^f = A_t a_t^{\alpha} n_t^{1-\alpha}, \tag{40}$$

where a_t is the quantity of material inputs and n_t is labor inputs that are hired in the last period. A_t is total factor productivity. For the sake of comparison, I assume A_t follows a VAR process with the money growth rate γ as in Wang and Shi (2006):

$$\begin{pmatrix} \gamma_{t+1} \\ lnA_{t+1} \end{pmatrix} = N_1 + N_2 \begin{pmatrix} \gamma_t \\ lnA_t \end{pmatrix} + \begin{pmatrix} \epsilon_{m,t+1} \\ \epsilon_{A,t+1} \end{pmatrix} \tag{41}$$

where N_1 is a 2×1 vector and $N2$ is a 2×2 matrix. ϵ_m is the shock to the money growth and ϵ_A is the productivity shock. Let us denote the price level in the intermediate goods market by $P_t^i = m_t^i/q_t^i$, and $P_t^f = m_t^f/q_t^f$ for the price level in the finished goods market.

The household's new decision problem is altered as follows. The representative household taking the sequence $\{\hat{q}_t^i, \hat{m}_t^i, \hat{q}_t^f, \hat{m}_t^f, \hat{W}_t\}_{t\geq 0}$ and initial conditions $\{M_0, i_0, n_0\}$ as given, chooses $\{C_t, a_t, s_t^i, s_t^f, \Delta_{t+1}, M_{t+1}, i_{t+1}, v_t, n_{t+1}\}_{t\geq 0}$ to maximize its expected lifetime utility:

$$\max \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{-1}[U(c_t) - g_{st}^i a_p^i \varphi(\hat{q}_t^i) - a_p \hat{n}_t \varphi^f - a_b^i \Phi^i(s_t^i) - a_b^f \Phi^f(s_t^f) - a_p^f K(v_t)] \tag{42}$$

subject to the following constraints for all $t \geq 0$:

$$\begin{aligned} c_t &\leq (1 - FI) s_t^f g_{bt}^f a_b^f \hat{q}_t^f, && (43)\\ \frac{(1-\Delta_{t+1})M_{t+1}}{a_b^f} &\geq \hat{m}_{t+1}^f, \quad \forall F_{bt+1}{}^* && (44)\\ q_t^f &= A_t a_t^\alpha n_t^{1-\alpha}, && (45)\\ a_t &\leq i_t + \frac{1}{a_p^f} s_t^i g_{bt}^i a_b^i \hat{q}_t^i, \quad \forall F_{pt}{}^* && (46)\\ q_t^f &\geq \hat{q}_t^f, \quad \forall F_{pt}{}^* && (47)\\ \frac{\Delta_{t+1} M_{t+1}}{a_b^i} &\geq \hat{m}_{t+1}^i, \quad \forall I_{bt}{}^* && (48) \end{aligned}$$

$$
\begin{aligned}
M_{t+1} &\le M_t + \tau_t - s^i_t g^i_{bt} a^i_b \hat{m}^i_t + g^i_{st} a^i_p \hat{m}^i_t + a^f_p \hat{n}_t \hat{P}_t \hat{W}_t \\
&- s^f_t g^f_{bt} a^f_b \hat{m}^f_t + g^f_{st} a^f_p \hat{m}^f_t - \hat{P}_t a^f_p \hat{W}_t n_t, \qquad (49)\\
0 &\le a^f_p[(1-\delta_n)n_t + v_t\mu_t - n_{t+1}], \qquad (50)\\
a^f_p i_{t+1} &\le (1-\delta_i)[a^f_p i_t + s^i_t g^i_{bt} a^i_b \hat{q}^i_t - g^f_{st} a^f_p a_t]. \qquad (51)
\end{aligned}
$$

Functions $U(\cdot), \Phi^f(\cdot)$ and $K(\cdot)$ have the same properties as in the benchmark model. A buyer's disutility of searching in the intermediate goods market is denoted by $\Phi^i(s^i_t)$. The function Φ^i satisfies $\Phi^{i'} > 0$ and $\Phi^{i''} > 0$ for $s^i > 0$, and $\Phi^i(0) = \Phi^{i'}(0) = 0$. Moreover, as discussed in Shi (1998),[20] I modify the model to incorporate fixed investment which is a constant fraction of aggregate sales.

I^*_{bt} (with measure $s^i_t g^i_{bt} a^i_b$) is the set of matched intermediate goods buyers in period t, and I^*_{bt} (with measure $s^f_t g^f_{bt} a^f_b$) is its finished goods counterparts. Similarly, F^*_{pt} (with measure $g^f_{st} a^f_p$) is the set of matched entrepreneurs in the finished goods market in period t.

Denote the multipliers of money constraint (44) and (48) by Λ^f_t and Λ^i_t respectively. All of the multipliers of the rest conditions are the same as in the benchmark model. Since condition (44), (46) and (48) are restricted to being binding in equilibrium and conditions (47), (49), (50) and (51) are binding in equilibrium, we can derive the first-order conditions with respect to $(M_{t+1}, i_{t+1}, n_{t+1}, s^i_t, s^f_t, \Delta_{t+1}, v_t, a_t)$ as follows:

$$
\begin{aligned}
\Omega_{Mt} &= \beta\mathbb{E}[\Omega_{Mt+1} + s^f_{t+1} g^f_{bt+1} \Lambda^f_{t+1}(1-\Delta_{t+1}) \qquad (52)\\
&+ s^i_{t+1} g^i_{bt+1} \Lambda^i_{t+1} \Delta_{t+1}],
\end{aligned}
$$

[20] Also see Wang and Shi (2006)

$$
\begin{aligned}
\Omega_{it} &= \beta\mathbb{E}[(1-\delta_i)\Omega_{it+1} + a_p^f g_{st+1}^f \Omega_{at+1}], && (53)\\
\Omega_{nt} &= \beta\mathbb{E}[(1-\alpha) g_{st+1}^f A_{t+1} a_{t+1}^{\alpha} n_{t+1}^{-\alpha}\Omega_{ft+1} && (54)\\
&+ (1-\delta_n)\Omega_{nt+1} - \hat{P}_{t+1}\hat{W}_{t+1}\Omega_{Mt+1}], &&\\
\Phi^{i'}(s_t^i) &= g_{bt}^i[g_{st}^f a_p^f \Omega_{at}\hat{q}_t^i - \Omega_{Mt}\hat{m}_t^i + (1-\delta_i)\Omega_{it}\hat{q}_t^i], && (55)\\
\Phi^{f'}(s_t^f) &= g_{bt}^f[(1-FI)U'(C_t) - \omega_t^f]q_t^f, && (56)\\
\Lambda_t^f s_t^f g_{bt}^f &= \Lambda_t^i s_t^i g_{bt}^i, && (57)\\
\Omega_{nt} &= K'(v_t)/\mu(\hat{v}_t), && (58)\\
a_p^f \Omega_{at} &= A_t \alpha a_t^{\alpha-1} n_t^{1-\alpha}\Omega_{ft} - (1-\delta_i)\Omega_{it}. && (59)
\end{aligned}
$$

The terms of trade in the intermediate goods market are determined by Nash bargaining. As in the benchmark model, I assume the intermediate goods buyers and sellers have the same bargaining powers, and the terms of trade is computed by treating each agent negligible in his household. I only specify the terms of trade in the intermediate goods market, the counterparts in the finished goods market and the labor market are the same as in Chapter 4.

The terms of trade in the intermediate goods market are denoted by $(q_t^i\varepsilon, \bar{m}_t^i\varepsilon)$, where $q_t^i\varepsilon$ is the quantity of intermediate goods and $\bar{m}_t^i\varepsilon$ is the quantity of money. Thus the trading surpluses of these two agents to their households are:

$$
\text{Seller's trade surplus:} \quad \Omega_{Mt}\bar{m}_t^i\varepsilon - [\varphi(\bar{q}_t^i + q_t^i\varepsilon) - \varphi(\bar{q}_t^i)], \tag{60}
$$

$$
\text{Buyer's trade surplus:} \quad [\bar{\Omega}_{at} + (1-\delta_i)\bar{\Omega}_{it}]q_t^i\varepsilon - (\bar{\Lambda}_t^i + \bar{\Omega}_{Mt})m_t^i\varepsilon. \tag{61}
$$

After normalizing surpluses by ε, the terms of trade are then determined by Nash bargaining between buyer and seller with equal weights, as demonstrated in

Shi (1998):

$$\max_{\bar{m}_t^i, q_t^i} \left[\Omega_{Mt}\bar{m}_t^i - \frac{\varphi(\bar{q}_t^i + q_t^i\varepsilon) - \varphi(\bar{q}_t^i)}{\varepsilon}\right]^{1/2} \left[[\bar{\Omega}_{at} + (1-\delta_i)\bar{\Omega}_{it}]q_t^i - (\bar{\Lambda}_t^i + \bar{\Omega}_{Mt})m_t^i\right]^{1/2}. \quad (62)$$

After substituting $\bar{m}_t^i = P_t^i q_t^i$ and taking the limit $\varepsilon \to 0$ on the first-order conditions, we can get the following two conditions:

$$P_t^i(\bar{\Omega}_{Mt} + \bar{\Lambda}_t^i) = \bar{\Omega}_{at} + (1-\delta_i)\bar{\Omega}_{it}, \quad (63)$$

$$\varphi'(q_t^i) = P_t^i \Omega_{Mt}. \quad (64)$$

The first condition equates the opportunity cost of spending money, $P_t^i(\bar{\Omega}_{Mt} + \bar{\Lambda}_t^i)$, with the benefits of obtaining ε additional units of intermediate goods, which includes relaxing the intermediate goods usage constraint and increasing the shadow value of inventories. The second condition states that the marginal cost of production equals the shadow value of real money balance in the intermediate goods market. Denote the shadow value of real money balance in the finished goods market by $\omega_t^i = P_t^i \Omega_{Mt}$.

The gross rate of money growth is defined by $\gamma_t \equiv M_{t+1}/M_t = (M_t + \tau_t)/M_t$. By substituting (57) into (52), dividing (44) by (48), denoting $k(v_t) = K'(v_t)/\mu(\hat{v}_t)$ and eliminating $(\Omega_n, W, \omega^i, \lambda^f, \Omega_a, \Omega_f)$ by using conditions (25), (26), (29), (58), (63), (64) and $P_t^f = M_t^f/q_t^f$, the dynamic system is characterized in terms of $(q^i, q^f, v, s^i, s^f, \omega^f, \Delta, \Omega_i, \lambda^i, a)$ by equations (43), (50)-(56) and (59):

$$n_{t+1} = (1-\delta_n)n_t + v_t\mu(v_t), \quad (65)$$

$$a_p^f i_{t+1} = (1-\delta_i)[a_p^f i_t + s_t^i g_{bt}^i a_b^i q_t^i - g_{st}^f a_p^f a_t], \quad (66)$$

$$
\begin{aligned}
i_t &= a_t - s^i_t g^i_{bt} a^i_b q^i_t / a^f_p, && (67)\\
\mathbb{E}[\frac{1-\Delta_{t+1}}{\Delta_{t+1}}] &= \mathbb{E}[\frac{\omega^f_{t+1} q^f_{t+1} a^f_b}{\varphi'(q^i_{t+1}) q^i_{t+1} a^i_b}], && (68)\\
\mathbb{E}\Big[\frac{(1-\Delta_{t+1})\gamma_t}{(1-\Delta_t)}\omega^f_t \frac{q^f_t}{q^f_{t+1}}\Big] &= \beta\mathbb{E}\Big[\{\omega^f_{t+1} + z^f (s^f_{t+1})^{\xi}[(1-FI)U'(c_{t+1} - \omega^f_{t+1})]\}\Big], && (69)\\
\Omega_{it} &= \beta\mathbb{E}\{(1-\delta_i)\Omega_{it+1} + g^f_{st+1} a^f_p [\varphi'(q^i_{t+1})\\
&+ \lambda^i_{t+1} - (1-\delta_i)\Omega_{it+1}]\}, && (70)\\
k(v_t) &= \beta\mathbb{E}[\sigma g^f_{st+1} A_{t+1} a^{\alpha}_{t+1} n^{-\alpha}_{t+1} \omega^f_{t+1}(1-\alpha)\\
&- \sigma\varphi^f + (1-\delta_n)\Omega_{nt+1}], && (71)\\
\Phi^{i'}(s^i_t) &= g^i_{bt}[g^f_{st} a^f_p \lambda^i_t - (1 - g^f_{st} a^f_p)\varphi'(q^i_t)\\
&+ (1 - g^f_{st} a^f_p)(1-\delta_i)\Omega_{it}] q^i_t, && (72)\\
\Phi^{f'}(s^f_t) &= z^f (s^f_t)^{\xi-1}[(1-FI)U'(c_t) - w^f_t] q^f_t, && (73)\\
c_t &= (1-FI) a^f_p B^f z^f (s^f_t)^{\xi} q^f_t, && (74)\\
A_t \alpha a^{\alpha-1}_t n^{1-\alpha}_t \omega^f_t &= a^f_p[\varphi'(q^i_t) + \lambda^i_t] + (1 - a^f_p)(1-\delta_i)\Omega_{it}. && (75)
\end{aligned}
$$

5.2 Calibration

The model is log-linearized and calibrated to match the quarterly US data. The sample period is 1967:I - 2010:IV. To be comparable to the literature, the inventory data are from the manufacturing sector. I will briefly summarize my calibration strategy before I describe the detailed procedures. The parameters in my model can be grouped into three categories. The first set of parameters ($\gamma^*, \hat{N}1, \hat{N}2, \sigma_m, \sigma_A, \sigma_{mA}, \rho, \sigma_g$) is calculated directly from data. The second set of parameters ($\phi, \sigma, \delta_n, \alpha, u, \delta_i, z^f_1, a^f_p, a^f_b, FI,$ $b, K_0, \varphi^i, \varphi^i_0, \varphi^f, \varphi^f_0, \bar{\mu}$) is pinned down by jointly matching a set of targets . Most of my targets are calculated from my samples, while some of them are taken from other work in the literature. The last set of parameters $(\eta, \epsilon_i, \epsilon_f, \xi, B_i)$ is hard to determine, and I pin them down by jointly minimizing the difference between the simulated second moments and the observed ones.

The discount factor is set at $\beta = 0.995$, which implies that the annual interest rate is 2%. In order to calibrate the model, we assume the utility function, the disutility functions of searching in the goods markets, the disutility function of producing intermediate goods, and the disutility function of posting vacancies have the following functional forms:

$$\begin{aligned} U(c_t) &= \frac{c_t^{1-\eta} - 1}{1-\eta}; \qquad (76) \\ \Phi^i(s_t^i) &= \varphi^i(\varphi_0^i s_t^i)^{1+1/\epsilon_i}; \qquad (77) \\ \Phi^f(s_t^f) &= \varphi^f(\varphi_0^f s_t^f)^{1+1/\epsilon_f}; \qquad (78) \\ \varphi(q_t^i) &= \frac{b}{2}(q_t^i)^2; \qquad (79) \\ K(v_t) &= K_0 v_t^2; \qquad (80) \end{aligned}$$

where, $\eta, \varphi^i, \varphi_0^i, \epsilon_i, \varphi^f, \varphi_0^f, \epsilon_f, b, K_0$ are constants. The gross rate of money growth is defined by $\gamma_t = M_{t+1}/M_t$, and assumed to follow an VAR(1) process jointly with productivities A_t. The estimated coefficients and the standard error of both shocks are the following:

$$\hat{N}1 = \begin{pmatrix} 0.738 \\ -0.958 \end{pmatrix}, \quad \hat{N}2 = \begin{pmatrix} 0.274 & -0.0773 \\ 0.845 & 0.942 \end{pmatrix},$$

$$\sigma_m = 0.0064, \quad \sigma_A = 0.0138, \quad \sigma_{mA} = 0.0045.$$

The time series of total factor productivity are calculated as the Solow residual. Since the production function is in the individual level, it has to be aggregated in order to match the data. The expression for the log of productivity is the following:

$$\begin{aligned} lnA &= ln(c^f) - ln(a_b^f v_c^f B^f) - \alpha ln(inputs/P_i) \\ &+ \alpha ln(a_b^f v_c^f B^f/(1-FI)) - (1-\alpha) ln(empl/a_p^f), \end{aligned} \tag{81}$$

where v_c^f is the velocity of money. The variable *inputs* are the total cost of material inputs. P_i is the price deflator for material costs. The variable *empl* is the aggregate employment in the manufacturing sector. These series come from NBER databases.

In order to highlight the role of multi-stage production in the effects of money growth, I also report the impulse response functions by assuming that no technology shocks exist and that the money growth rate follows a AR(1) process with parameters $\rho = 0.569$ and $\sigma_g = .00789$:[21]

$$\gamma_t = (1-\rho)\gamma^* + \rho\gamma_{t-1} + e_t, \qquad e \sim (0, \sigma_e^2) \tag{82}$$

I use M2 as the money stock since it is stable within the sample period. The steady state money growth rate is calculated as the sample average. We also need to determine the following parameters: $(\phi, \eta, \sigma, \delta_n, \xi, u, \delta_i, z_1^i, z_1^f, a_p^i, a_p^f, a_b^i, a_b^f, FI, b, K_0, \varphi^i, \varphi_0^i, \varphi^f, \varphi_0^f, \epsilon_i, \epsilon_f, \alpha, B_i, \bar{\mu})$.

Parameters $(\phi, \sigma, \delta_n, \alpha, u, \delta_i, z_1^f, a_p^f, a_b^f, FI, b, K_0, \varphi^i, \varphi_0^i, \varphi^f, \varphi_0^f, \bar{\mu})$ are jointly calibrated to match the following targets: (1) the average labor participation rate is 0.6445 over the same sample period. (2) The average unemployment rate is 0.061.

[21]See Hornstein and Sarte (1998) and Menner (2006) for similar estimation results.

(3) The average velocity of M2 money stock is 1.8236. (4) The average input inventory to final sales ratio is 0.984. (5) The intermediate inputs to final sales ratio is 0.549. (6) The inventory to output ratio is 0.981 and the inventory investment to output ratio is 0.0038. (7) The shopping time of the population is 11.17% of the working time and the working time is 30% of agents' discretionary time. (8) The vacancy posting cost is 3.72×10^{-4}. (10) The average monthly separation rate from employment to unemployment is 0.034. (11) The average markup is 70%.

The first six targets are calculated from my samples. The seventh target is used to compute the goods markets' search intensities and is taken from Shi (1998) (Wang and Shi (2006) use the same target). The target value for the vacancy posting cost is taken from Berentsen, Menzio, and Wright (2011). The target value for the average monthly separation rate is taken from Shimer (2005). Since the markup is hard determine, I do sensitivity analysis for the target value of the markup in Section 5.6.

The finished good producers' bargaining power (σ), in the labor market, is set to the same value as the elasticity of the labor market matching function (ϕ) to give workers 72% of the rent, as such the Hosios condition holds.[22] The elasticity of the labor market matching function (ϕ) and the associate constant are estimated by the ordinary least squares, regressing the log of the job-finding rate on the log of market tightness. For the sake of comparison, FI is set to 0.269 as outlined in Shi (1998) and Wang and Shi (2006).[23]

Since I have an intermediate goods market in my model, I make the following assumptions in order to pin down the parameters (a_p^i, a_b^i, z_1^i). First, I assume the intermediate goods market tightness equals B_f. Second, I assume the time spent on

[22] Also see Hosios (1990); Shi (2006); Shimer (2005) and Rogerson and Shimer (2010).
[23] Also see Christiano (1988).

Table 9: Parameter Values and Targets

Parameters	Values	Targets
β	0.995	Annual interest rate: 4%
A^*	1	Normalization
Unemployment u	0.0393	Avg. LP: 0.6445
i-sellers a_p^i	0.2010	Avg. UR: 0.061
f-sellers a_p^f	0.2010	Avg. Velocity of M2: 1.8236
i-buyers a_b^i	0.1005	Avg. ISR: 0.984
f-buyers a_b^f	0.0515	Avg. IPS: 0.549
δ_i	0.0038	Avg. INV/GDP: 0.981
z_1^i	0.0934	Avg. NII/GDP: 0.0038
z_1^f	3.0524	Shopping time/Working time: 11.17%
b	0.6706	Working time/Discretionary time: 30%
K_0	0.0006	Vacancy posting cost: 3.72×10^{-4}
B_f	0.1947	Avg. markup: 70%
α	0.4156	
φ^i	14.2091	
φ_0^i	0.1104	
φ^f	1.6025	
φ_0^f	1.8043	
ϕ	0.28	OLS estimation
$\bar{\mu}$	0.364	OLS estimation
σ	0.28	Give workers 72% of the rent
δ_n	0.105	Avg. monthly separation rate: 0.034
FI	0.269	Shi (1998)

searching intermediate goods is also 11.17% of the working time. Third, I assume the numbers of sellers in both goods markets are equal. Finally, I assume the velocity of the money stock in the intermediate goods market is 0.2. The velocity is very low in the intermediate goods market, but 0.2 is the upper bound that works for this model.

Since $(\eta, \epsilon_i, \epsilon_f, \xi, B_i)$ cannot be determined, sensitivity analysis on these parameters will be discussed in Section 5.6. Right now, I only provide the parameter values which best fit the data. I set the elasticity of goods market matching functions ξ

Table 10: Parameter Values and Targets (cont'd)

Parameters	Benchmark Values
ξ	0.8
ϵ_i	0.4
ϵ_f	0.01
η	0.8
B_i	0.2

to be 0.8, the elasticities of the disutility functions of searching ϵ_i and ϵ_f to be 0.4 and 0.01 respectively. I set the relative risk aversion $\eta = 0.8$ and the markup to be 50%. I normalize the number of workers hired by each firm to $n = 1$. The parameter values and corresponding targets are summarized in Table 9. The benchmark values for assumed parameters are summarized in Table 10. The strategy of calibration is described in detail in Appendix D.

5.3 Model Predictions

By using the calibrated parameter values, I give a numerical example in Figure 7 to illustrate Proposition 1-3 which shows that, for a low level of money growth, GDP, the net inventory investment, the inventory-to-sales ratio and the quantity of finished goods per trade increase with the money growth rate, but decrease with the money growth rate if it reaches a high growth threshold. The critical money growth rate for each variable is slightly different, ranging from 5.68% to 7.98%.

Now let us look at the quantitative performance of the multi-stage production model. Table 11 reports the model prediction for stylized facts of input inventories. By assuming the economy is hit by a positive money growth shock, the stylized facts of input inventories can be quantitatively well reproduced by my model, such

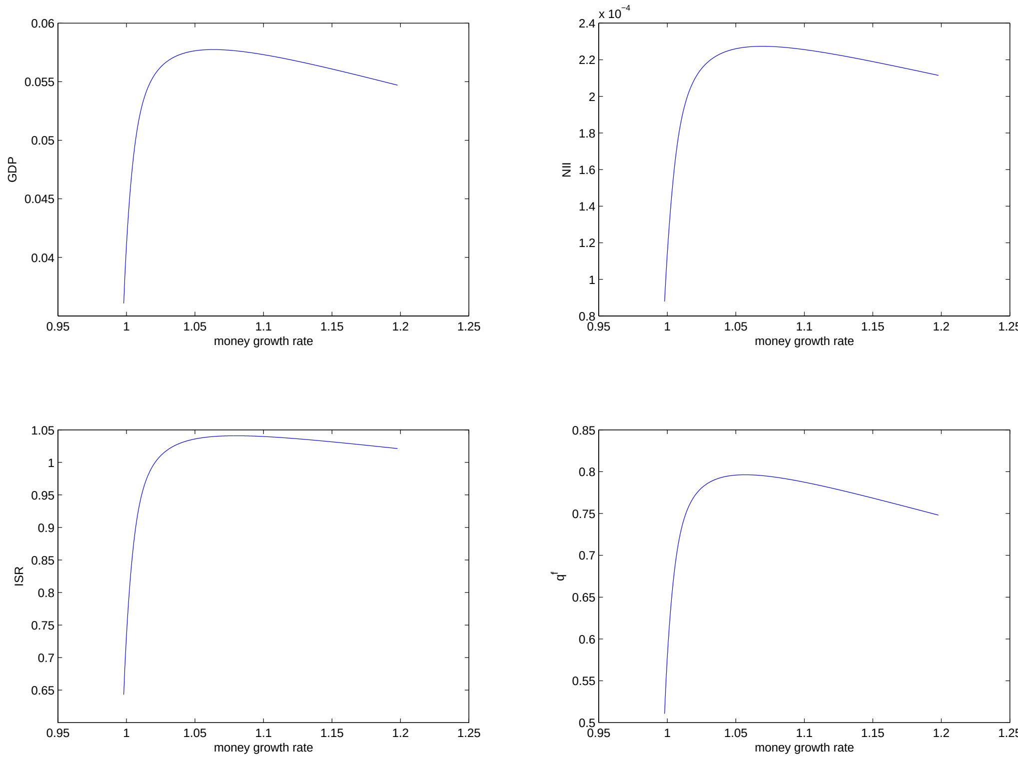

Figure 7: Long Run Effects of Money Growth

as procyclical inventory investment, countercyclical inventory-to-sales ratio, negative correlation between final sales and inventory-to-sales ratio and more volatile output relative to final sales.

The most striking result is that the model produces 96% of the observed correlation between final sales and inventory investment. As tested in Khan and Thomas (2007b), neither the (S, s) model nor the basic stockout avoidance model can reproduce the positive relationship between final sales and net inventory investment under a preference shock. Even after introducing idiosyncratic shocks, the generalized stockout avoidance model still can only generate a very weak positive correlation, and this slight improvement comes as a result of severely sacrificing the ability to match the

Table 11: Model Predictions

	Data*	Model	Standard Deviations
corr(GDP, IS)	-0.756	-0.798	(0.011)
corr(GDP, NII)	0.789	0.882	(0.008)
corr(FS, IS)	-0.756	-0.665	(0.018)
corr(FS, GDP)	0.987	0.957	(0.001)
corr(FS, NII)	0.680	0.708	(0.014)
$\sigma(FS)/\sigma(GDP)$	0.837	0.666	(0.008)
$\sigma(NII)/\sigma(GDP)^{**}$	0.220	0.411	(0.005)
$\sigma(IS)/\sigma(GDP)$	0.843	0.794	(0.011)

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales. I use quarterly U.S. data for the period 1967:Q1 to 2010:Q4. All data are real series, end of period, seasonally adjusted and chained in 2005 dollars. Net inventory investment is detrended as a share of GDP. Other series are detrended using a HP filter ($\lambda = 1600$).

** NII are calculated as a share of GDP in order to compare the results with that in Khan and Thomas (2007b).

long run average inventory-to-sales ratio. Moreover, as mentioned in the introduction, a one sector search model is unable to replicate the stylized facts of inventories without technology shocks. Since money growth shocks can be viewed as a demand shock, it can be understood that demand shocks help to explain inventory behaviors and are important for studying business cycles.

Unlike the models of Kryvtsov and Midrigan (2010a,b) and Jung and Yun (2006), my model replicates the stylized facts of input inventories with calibrated depreciation rate that is as low as the empirical one. Please note that the positive correlation holds even without the modeling of technology shocks.[24] In this paper, I use a generalized model with productivity shocks, because I want to match the estimated empirical impulse response functions qualitatively.

[24]Simulation results with Leontief production function are reported in Table 2 in the previous version of this paper. Also see Figure 10 for the impulse response functions.

The positive responses of q^f are essential for being able to match the stylized facts. In a standard search model, buyers search more in instances of higher money growth rate; in such instances, the number of matched buyers increases, and output inventories decrease, since the quantity of goods per match decreases. In a multi-stage production model, the positive responses of q^f are strong enough to push input inventories to move during the transition in the same direction as final sales. In this paper, I only model input inventories, but this model is able to also fit data with a broader concept of inventories, since input inventories are empirically more important than output inventories.

The model predicts a higher standard deviation of inventory investment relative to GDP, since there is no adjustment cost in this model. By introducing adjustment cost, the response of inventory investment will be smoother. Of course, it may sacrifice the accurateness of the stylized facts in other dimensions. In order to keep the model simple, I abstain from using the adjustment cost. Since the model reproduce the focused long run relations, the analysis of short run dynamic responses is credible.

5.4 Impulse Responses

In this section, I show how multi-stage production affects the short run dynamic responses of the equilibrium. Figure 8 depicts the impulse responses functions to a one positive standard deviation shock to the money growth rate. The most striking results are that both the responses of employment and q^f are hump-shaped, and that both variables stay above the steady state during the entire transition. The hump-shaped response of q^f is consistent with the model's long run predictions in the sense that the quantity of finished goods per match response positively to the money growth shock in the short run, given that the calibrated money growth rate is lower than the threshold.

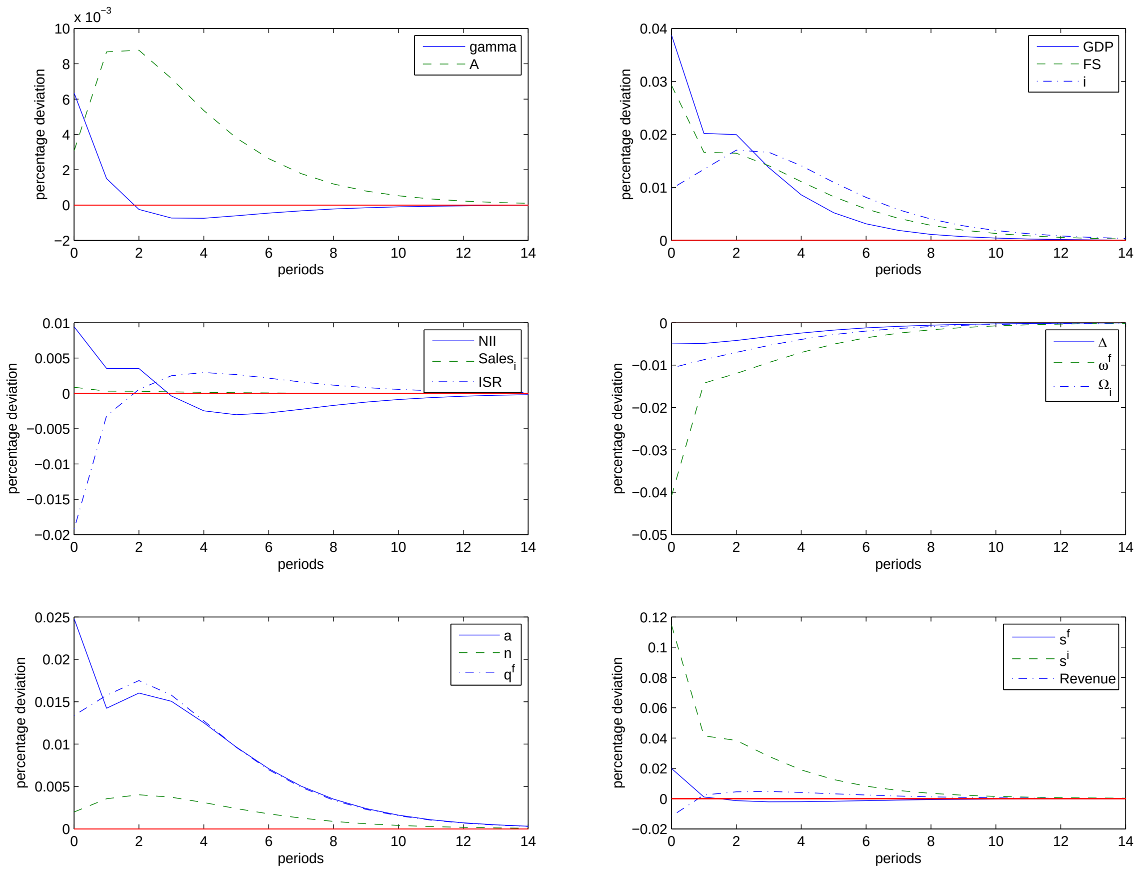

Figure 8: Impulse Response Functions to a Positive Money Growth Shock

Now let us look at the details of the propagation mechanism. Figure 8 depicts the short run dynamic responses of the equilibrium to a one positive standard deviation shock to the money growth rate. First, when the shock hits the economy, the money growth rate increases and real money balances fall immediately, which stimulates buyers to search more intensively in both goods markets. Since inventories depreciate each period and households are eager to consume, households allocate proportionally more money to the finished goods market, as demonstrated by the fact that Δ_t drops immediately. Given the low money growth rate, households anticipate higher final sales and immediately increase intermediate goods buyers' search intensities in order to obtain more intermediate goods. Since households do not know which finished goods producers would get a match in the second sub-period, they increase the level of intermediate goods for all finished goods producers, as demonstrated by the fact that a jumps immediately. With more intermediate goods, finished goods producers produce more if they matched and q^f jumps immediately.

Second, the money growth shock also induces a positive response of productivity, because I assume that monetary shocks affect productivity contemporarily and past money growth has a positive correlation with current productivity. Since the effects on productivity are very persistent, final sales keep above the steady state for more than twelve quarters. Such effects on final sales transfer back to the intermediate market and keep intermediate sales above the steady state. The positive response of final sales also increases future revenues. Since revenues stay above the steady state from the second period, households post more vacancies immediately, and n_{t+1} stays above the steady state for more than ten periods. As a result, q^f continues to rise before slowly going back to the steady state as the effects of technology shock diminishing. The multi-stage production enables q^f to synchronize with the responses of employment and material inputs during the transition. Thus the positive response of q^f arises from the interaction between different production stages which is different from the standard search model.

Finally, inventories stays above the steady state since unmatched finished goods producers hold more intermediate goods during the transition, which is shown by the decreased shadow value of inventories. The sluggish responses of inventories shed light on the "slow speed of adjustment" puzzle in the literature. Both search frictions and persistent shocks contribute to the hump-shaped response.

Figure 9 depicts the short run dynamic responses of the equilibrium to one positive standard deviation shock to productivity. The same qualitative responses were obtained as in the case of the money growth shock, except for the responses related to employment and q^f, which decrease monotonically toward the steady state instead of hump-shaped responses. Intuitively speaking, this is because the technology shock decreases monotonically during the transition. The positive technology shock induces a negative response of money growth shock, because current money growth is negatively correlated with past productity. As a result, the responses of both shocks are qualitatively the same as those in Figure 8 from the second period. Quantitatively speaking, the responses are stronger in the case of the technology shock, since it is bigger than the money growth shock.

Compared to the structural impulse response functions estimated in Chapter 3, the theoretical impulse response functions are more well-matched to the empirical response functions if the aggregate fluctuations originated from productivity shocks. The theory predicts that real variables respond contemporarily to the money growth shock, because the shocks are realized at the beginning of each period. Households are forward looking, thus they immediately increase search intensities whenever the money growth rate increases. As a result, the velocity of money and the total number of matched buyers increases in both goods markets, which leads to higher intermediate goods sales and higher inventory investments. In contrast, real variables respond

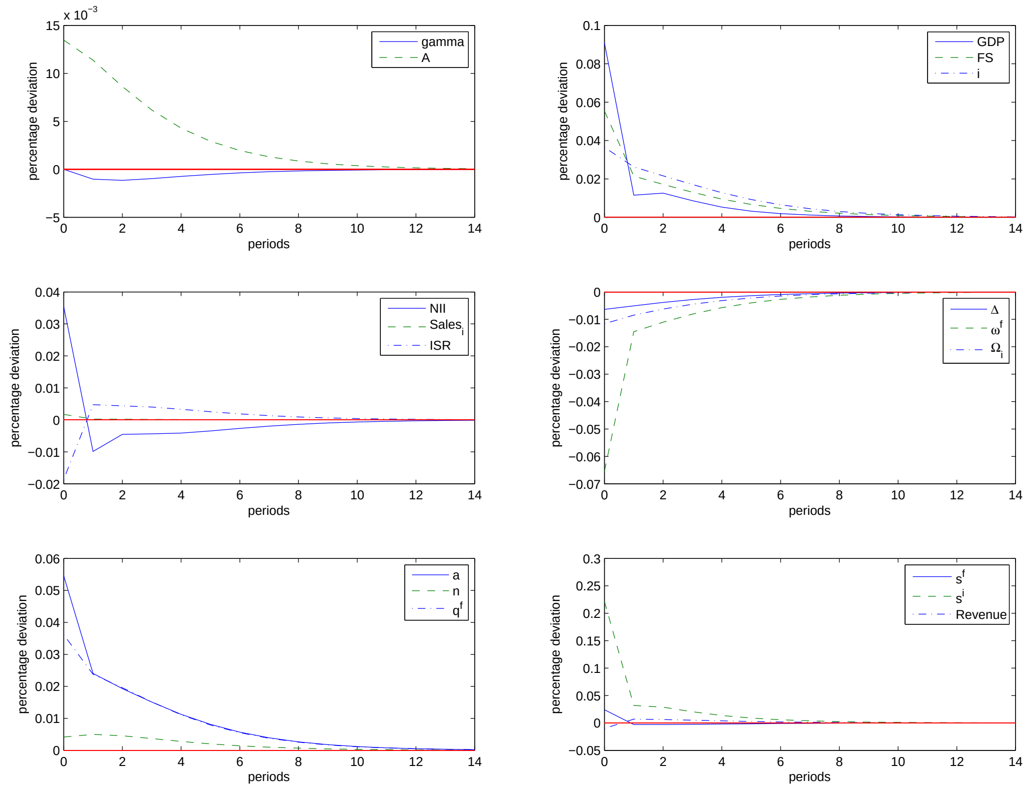

Figure 9: Impulse Response Functions to a Positive Productivity Shock

to the money growth rate shock with one lag in the data. Moreover, the inventory investment and employment drop at the beginning in the data. The theoretical impulse response functions would have matched the data more effectively if I had assumed that the shock was realized at the end of each period. I kept my assumptions regarding timing consistent with Menner (2006) and Wang and Shi (2006), to enable comparison with their models.

Multi-stage production implies that input inventories move with final sales in the same direction during the transition and a lower inventory level mutes the effects of money growth shocks on aggregate variables. This is in sharp contrast to the model put forth by Khan and Thomas (2007b), in which there is a tradeoff between final sales and inventories and inventory accumulation dampens the response of final sales.

I will discuss this in more detail in the next section.

The responses of input inventories in my model and that of output inventories in Shi (1998) are very different. Shi's model is similar to the production smoothing model, which always has a tradeoff between the inventory and output. Thus, inventories decrease whenever output increases during the transition, and it is difficult to reproduce the stylized facts of inventories without technology shocks.[25]

Building on Shi's (1998) model, Wang and Shi (2006) predict positive response of inventories with the money growth shock. Their result heavily relies on the positive correlation between money growth shock and technology shock. But in my model, the positive correlation between the two shocks is not essential for generating the positive response of inventories. Figure 10 shows that, without technology shocks, input inventories still respond positively to the monetary shocks except for one period drop below the steady state. Moreover, employment responds negatively to both shocks in their paper, because abundant goods reduce households' profitability to hire labor. Contrary to their results, employment increases in response to both shocks in my paper, because input inventories work differently through the propagation mechanism. Input inventories are a part of the next period's material inputs in the multi-stage production model, thus higher input inventories have positive effects on final sales and revenues. As a result, households hire more labor if future revenues increase. In Wang and Shi (2006), inventories are important for propagation mechanism because output inventories induce a shortage of future goods supply, which keeps buyers' search intensities above the steady state. Therefore, the short run dynamics of employment are very different in these two models.[26]

[25]The responses of inventories and final sales move in the opposite directions during the transition in Menner (2006); Shi (1998); Wang and Shi (2006).

[26]Our findings are consistent with Chang, Hornstein, and Sarte (2006) in which employment (and

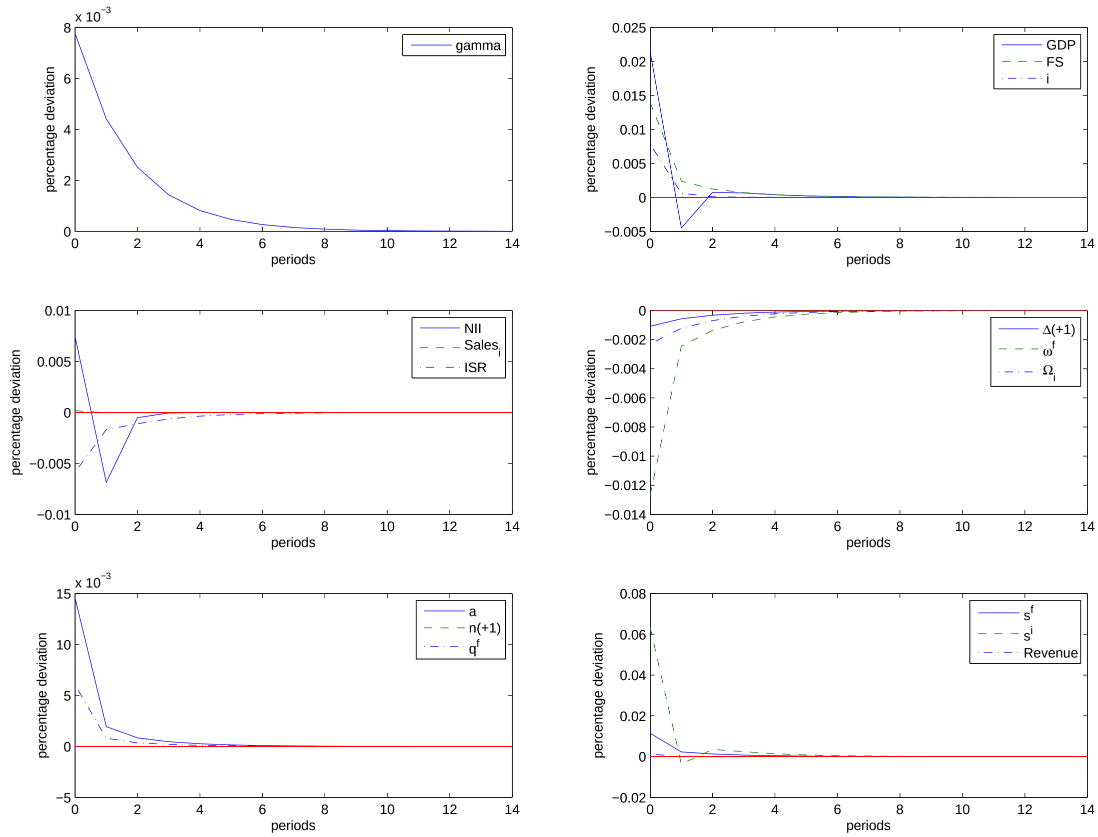

Figure 10: Impulse Response Functions to a Positive Money Growth Shock: Without Productivity Shock

Shibayama (2008) models input inventories as unsold goods by assuming price posting, because he argues that firms have little incentive to hold inventories when they make sale decisions. The friction in the intermediate goods market is the reason for the existence of inventories in his model, while search friction in the finished goods market is the key in my model. Moreover, the adjustment of inventories is slow in his model, because the stockout constraint in the intermediate goods market and the production chain prevent firms adjust inventories in time. In my model, the slow speed of adjustment is due to the search friction in the labor market. Since the

inventories) increases in response to a permanent and positive shock to productivity in a model with inventories, if the costs of holding inventories are sufficiently low.

response of employment is persistent, intermediate goods sales stay above the steady state for a long period and so do the inventories.

5.5 The Role of Input Inventories over Business Cycles

There has long been a debate within the inventory literature about the role of inventories over business cycles. Since GDP is more volatile than final sales in the data, most researchers believe that inventories amplify aggregate fluctuations over business cycles. Others argue that inventories smooth business cycles because they smooth productions. In this section, I revisit this debate and use the multi-stage production to examine the role of input inventories over business cycles.

I compare two pairs of results by targeting different inventory-to-sales ratios and holding other parameters unchanged. I divide the sample period into two sub periods. The first inventory-to-sales ratio is 1.0402, which is calculated from the first sub period: 1967:I-1983:IV, and the second inventory-to-sales ratio is 0.9499, which is calculated from the second sub period: 1984:I-2010:IV. I choose the year 1984 as a break point for my sample because the (input) inventory-to-sales ratio experiences a declining trend beginning in 1984.[27] Some researchers argue that this declining trend is one of the reasons for "Great Moderation". My model predictions support this argument.

The calibration results are reported in Table 12. The stylized facts calculated from these two sub samples are similar except for the relative standard deviations

[27]The (output) inventory-to-sales ratio exhibits an opposite trend. See Iacoviello, Schiantarelli, and Schuh (2011) for details.

of the inventory investment and the inventory-to-sales ratio relative to GDP. The most striking results are that the standard deviations of GDP, the input inventory investment, the inventory-to-sales ratio and final sales are lower in the second sub sample. Thus, my model predicts that lowing input inventories smooth aggregate fluctuations in the second sub sample.

Table 12: Role of Inventories

	$ISR = 1.0402^*$		$ISR = 0.9499^*$		*Data*
corr(GDP, IS)	-0.774	(0.014)	-0.801	(0.011)	-0.756
corr(GDP, NII)	0.896	(0.007)	0.874	(0.008)	0.789
corr(FS, IS)	-0.599	(0.023)	-0.690	(0.017)	-0.756
corr(FS, GDP)	0.940	(0.001)	0.965	(0.001)	0.987
corr(FS, NII)	0.693	(0.014)	0.717	(0.014)	0.680
$\sigma(FS)/\sigma(GDP)$	0.613	(0.008)	0.696	(0.007)	0.837
$\sigma(NII)/\sigma(GDP)$	0.473	(0.005)	0.376	(0.004)	0.220
$\sigma(IS)/\sigma(GDP)$	0.670	(0.009)	0.860	(0.013)	0.843
$\sigma(GDP)$	0.112	(0.003)	0.088	(0.002)	-
$\sigma(NII)$	0.053	(0.001)	0.033	(0.001)	-
$\sigma(INV)$	0.016	(0.001)	0.012	(0.001)	-
$\sigma(FS)$	0.069	(0.002)	0.061	(0.002)	-

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. The model is calibrated to two different ISR targets while keeping all the other parameters unchanged. $ISR = 1.0402$ is calculated from the first sub-sample: 1967:I - 1983:IV and $ISR = 0.9499$ is calculated from the second sub-sample: 1984:I - 2010:IV.

Figure 11 and 12 depict how the short run dynamic systems change with different inventory-to-sales ratios to both monetary and technology shocks. Both figures show that input inventories amplify aggregate fluctuations over business cycles. The model predicts that GDP, final sales the inventory investment and employment respond stronger to both shocks in cases of a higher inventory-to-sales ratio. Households hold more inventories during the transition and use more material inputs for finished goods production.

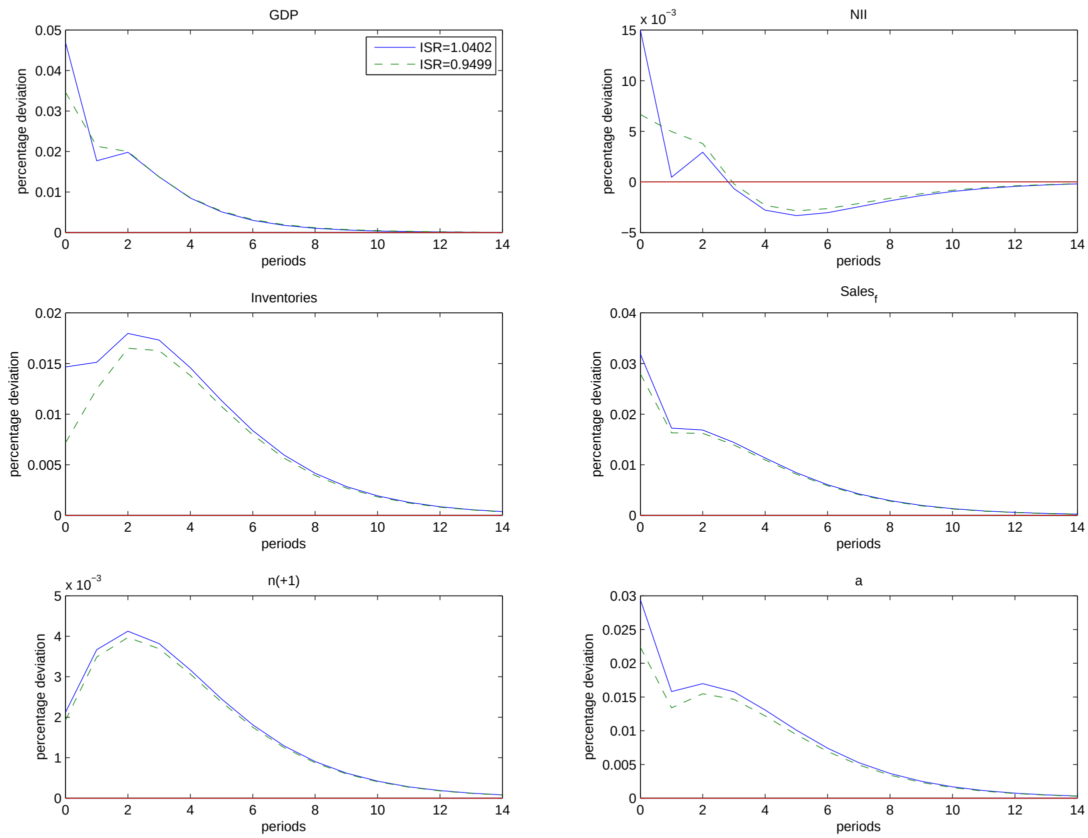

Figure 11: Role of Inventories: Money Growth Shock

The intuition behind these results is straightforward. A lower inventory-to-sales ratio implies that households hold a lower inventory level. Since inventories enter the production function, a lower inventory level has negative effect on q^f and revenues. Therefore, q^f, employment and final sales respond at a lower magnitude to both shocks. Inventory investment becomes less volatile, because households do not need to adjust a high level of intermediate goods for each finished goods producers, for example material inputs(a) becomes less volatile. Overall, GDP is less volatile to shocks with a lower inventory-to-sales ratio.

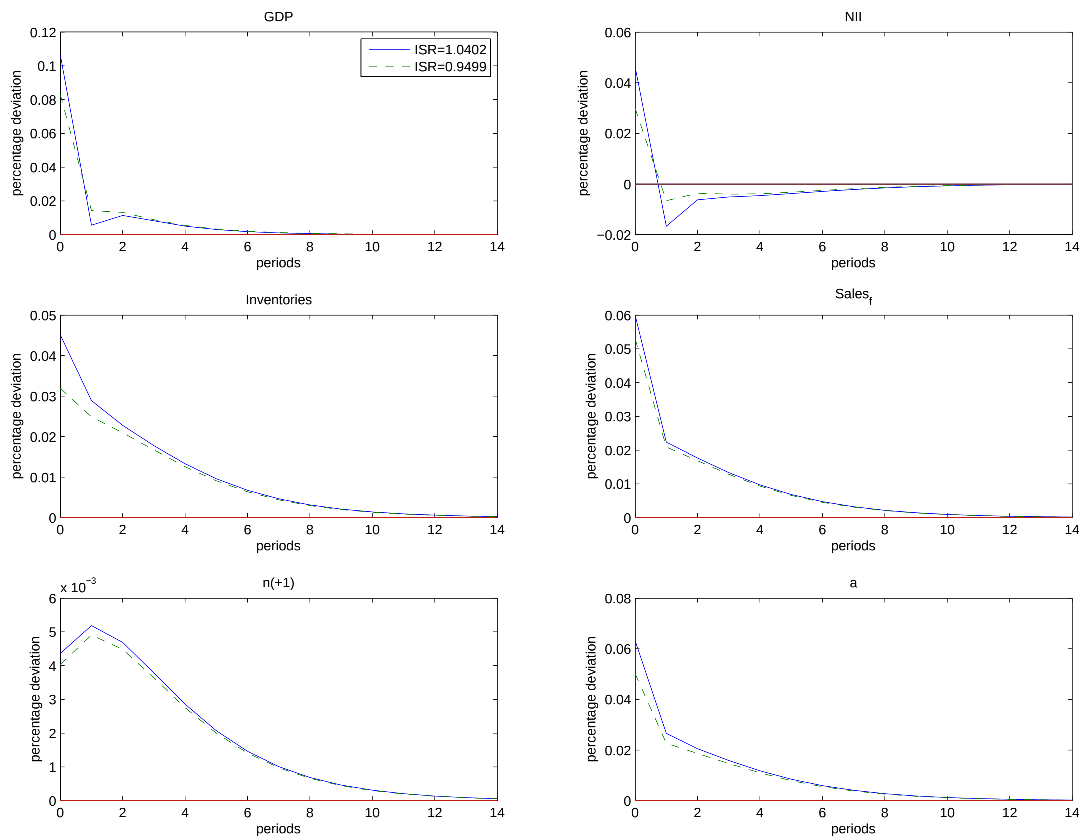

Figure 12: Role of Inventories: Productivity Shock

Since inventories amplify aggregate fluctuations over business cycles, the policy implications are different from that in a standard search model. If a central bank decreases the money growth rate permanently, the long run GDP, employment, final sales and input inventory investment decrease. Finished goods producers also hold lower level of inventories in the steady state. In the short run, a contractionary monetary policy leads to decreases in employment and GDP. Final sales also decrease, as a result of which it is optimal for households to cut input inventories. Therefore, the multi-stage production model supports the argument that the declining trend of inventory-to-sales ratio is one of the reasons for "Great Moderation". Moreover, this model also suggests that, besides the better inventory management technology, declines in the money growth rate would be one of the reasons for the decline of the inventory-to-sales ratio since the mid-1980.

Herrera and Pesavento (2005) argue that the better inventory management technology cannot account for most of the decline in volatility of output, because both the volatility of sales and inventories decreased since mid-1980s and most of the decline is due to the decline in that of input inventories.[28] Consistent with Stock and Watson (2002), my model suggests that besides the better inventory management technology, reduction in volatility of money growth shocks may be another promising reason for explaining the “Great Moderation”.

5.6 Sensitivity Analysis

In the preceding sections, we have seen that the search model with shocks to the money growth rate can reproduce stylized facts of inventories. The model also suggests that input inventories amplify aggregate fluctuations over business cycles. Since parameters $\epsilon_i, \epsilon_f, \alpha, \eta, B_i$ and markup are hard calibrate from the data, I assumed their values for the best fit to data in the benchmark model. In this section, I will examine the sensitivity of the quantitative results to different values of these parameters. Each parameter will be analyzed separately by holding other parameters unchanged, and the model is recalibrated to the data for each analysis. The results of sensitivity analysis are reported in Table 13 -18.

Table 13 shows that the correlations predicted by this model are sensitive to

[28]Herrera, Murtazashvili, and Pesavento (2008) show that the cross-section correlation among manufacturing inventories and sales increased since the ”Great Modaration”.

changes in the value of ϵ_i, except for the correlation between final sales and GDP. Other correlations match the data more effectively with low value of ϵ_i. In particular, if ϵ_i were large, (for example, ten times larger than the benchmark value,) the correlation between inventory-to-sales ratio and GDP (or FS) would turn positive which is not consistent with data. Moreover, the model predicts that the inventory investment is much volatile relative to GDP and the inventory-to-sales ratio is so much less volatile (relative to GDP) compared to the data.

The intuitions of these results are as follows. A higher value of ϵ_i implies that intermediate goods buyers are more responsive to both shocks, which is demonstrated by larger standard deviations of aggregate variables. Intermediate goods buyers search more intensively, while the effect on finished goods buyers' search intensities is limited because ϵ_f is unchanged. Therefore, the increase in intermediate goods is larger than the increase in final sales, so households accumulate more inventories, which drives the inventory-to-sales ratio up and generates counterfactual results.

Table 14 shows that the model matches the data effectively with low ϵ_f. A higher value of ϵ_f implies that finished goods producers are more responsive to both shocks. Since finished goods buyers search more intensively in this case, q^f responds at a lower magnitude to the shocks and GDP and final sales become less volatile. Despite the decreasing volatilities of GDP and final sales, the inventory investment responds more strongly to the shocks. Therefore, the relative standard deviation of the inventory-to-sales ratio (relative to GDP) increases with ϵ_f, and the correlation between inventory-to-sales ratio and GDP (or FS) are underestimated.

An interesting result is that the value of ϵ_f should be relatively lower than the value of ϵ_i in order to match the data. This result implies that intermediate goods buyers are more responsive than finished goods buyers. This is consistent with the empirical facts that most downstream firms sign long term contracts with upstream firms instead of searching for suppliers every period.

Table 15 shows that the predicted correlations are relatively stable in response to a wide range of ξ. The model tends to overestimate the correlations with a low value of ξ, except for the correlation between final sales and the inventory investment, which is underestimated. Moreover, the relative standard deviation of inventory investment is overshot by the model with a large value of ξ, and the relative standard deviation of the inventory-to-sales ratio is overshot with a low value of ξ. Intuitively speaking, as ξ decreases, the matching rates decrease for both sellers and buyers in both goods markets. Because of input inventory the negative effects on final sales is stronger than those on intermediate goods sales; as a result, inventories become too volatile to match the data.

Table 16 shows that the quantitative results are sensitive to changes in the value of relative risk aversion. The model matches the data well with low η. If η were high, (for example, greater than one), both the correlation between inventory-to-sales ratio and GDP (or final sales) and the relative standard deviations would be mismatched. For the cyclical behavior, the model becomes more volatile to shocks with higher η. These results are due to the fact that the motivation for smoothing consumption is strong with high η, so final sales respond to the shocks at a lower magnitude. Thus, the relative standard deviation of final sales is much lower than the value observed in the data. Moreover, in order to smooth consumption, households use more material inputs during the transition and hold more inventories at the end of each period. As a result, the response of inventories is too volatile such that the inventory-to-sales ratio is positively correlated with GDP (or final sales) and the relative standard deviation of inventory investment is overestimated.

Similar to the findings of Wang and Shi (2006), the inventory regularities are insensitive to changes in B_i (see Table 17), the intermediate goods market's buyer/seller ratio. The intuition is the following. Increased market tightness on one hand generates negative externalities, hence has a negative effect, on buyers' matching probabilities and, on the other hand, it also generates positive externalities, hence has a positive effect, on sellers' matching probabilities. Since positive effects cancel out negative effects, the overall results are insensitive to market tightness.

Table 18 shows that the predicted results are sensitive to the changes in markups, and the model would fit the data more effectively with high markups. If the markup is low, households can consume more in the long run and final sales would respond more strongly to the shocks. More final sales require more material inputs, as a result of which inventories become too volatile and the inventory-to-sales ratio is positively correlated with GDP (or final sales). Moreover, since the effects on GDP are bigger than the effects on inventories, the relative standard deviation of the inventory-to-sales ratio stays within an acceptable region. But, the inventory investment is much more volatile than the inventories because it is a flow concept, thus the relative standard deviation of inventory investment is overestimated by the model.

Table 13: Sensitivity analysis: ϵ_i

ϵ_i	0.1	0.4*	1	4	8	Data
corr(GDP, IS)	-0.745	-0.798	-0.413	0.631	0.733	-0.756
	(0.006)	(0.011)	(0.028)	(0.011)	(0.009)	
corr(GDP, NII)	0.507	0.882	0.917	0.937	0.940	0.789
	(0.020)	(0.008)	(0.006)	(0.004)	(0.004)	
corr(FS, IS)	-0.858	-0.665	-0.157	0.821	0.891	-0.756
	(0.006)	(0.018)	(0.038)	(0.010)	(0.007)	
corr(FS, GDP)	0.940	0.957	0.908	0.872	0.866	0.987
	(0.004)	(0.001)	(0.002)	(0.003)	(0.003)	
corr(FS, NII)	0.184	0.708	0.667	0.655	0.654	0.680
	(0.031)	(0.014)	(0.014)	(0.014)	(0.013)	
$\sigma(FS)/\sigma(GDP)$	0.877	0.666	0.532	0.444	0.427	0.837
	(0.007)	(0.008)	(0.009)	(0.008)	(0.007)	
$\sigma(NII)/\sigma(GDP)$	0.346	0.411	0.566	0.663	0.681	0.220
	(0.010)	(0.005)	(0.005)	(0.005)	(0.006)	
$\sigma(IS)/\sigma(GDP)$	2.321	0.794	0.300	0.265	0.294	0.843
	(0.049)	(0.011)	(0.008)	(0.008)	(0.007)	
$\sigma(GDP)$	0.043	0.096	0.161	0.247	0.270	-
	(0.002)	(0.002)	(0.004)	(0.006)	(0.007)	
$\sigma(NII)$	0.015	0.039	0.091	0.164	0.184	-
	(0.000)	(0.001)	(0.002)	(0.005)	(0.005)	
$\sigma(INV)$	0.005	0.013	0.023	0.035	0.039	-
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(FS)$	0.038	0.064	0.086	0.110	0.116	-
	(0.001)	(0.002)	(0.002)	(0.003)	(0.003)	

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

Table 14: Sensitivity analysis: ϵ_f

ϵ_f	0.01*	0.03	0.1	1	2	Data
corr(GDP, IS)	-0.798	-0.749	-0.576	-0.304	-0.280	-0.756
	(0.011)	(0.014)	(0.020)	(0.021)	(0.020)	
corr(GDP, NII)	0.882	0.889	0.888	0.939	0.943	0.789
	(0.008)	(0.006)	(0.006)	(0.003)	(0.003)	
corr(FS, IS)	-0.666	-0.716	-0.591	-0.352	-0.331	-0.756
	(0.018)	(0.020)	(0.027)	(0.030)	(0.028)	
corr(FS, GDP)	0.957	0.978	0.966	0.950	0.949	0.987
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	
corr(FS, NII)	0.709	0.774	0.739	0.784	0.790	0.680
	(0.014)	(0.012)	(0.013)	(0.010)	(0.010)	
$\sigma(FS)/\sigma(GDP)$	0.666	0.722	0.683	0.556	0.542	0.837
	(0.008)	(0.005)	(0.005)	(0.005)	(0.005)	
$\sigma(NII)/\sigma(GDP)$	0.412	0.331	0.383	0.503	0.516	0.220
	(0.005)	(0.004)	(0.004)	(0.005)	(0.005)	
$\sigma(IS)/\sigma(GDP)$	0.794	1.454	1.796	2.128	2.174	0.843
	(0.011)	(0.019)	(0.020)	(0.049)	(0.052)	
$\sigma(GDP)$	0.096	0.096	0.087	0.074	0.073	-
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	
$\sigma(NII)$	0.039	0.032	0.033	0.037	0.038	-
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(INV)$	0.013	0.013	0.014	0.016	0.016	-
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(FS)$	0.064	0.069	0.059	0.041	0.040	-
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

Table 15: Sensitivity analysis: ξ

ξ	0.2	0.4	0.6	0.8*	0.9	Data
corr(GDP, IS)	-0.901	-0.874	-0.833	-0.798	-0.780	-0.756
	(0.005)	(0.004)	(0.008)	(0.011)	(0.013)	
corr(GDP, NII)	0.663	0.809	0.855	0.882	0.892	0.789
	(0.015)	(0.011)	(0.010)	(0.008)	(0.007)	
corr(FS, IS)	-0.912	-0.804	-0.722	-0.665	-0.639	-0.756
	(0.002)	(0.006)	(0.012)	(0.018)	(0.020)	
corr(FS, GDP)	0.981	0.975	0.965	0.957	0.953	0.987
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
corr(FS, NII)	0.504	0.659	0.690	0.708	0.714	0.680
	(0.018)	(0.016)	(0.015)	(0.014)	(0.013)	
$\sigma(FS)/\sigma(GDP)$	0.867	0.782	0.716	0.666	0.645	0.837
	(0.006)	(0.007)	(0.008)	(0.008)	(0.008)	
$\sigma(NII)/\sigma(GDP)$	0.226	0.294	0.362	0.411	0.432	0.220
	(0.006)	(0.005)	(0.005)	(0.005)	(0.004)	
$\sigma(IS)/\sigma(GDP)$	1.731	1.148	0.927	0.794	0.741	0.843
	(0.020)	(0.018)	(0.013)	(0.011)	(0.010)	
$\sigma(GDP)$	0.035	0.054	0.075	0.096	0.107	-
	(0.001)	(0.002)	(0.002)	(0.002)	(0.003)	
$\sigma(NII)$	0.008	0.016	0.027	0.039	0.046	-
	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	
$\sigma(INV)$	0.004	0.007	0.010	0.013	0.015	-
	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	
$\sigma(FS)$	0.030	0.042	0.053	0.064	0.069	-
	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

Table 16: Sensitivity analysis: η

η	0.2	0.4	0.8*	2	3	Data
corr(GDP, IS)	-0.857	-0.855	-0.798	0.520	0.685	-0.756
	(0.005)	(0.006)	(0.011)	(0.009)	(0.007)	
corr(GDP, NII)	0.825	0.845	0.882	0.954	0.974	0.789
	(0.012)	(0.011)	(0.008)	(0.003)	(0.002)	
corr(FS, IS)	-0.785	-0.770	-0.666	0.798	0.943	-0.756
	(0.009)	(0.010)	(0.018)	(0.010)	(0.004)	
corr(FS, GDP)	0.978	0.974	0.957	0.871	0.826	0.987
	(0.001)	(0.001)	(0.001)	(0.004)	(0.006)	
corr(FS, NII)	0.691	0.703	0.709	0.686	0.682	0.680
	(0.017)	(0.017)	(0.014)	(0.013)	(0.013)	
$\sigma(FS)/\sigma(GDP)$	0.781	0.752	0.666	0.408	0.299	0.837
	(0.007)	(0.007)	(0.008)	(0.007)	(0.005)	
$\sigma(NII)/\sigma(GDP)$	0.286	0.317	0.412	0.678	0.776	0.220
	(0.004)	(0.004)	(0.005)	(0.004)	(0.003)	
$\sigma(IS)/\sigma(GDP)$	1.210	1.102	0.794	0.606	0.965	0.843
	(0.022)	(0.018)	(0.011)	(0.019)	(0.023)	
$\sigma(GDP)$	0.082	0.087	0.096	0.111	0.117	-
	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	
$\sigma(NII)$	0.023	0.027	0.039	0.075	0.091	-
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	
$\sigma(INV)$	0.011	0.012	0.013	0.018	0.021	-
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(FS)$	0.064	0.065	0.064	0.045	0.035	-
	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

Table 17: Sensitivity analysis: B_i

B_i	0.1	0.2*	0.4	0.6	0.8	Data
corr(GDP, IS)	-0.797	-0.798	-0.798	-0.798	-0.798	-0.756
	(0.011)	(0.011)	(0.012)	(0.011)	(0.011)	
corr(GDP, NII)	0.882	0.882	0.882	0.882	0.882	0.789
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)	
corr(FS, IS)	-0.664	-0.665	-0.665	-0.665	-0.665	-0.756
	(0.018)	(0.018)	(0.019)	(0.018)	(0.018)	
corr(FS, GDP)	0.957	0.957	0.957	0.957	0.957	0.987
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
corr(FS, NII)	0.708	0.708	0.709	0.708	0.709	0.680
	(0.014)	(0.014)	(0.015)	(0.015)	(0.014)	
$\sigma(FS)/\sigma(GDP)$	0.666	0.666	0.665	0.666	0.666	0.837
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)	
$\sigma(NII)/\sigma(GDP)$	0.412	0.411	0.411	0.411	0.411	0.220
	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	
$\sigma(IS)/\sigma(GDP)$	0.794	0.794	0.794	0.794	0.794	0.843
	(0.011)	(0.011)	(0.010)	(0.011)	(0.011)	
$\sigma(GDP)$	0.096	0.096	0.096	0.096	0.096	-
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	
$\sigma(NII)$	0.040	0.039	0.040	0.040	0.040	-
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(INV)$	0.013	0.013	0.013	0.013	0.013	-
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(FS)$	0.064	0.064	0.064	0.064	0.064	-
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

Table 18: Sensitivity analysis: *markup*

markup	0.2	0.4	0.5	0.6	0.7*	Data
corr(GDP, IS)	0.7692	-0.183	-0.648	-0.766	-0.798	-0.756
	(0.005)	(0.029)	(0.021)	(0.015)	(0.011)	
corr(GDP, NII)	0.931	0.921	0.905	0.892	0.882	0.789
	(0.005)	(0.005)	(0.006)	(0.007)	(0.008)	
corr(FS, IS)	0.981	0.171	-0.403	-0.591	-0.666	-0.756
	(0.001)	(0.035)	(0.031)	(0.023)	(0.018)	
corr(FS, GDP)	0.855	0.902	0.924	0.943	0.957	0.987
	(0.005)	(0.002)	(0.002)	(0.001)	(0.001)	
corr(FS, NII)	0.664	0.665	0.675	0.691	0.709	0.680
	(0.014)	(0.014)	(0.014)	(0.015)	(0.014)	
$\sigma(FS)/\sigma(GDP)$	0.365	0.516	0.575	0.624	0.666	0.837
	(0.006)	(0.008)	(0.008)	(0.009)	(0.008)	
$\sigma(NII)/\sigma(GDP)$	0.797	0.584	0.519	0.462	0.412	0.220
	(0.014)	(0.004)	(0.005)	(0.005)	(0.005)	
$\sigma(IS)/\sigma(GDP)$	0.527	0.262	0.402	0.596	0.794	0.843
	(0.012)	(0.008)	(0.006)	(0.008)	(0.011)	
$\sigma(GDP)$	0.549	0.174	0.134	0.111	0.096	-
	(0.014)	(0.004)	(0.003)	(0.003)	(0.002)	
$\sigma(NII)$	0.437	0.102	0.070	0.051	0.039	-
	(0.016)	(0.003)	(0.002)	(0.001)	(0.001)	
$\sigma(INV)$	0.058	0.020	0.016	0.014	0.013	-
	(0.002)	(0.006)	(0.001)	(0.001)	(0.001)	
$\sigma(FS)$	0.200	0.090	0.077	0.069	0.064	-
	(0.005)	(0.002)	(0.001)	(0.002)	(0.002)	

* Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

Chapter 6

Conclusion

In this paper, I propose a multi-stage production model with input inventories to investigate the monetary transmission mechanism. I find that the multi-stage production influences both the long run and short run effects of money growth. In particular, money growth has non-monotonic real effects on the quantity of finished goods per match and input inventory investment, because multi-stage production provides an additional channel for money growth to have effects. I also highlight the difference between the response of input inventories and that of output inventories to shocks.

Since researchers do not distinguish input from output inventories when calculating GDP, it is worthwhile to explore the behavior of inventories more generally by modeling both input and output inventories in a multi-stage production model. Contrary to the response of input inventory investment, output inventory investment responds monotonically in the long run to the money growth rate, because output inventories serve as buffer stocks and decrease whenever sales increase. By putting these two types of inventories in one model, the contribution of input inventories may be partially offset by movements in output inventories. Quantitatively speaking, it would be interesting to investigate further to what extent general inventories can then still affect aggregate fluctuations in the type of large household model with search that I am using.

Besides my theoretical work, I have calibrated my model to quarterly US data in a slightly richer environment. Unlike other work, my model is able to replicate the stylized facts on inventory movements over the business cycle by relying solely on monetary shocks. Multi-stage production is essential for matching the data, because it induces a positive response in the quantity that is produced per match when looking at monetary shocks. Furthermore, this positive response is strong enough to push input inventories to move in the same direction as final sales during the transition, which is the key to replicate the stylized facts. My quantitative analysis also provides support for the argument that input inventories amplify aggregate fluctuations over business cycles.

It would be interesting to calibrate my model to other countries' data, especially China. Because I assume producers produce only if they are matched, this characteristic of my model resembles the organization of original equipment manufacturers which produce only after they have received orders. This is a major industry in export driven economies, such as China and other countries of the Asia-Pacific region. Inventory behavior and its importance for business cycles may very well be different from the US experience.

Finally, I conduct a sensitivity analysis of some parameters relative to the baseline calibration. In order to match the data, the model requires that finished goods buyers are less responsive to the shock in comparison to intermediate goods buyers in order to keep the relative standard deviation of the inventory-to-sales ratio (relative to GDP) within a reasonable range. Moreover, my results are sensitive to the degree of relative risk aversion and the markup that is charged in intermediate goods transactions. The model matches the data better with a low value of relative risk aversion and a high value for the markup.

To conclude, my thesis sheds light on the importance of multi-stage production for understanding the monetary transmission mechanism. It also provides important clues for understanding the different behaviors of input and output inventories over the cycle. In the future, I plan to explore further the effects of technology shocks on different stages of the production process. First this might help us to understand better how technology shocks work their way through the economy. I would expect a very different response of employment and other variables, if the productivity shock hits the intermediate goods sector rather than the final goods sector. This could lead to a better understanding of linkages across industries and the persistence of structural unemployment. The multi-stage production model as developed here could be employed also to study the effect of technology shocks on international trade, where trade is mainly in intermediate goods so that global supply chains are at the center of international spillovers of shocks.

Finally, it is also interesting to study optimal monetary policy in this environment, in particular, inflation targeting. Central banks usually target CPI, while I argue that PPI is important because of the cyclical behavior of input inventories. For example, central banks can smooth fluctuations by reducing the money growth rate during a boom. But as suggested by my theory, the economy could contract deeper than expected if monetary policy makers were to target exclusively CPI.

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Appendix A

Data Sources

1. Underlying Detail - NIPA Tables, The Bureau of Economic Analysis

- Table 1AU. Real Manufacturing and Trade Inventories, Seasonally Adjusted, End of Period [Chained 1996 dollars, 1967-96, SIC] (Q)

- Table 1AU2. Real Manufacturing and Trade Inventories, Seasonally Adjusted, End of Period [Chained 2005 dollars, 1967-97, SIC] (Q)

- Table 1BU. Real Manufacturing and Trade Inventories, Seasonally Adjusted, End of Period [Chained 2005 dollars, 1997 forward, NAICS] (Q)

- Table 2AU. Real Manufacturing and Trade Sales, Seasonally Adjusted at Monthly Rate [Chained 1996 dollars, 1967-96, SIC] (Q)

- Table 2AUI. Implicit Price Deflators for Manufacturing and Trade Sales [Index base 1996, 1967-96, SIC] (Q)

- Table 2BU. Real Manufacturing and Trade Sales, Seasonally Adjusted at Monthly Rate [Chained 2005 dollars, 1997 forward, NAICS] (Q)
- Table 2BUI. Implicit Price Deflators for Manufacturing and Trade Sales [Index base 2005, 1997 forward, NAICS] (Q)
- Table 4AU1. Real Manufacturing Inventories, by Stage of Fabrication (Materials and supplies), Seasonally Adjusted, End of Period [Chained 2005 dollars, 1967-97, SIC] (Q)
- Table 4AU2. Real Manufacturing Inventories, by Stage of Fabrication, Seasonally Adjusted (Work-in-process), End of Period [Chained 2005 dollars, 1967-97, SIC] (Q)
- Table 4BU1. Real Manufacturing Inventories, by Stage of Fabrication (Materials and supplies), Seasonally Adjusted, End of Period [Chained 2005 dollars, 1997 forward, NAICS] (Q)
- Table 4BU2. Real Manufacturing Inventories, by Stage of Fabrication (Work-in-process), Seasonally Adjusted, End of Period [Chained 2005 dollars, 1997 forward, NAICS] (Q)

2. Databases, the Federal Reserve Bank of St. Louis

- M2 Money Stock, seasonally adjusted, end of period, quarterly
- Velocity of M2 Money Stock, seasonally adjusted, end of period, quarterly

3. Databases, Bureau of Labor Statistics

- Civilian Labor Force (Seasonally Adjusted) - LNS11000000
- Civilian Employment (Seasonally Adjusted) - LNS12000000
- Civilian Unemployment (Seasonally Adjusted) - LNS13000000
- Manufacturing Employment - CES3000000001

3. Manufacturing Industry Productivity Database, The National Bureau of Economic Research

- emp: Total employment in 1000s, 1987 SIC version
- matcost: Total cost of materials in $1,000,000, 1987 SIC version
- pimat: Deflator for MATCOST 1987=1.000, 1987 SIC version

Appendix B

Proof of Existence of Steady States

In this section, I prove that the model economy exists at least one steady state, which satisfies $(\lambda^f; \Omega_a) > 0$. Denote the steady state values with an asterisk, which can be rewritten by the dynamic system (4.4.1):

$$
\begin{aligned}
z^f(s^{f*})^\xi &= \frac{\gamma-\beta}{\beta}\frac{\omega^{f\star}}{U'(c^*)-w^{f\star}}, &(83)\\
\Omega_i^* &= \beta\varphi'(q^{i*}), &(84)\\
(1-\beta(1-\delta_n))k(v^*) &= \beta\{\sigma z^f B^f(s^{f*})^\xi\omega^{f*}+[(1-z^f B^f(s^{f*})^\xi)\sigma(1-\delta_i)\beta\\
&- \sigma]\varphi'(q^{i*})-\sigma\varphi^f\}, &(85)\\
\Phi^{f\prime}(s^{f*}) &= z^f(s^{f*})^{\xi-1}[U'(c^*)-\omega^{f*}]q^{f*}, &(86)\\
v^*\mu(v^*) &= \delta_n q^{f*}, &(87)\\
q^{i*} &= \{1-(1-\delta_i)[1-z^f B^f(s^{f*})^\xi]\}q^{f*}, &(88)\\
c^* &= a_p^f B^f z^f(s^{f*})^\xi q^{f*}. &(89)
\end{aligned}
$$

As demonstrated in Chapter 4, the steady state system can be reduced to two equations which are repeated here for future use:

$$z^f[s^f(\omega^{f*}, q^{f*})]^\xi = \frac{\gamma - \beta}{\beta} \frac{\omega^{f*}}{U'(c(\omega^{f*}, q^{f*})) - w^{f*}}, \tag{90}$$

$$\begin{aligned}(1 - \beta(1 - \delta_n))k(v(q^{f*})) + \beta\sigma\varphi^f &= \beta\{\sigma z^f B^f(s^f(\omega^{f*}, q^{f*}))^\xi \omega^{f*} \\ &+ [(1 - z^f B^f(s^f(\omega^{f*}, q^{f*}))^\xi)\sigma(1 - \delta_i)\beta - \sigma]\varphi'(q^i(\omega^{f*}, q^{f*}))\}. \end{aligned} \tag{91}$$

Similarly to Shi (1998), Equation(90) and (91) give a relationship between ω^f and q^f, denote $q^{f*} = q^f1(\omega^{f*})$ and $q^{f*} = q^f2(\omega^{f*})$. The steady state value ω^{f*} is a solution to $Q^f1(\omega^{f*}) = Q^f2(\omega^{f*})$. To ensure $\lambda^f > 0$, the solution must satisfy $U'(c^*) \geq \omega^{f*} + \Delta$, where $\Delta > 0$ is an arbitrarily small number. That is, we require $q^{f*} \leq q^f(\omega^{f*}, \Delta)$,[29] where $q^f(\omega^f, \Delta)$ is defined by:

$$U'(c(\omega_f, q^f(\omega^f, \Delta))) = \omega^f + \Delta. \tag{92}$$

Using Lemma 3.2 as explained in Shi (1998), I can prove that the function $Q^f(\omega^f, \lambda)$ is well defined and has the following properties for sufficiently small $\Delta > 0$: $Q^f_{\omega^f}(\omega^f, \Delta) < 0, Q^f(\infty, \Delta) = 0$, and $\lim_{\Delta\to 0} Q^f(0, \Delta) = \infty$. The function $q^f1(\omega^f)$ satisfies $q^f1'(\omega^f) < 0, q^f1(0) = \infty$ and $q^f1(\infty) = 0$. Furthermore, the two curves $q^f1(\omega^f)$ and $Q^f(\omega^f, \Delta)$ have a unique intersection at a level denoted $\omega^f_1(\Delta)$ which satisfies $\lim_{\Delta\to 0} \omega^f1(\Delta) = 0$.[30]

In order to prove the uniqueness, we also need to know the properties of q^f2.

[29] First, positive nominal interest rate implies $\gamma > \beta$ and is enough to ensure $\lambda^f > 0$. If $q^{f*} > Q^f(\omega^{f*}, \Delta)$, $U''(c) < 0$ implies $U'(c(\omega^{f*}, q^{f*})) < U'(c(\omega^{f*}, Q^f(\omega^{f*}, \Delta)))$, which violates $U'(c^*) \geq \omega^{f*} + \Delta$.

[30] Since equation (90) is identical to the steady state equation (3.4) in Shi (1998), I omit the proof here.

Although the properties of q^f2 are the same as what is described in Lemma 3.3 in Shi (1998), the proof is not the same due to different function forms.

We are going to prove that q^f2 has the following properties: $q^f2(0) = 0, q^f2(\infty) = 0$, and $q^f2'(\omega^f) < 0$ for sufficiently large ω^f. The two curves $q^f2(\omega^f)$ and $Q(\omega^f, \Delta)$ have a unique intersection at a level denoted $\omega^f2(\Delta)$ which approaches infinity when Δ approaches zero.

First, let us show $q^f2(0) = 0$ by rearranging equation (91):

$$\begin{aligned}
\varphi'(q^i(\omega^{f*}, q^{f*})) &= \frac{\beta[\sigma z^f B^f (s^f(\omega^{f*}, q^{f*}))^\xi \omega^{f*} - \sigma\varphi^f] - (1-\beta(1-\delta_n))k(v(q^{f*}))}{\beta[1-(1-z^f B^f(s^f(\omega^{f*}, q^{f*}))^\xi)\sigma(1-\delta_i)\beta]\sigma} \\
&\leq \frac{\sigma z^f B^f (s^f(\omega^{f*}, q^{f*}))^\xi \omega^{f*}}{1-(1-z^f B^f(s^f(\omega^{f*}, q^{f*}))^\xi)\sigma(1-\delta_i)\beta]\sigma} \\
&\leq \frac{\sigma z^f B^f \omega^{f*}}{\{[1-\sigma(1-\delta_i)\beta]/(s^f(\omega^{f*}, q^{f*}))^\xi + z^f B^f \sigma(1-\delta_i)\beta\}\sigma} \qquad (93)
\end{aligned}$$

The right-hand side of (93) approaches zero as ω^f approaches zero, becuase $\lim_{\omega^f \to 0}$ numerator $= 0$ and $\lim_{\omega^f \to 0}$ denominator $= z^f B^f \sigma(1-\delta_i)\beta$. Since $\varphi'(\cdot) > 0$, equation (93) implies $\lim_{\omega^f \to 0} q^i(\omega^{f*}, q^{f*}) = 0$. Finally, $\lim_{\omega^f \to 0} q^f2(\omega) = 0$ is implied by the steady state equation $q^{i*} = \{1-(1-\delta_i)[1-z^f B^f(s^{f*})^\xi]\}q^{f*}$.

Second, let us prove that the two curves $q^f2(\omega^f)$ and $Q(\omega^f, \Delta)$ have a unique intersection. By plugging the equation of c^* into the fourth equation of the steady state system, we can get a useful equation:

$$\Phi^{f'}(s^{f*})s^{f*} = [U'(c^*) - \omega^{f*}]\frac{c^*}{a_p^f B^f}. \qquad (94)$$

As the definition of $Q^f(\omega^f, \Delta)$, set $\omega^f = u'(c) - \Delta$. Then equation (94) implies that s^f is a function of (c, Δ): $\Phi^{f'}(s^{f*})s^{f*} = \Delta c^*/(a_p^f B^f)$. Denote the solution for s^f as $s^f(c, \Delta)$. Because $\Phi^{f'}(0) = 0$, $\Phi^{f'}(\cdot) > 0$ and $\Phi^{f''}(\cdot) > 0$, we can get $s^f(c, 0) = 0$, $s^f(\infty, \Delta) = \infty$, $s^f(0, \Delta) = 0$ and $s_c^f(c, \Delta) > 0$. By rearranging equation (94), I can prove that $c/(s(c, \Delta))^\xi$ is an increasing function of c.

Rearranging the steady state equation of c^*, it is easy to see that $q^f(c, \Delta)$ is also an increasing function of c: $q^f(c, \Delta) = c/a_p^f B^f z^f [s^f(c, \Delta)]^\xi$. Similarly, q^i can be rewritten as a function of (c, Δ): $q^i(c, \Delta) = \{1 - (1 - \delta_i)[1 - z^f B^f (s^f(c, \Delta))^\xi]\} q^f(c, \Delta)$. Since both $s^f(c, \Delta)$ and $q^f(c, \Delta)$ are increasing in c, $q^i(c, \Delta)$ is an increasing function of c.

Now we are ready to prove that the two curves $q^f 2(\omega^f)$ and $Q(\omega^f, \Delta)$ have a unique intersection. Rewrite equation (91) in terms of (c, Δ):

$$\text{LHS}(91) = (1 - \beta(1 - \delta_n))k(v(q^f(c, \Delta))) + \beta\sigma\varphi^f \tag{95}$$

$$\begin{aligned} \text{RHS}(91) &= \beta\{\sigma z^f B^f (s^f(c, \Delta))^\xi \omega^{f*} \\ &+ [(1 - z^f B^f (s^f(c, \Delta))^\xi)\sigma(1 - \delta_i)\beta - \sigma]\varphi'(q^i(c, \Delta))\} \end{aligned} \tag{96}$$

The left-hand side of (91) is an increasing function of c, and the right-hand side of (91) is a decreasing function of c, because $s^f(c, \Delta)$, $q^f(c, \Delta)$ and $q^i(c, \Delta)$ are increasing in c, $k'(v) > 0$ and $\varphi'(q^i) > 0$. Moreover, since $q^f(0, \Delta) = 0$, $k(v(q^f(0, \Delta))) = 0$, $q^f(\infty, \Delta) = \infty$ and $k(v(q^f(\infty, \Delta))) = \infty$ it is easy to see that $\lim_{c\to 0} \text{LHS}(91) = 0$ and $\lim_{c\to\infty} \text{LHS}(91) = \infty$. Similarly, the right-hand side has the following properties. $\lim_{c\to 0} \text{RHS}(91) = \infty$, because $q^i(0, \Delta) = 0$, $s^f(0, \Delta) = 0$ and $\lim_{c\to 0} cu'(c) = \infty$. And $\lim_{c\to\infty} \text{RHS}(91) = -\infty$. because $q^i(\infty, \Delta) = \infty$, $s^f(\infty, \Delta) = \infty$ and $\lim_{c\to\infty} cu'(c) =$

0.

Given these properties of (91), there is a unique solution for c to (91). Denote this solution by $c(\Delta)$, then $\omega^f 2(\Delta) = u'(c(\Delta)) - \Delta$ is unique. Thus there must be a unique intersection between the two curves $q^f 2(\omega^f)$ and $Q(\omega^f, \Delta)$.

Third, we are going to prove that $\lim_{\Delta \to 0} \omega^f 2(\Delta) = 0$, $q^f 2'(\omega^f) < 0$ and $q^f 2(\infty) = 0$. For fixed c, $\lim_{\Delta \to 0} \mathrm{LHS}(91) = \infty$ and $\lim_{\Delta \to 0} \mathrm{RHS}(91) = -\infty$, because $\lim_{\Delta \to 0} s(c, \Delta) = 0$ and $\lim_{\Delta \to 0} q^f(c, \Delta) = \infty$. Thus (91) is satisfied only when $\lim_{\Delta \to 0} c(\Delta) = 0$ and $\lim_{\Delta \to 0} \omega^f 2(\Delta) = \infty$. Next, $q^f 2'(\omega^f) < 0$ since $q^f 2(c, \Delta)$ is an increasing function of c and $\omega^f 2'(c(\Delta)) < 0$. This can be proved by plugging $\omega^f 2(c(\Delta))$ into $q^f 2(\omega^f)$ and analyzing $q^f 2(\omega^f 2(c(\Delta)))$.

Now, we are going to prove $q^f 2(\infty) = 0$. Because $Q^f(0, \Delta)$ is a positive constant and $q^f 2(0) = 0$ is proven, $q^f 2(0) < Q^f(0, \Delta)$ and the curve $q^f 2(w^f)$ must cross the curve $Q^f(\omega^f, \Delta)$ from below if the two have a unique intersection. Moreover, because $Q^f(\infty, \Delta) = 0$ is proven in Lemma 1 and $0 \leq q^f 2(\omega^f) < Q^f(\omega^f, \Delta)$ for $\omega^f < \omega^f 2(\Delta)$, $0 \leq q^f 2(\infty) < 0$ for $\omega^f < \omega^f 2(\Delta)$. Then $q^f 2(\infty) = 0$ since $q^f 2(\omega^f)$ is continuous and only has one intersection with $Q^f(\omega^f, \Delta)$.

Finally, given the properties of equations (90) and (91) proven, there exists at least one steady state for the model.

Appendix C

Proof of Propositions

C.1. *Proof of proposition* 1

Now we are going to prove that the long run effect of money growth on q_f is not monotonic. Equation (91) is independent of γ, while equation (90) will be shifted to the right as $\gamma \to \beta$ and to the left as $\gamma \to \infty$. Since $q^f 2(0) = 0$, $q^f 2(\infty) = 0$ and $q^f 2'(\omega^f) < 0$ for sufficiently large ω^f, equation (91) is hump-shaped. Thus steady state q^f decreases with γ if γ is high, but increases with γ if it is low.

C.2. *Proof of proposition* 2

Since the difference between steady state net inventory investment and steady state inventory level is just a constant multiplier $a_p^f \delta_i$, I only prove the long run response of inventory investment. Equation (39) implies that the steady state net inventory investment is the difference between the quantity of goods per match and the final sales discounted at a proper rate, namely,

$$NII^* = a_p^f \delta_i i = (1-\delta_i)\delta_i a_p^f q^f [1 - z^f B^f s^{f^{\alpha}}]. \tag{97}$$

The derivative of i with respect to q^f can be derived from this equation:

$$\begin{aligned} \frac{di}{dq^f} &= (1-\delta_i)[1 - z^f B^f s^{f^{\alpha}}] - (1-\delta_i) q^f \alpha z^f B^f (s^f)^{\alpha-1} \frac{ds^f}{dq^f} \\ &> 0. \end{aligned} \tag{98}$$

$di/dq^f > 0$ because $ds^f/dq^f < 0$. Since i is a function of q^f and ω^f, the effects of money growth on input inventory investment can be studied by taking the derivative of $a_p^f \delta_i i(q^f(\omega^f))$ with respect to γ:

$$\begin{aligned} a_p^f \delta_i \frac{di(q^f(\omega^f))}{d\gamma} &= \frac{di}{dq^f}\frac{dq^f}{d\omega^f}\frac{d\omega^f}{d\gamma} \\ &> 0, \quad \text{if } \gamma \text{ if low;} \\ &< 0, \quad \text{if } \gamma \text{ if high.} \end{aligned} \tag{99}$$

We can conclude that $dNII^*/d\gamma > 0$ if γ is low and $dNII^*/d\gamma < 0$ if γ is high, because $dq^f/d\omega^f < 0$ if γ is low, $dq^f/d\omega^f > 0$ if γ is high as claimed in proposition 1, $di/dq^f > 0$ and $d\omega^f/d\gamma < 0$.

C.3. *Proof of proposition* 3

By rearranging equation (39), we can get a expression for steady state inventory-to-sales ratio:

$$\begin{aligned} IS &= \frac{a_p^f i^*}{a_p^f B^f z^f (s^{f*})^\alpha q^{f*}} \\ &= (1-\delta_i)[\frac{a_p^f (q^{f*})}{a_p^f B^f z^f (s^{f*})^\alpha q^{f*}} - 1], \\ &= (1-\delta_i)[\frac{1}{B^f z^f (s^{f*})^\alpha} - 1] \end{aligned} \tag{100}$$

The effects of money growth on the inventory-to-sales ratio can be studied by taking the derivative with respect to γ:

$$\begin{aligned} \frac{dIS^*}{d\gamma} &= -(1-\delta_i)\frac{B^f z^f \alpha s^{f\,\alpha-1}}{[B^f z^f (s^{f*})^\alpha]^2}\frac{ds^f}{dq^f}\frac{dq^f}{d\omega^f}\frac{d\omega^f}{d\gamma}, \\ &> 0, \quad \text{if } \gamma \text{ if low;} \\ &< 0, \quad \text{if } \gamma \text{ if high.} \end{aligned} \tag{101}$$

evaluating at $\omega = \omega^*(\gamma)$.The inventory-to-sales ratio has a hump-shaped long run response to the money growth rate across steady states, because $dq^f/d\omega^f < 0$ if γ is low, $dq^f/d\omega^f > 0$ if γ is high as claimed in proposition 1, $d\omega^f/d\gamma < 0$ and $ds^f/dq^f < 0$.

Appendix D

Calibration Procedures

Now I am going to describe the steps to pinning down parameters $(u, a_p^i, a_p^f, a_b^i, a_b^f, z_1^i, z_1^f, b, \delta_i, K_0, \varphi^i, \varphi_0^i, \varphi^f, \varphi_0^f)$. The steady state equations can be derived from the dynamic system (ex. equations (65)-(75)):

$$
\begin{aligned}
v^*\mu(v^*) &= \delta_n n^*, && (102)\\
\delta_i a_p^f i^* &= (1-\delta_i)[z^i a_b^i (s^{i*})^\xi q^{i*} - z^f B^f (s^{f*})^\xi a_p^f a^*], && (103)\\
a^* &= i^* + z^i a_b^i (s^{i*})^\xi q^{i*}/a_p^f, && (104)\\
z^f (s^{f*})^\alpha &= \frac{\gamma-\beta}{\beta}\frac{\omega^{f\star}}{(1-FI)U'(c^*) - w^{f\star}}, && (105)\\
\Omega_i^* &= \frac{\beta z^f B^f (s^{f*})^\alpha a_p^f [\lambda^{i*} + \varphi'(q^{i*})]}{1-\beta(1-\delta_i)(1-z^f B^f (s^{f*})^\xi a_p^f)} && (106)\\
(1-\beta(1-\delta_n))k(v^*) &= \beta[(1-\alpha)\sigma z^f B^f (s^{f*})^\xi A^* (a^*)^\alpha (n^*)^{-\alpha}\omega^{f*} - \sigma\varphi^f], && (107)\\
\Phi^{i'}(s^{i*}) &= g_b^{i*}\{[g_s^{f*} a_p^f \lambda^{i*} - (1-g_s^{f*} a_p^f)[\varphi'(q^{i*}) \\
&- (1-\delta_i)\Omega_i^*]\}q^{i*}, && (108)\\
\Phi^{f'}(s^{f*}) &= z^f (s^{f*})^{\xi-1}[(1-FI)U'(c^*) - \omega^{f*}]q^{f*}, && (109)
\end{aligned}
$$

$$A^*\alpha(a^*)^{1-\alpha}(n^*)^{1-\alpha}\omega^{f*} = a_p^f[\varphi'(q^{i*})+\lambda^{i*}]+(1-a_p^f)(1-\delta_i)\Omega_i^*, \quad (110)$$

$$c^* = (1-FI)a_p^f B^f z^f (s^{f*})^\alpha q^{f*}. \quad (111)$$

The average labor participation rate ($LP = 0.6445$), the average unemployment rate ($UR = 0.061$) and the assumption $a_p^i = a_p^f$ can be used to pin down parameters (u, a_p^i, a_p^f):

$$u = LP \cdot UR = 0.0393. \quad (112)$$

Since the households have measure one, the labor participation rate equals $u + a_p^f(1+n)+a_p^i$. Using the assumption $a_p^i = a_p^f$, the number of sellers in the intermediate goods market a_p^i is equate to its counterparts in the finished goods market a_p^f, and the steady state vacancies can be calculated as the following:

$$a_p^f = (LP-u)/(2+n) = 0.2017, \quad (113)$$

$$a_p^i = LP-u-a_p^f(1+n) = 0.2017, \quad (114)$$

$$v^* = \left[\frac{\delta_n \cdot n}{\bar{\mu}(a_p^f/u)^{\phi-1}}\right]^{1/\phi}. \quad (115)$$

The depreciation rate of input inventory can be pinned down by matching the average inventory to output ratio and the average inventory investment to output ratio, which are $a_p^i i^*/GDP$ and $a_p^i\delta_i i^*/GDP$ respectively in the model.

$$\delta_i = \frac{NII/GDP}{INV/GDP} = 0.0038, \quad (116)$$

where GDP=NII+FS.

By matching the average velocity of M2 money stock ($v_c^{f*} = 1.836$), we can get

$z^f(s^{f*})^\xi = 2.4947$ which can be used later:

$$
\begin{aligned}
v_c^{f*} &= p^f c^{f*}/m^f \\
&= c^{f*}/(a_b^f q^{f*})
\end{aligned}
$$

$$
= (1 - FI) z^f (s^{f*})^\xi \tag{117}
$$

Similarly, $v_c^{i*} = z^i(s^{i*})^\xi$. I need two more targets to pin down B^f: the average input inventory to final sales ratio (ISR) and the intermediate inputs to final sales ratio (IPS). In this model:

$$
\begin{aligned}
ISR &= a_p^i i^*/c^{f*}, \qquad (118)\\
IPS &= \frac{z^f B^f (s^{f*})^\xi a_p^f a^*}{B^f v_c^f a_p^f q^f} \frac{P_i}{P^f}, \qquad (119)\\
\Rightarrow \frac{z^f B^f (s^{f*})^\xi a_p^f a^*}{B^f v_c^f a_p^f q^{f*}} &= IPS(1 + markup). \qquad (120)
\end{aligned}
$$

By plugging equation (104) into equation (103), I can get $i^*/a^* = (1-\delta_i)[1 - z^f B^f (s^{f*})^\xi]$. I also can rewrite i^*/a^* in terms of ISR and IPS:

$$
\begin{aligned}
\frac{i^*}{a^*} &= \frac{a_p^i i^*}{c^f} \frac{B^f v_c^{f*} a_p^f q^{f*}}{z^f B^f (s^{f*})^\xi a_p^f a^*} z^f B^f (s^{f*})^\xi, \\
&= \frac{ISR \cdot B^f v_c^f}{(1 - FI) IPS (1 + markup)} \quad . \qquad (121)
\end{aligned}
$$

After equalizing the two equations of i^*/a^*, B^f can be calculated in terms of ISP, IPS, v_c^{f*} and the markup:

$$\begin{aligned} B^f &= \frac{(1-\delta_i)}{\{(1-\delta_i)+ISR/[IPS(1+markup)]\}v_c^{f*}/(1-FI)} \quad , \\ &= 0.1947. \end{aligned} \tag{122}$$

Next, equations (105)-(107) are plugged into equation (110) to get rid of ω^f, λ^i and Ω_i; I can then pin down the parameter α. Rearranging equations (102)-(111), I can get:

$$\begin{aligned} \omega^{f*} &= \frac{\beta v_c^{f*}}{\gamma^* - \beta + \beta v_c^{f*}/(1-FI)} U'(c^*), \\ &\equiv E \cdot U'(c^*), \end{aligned} \tag{123}$$

$$\begin{aligned} \lambda^i &= \frac{\gamma^* - \beta}{\beta z^i (s^{i*})^\xi} \frac{\omega^{f*}}{1+markup}, \\ &= \frac{\gamma^* - \beta}{\beta v_c^{i*}(1+markup)} E \cdot U'(c^*), \\ &\equiv F \cdot U'(c^*), \end{aligned} \tag{124}$$

where,

$$\frac{\omega^{f*}}{\omega^{i*}} = \frac{P^{f*}\Omega_M^*}{P^{i*}\Omega_M^*} \quad \Rightarrow \omega^{i*} = \frac{\omega^{f*}}{1+markup} \quad , \tag{125}$$

and,

$$\begin{aligned} \Omega_i^* &= \frac{\beta B^f v_c^{f*} a_p^f/(1-FI)}{1-\beta(1-B^f v_c^{f*} a_p^f/(1-FI))(1-\delta_i)} \Big[\frac{\omega^{f*}}{1+markup} + \lambda^{i*}\Big], \\ &= \frac{\beta B^f v_c^{f*} a_p^f/(1-FI)}{1-\beta(1-B^f v_c^{f*} a_p^f/(1-FI))(1-\delta_i)} \Big[\frac{E}{1+markup} + F\Big] U'(c^*), \\ &\equiv G \cdot U'(c^*). \end{aligned} \tag{126}$$

Then α can be calculated by plugging the above equations into equation (110):

$$\begin{aligned}\alpha &= \frac{a_p^f[E/(1+markup)+F]+(1-a_p^f)(1-\delta^i)G}{E}\frac{a^*}{q^*} \quad , \\ &\equiv H\frac{a^*}{q^{f*}} \quad , \\ &= 0.4156,\end{aligned} \tag{127}$$

where $a^*/q^{f*} = 0.6822$ can be caluculated by rearranging $i^*/a^* = (1-\delta_i)[1 - z^f B^f (s^{f*})^\xi]$:

$$\begin{aligned} a_p^f i^* &= (1-\delta_i)[1-z^f B^f (s^{f*})^\xi] a_p^f a^*, \\ \Rightarrow \frac{a_p^f i^*}{B^f v_c^{f*} a_p^f (q^{f*})} &= (1-\delta_i)[1-z^f B^f (s^{f*})^\xi]\frac{a_p^f a^*}{B^f v_c^{f*} a_p^f q^{f*}}, \\ \Rightarrow ISR &= (1-\delta_i)\frac{1}{B^f v_c^{f*}}\frac{a^*}{q^{f*}} - (1-\delta_i) z^f B^f (s^{f*})^\xi \frac{a_p^f a^*}{B^f v_c^{f*} a_p^f q^{f*}}, \\ \Rightarrow a^*/q^{f*} &= [ISR + (1-\delta_i) IPS(1+markup)] B^f v_c^{f*}/(1-\delta_i). \end{aligned} \tag{128}$$

Since α is known, we can calculate $a^* = 0.5198$ by rearranging the production function $q^{f*} = A^* a^{*\alpha} n^{*(1-\alpha)}$:

$$\begin{aligned} \frac{q^{f*}}{a^*} &= A^* a^{*(\alpha-1)} n^{*(1-\alpha)}, \\ \Rightarrow a^* &= [\frac{q^{f*}}{a^*} A^* n^{*(1-\alpha)}]^{1/(\alpha-1)} \quad . \end{aligned} \tag{129}$$

Then q^{f*}, c^{f*}, i^* and a_b^f can be calculated:

$$\begin{aligned} q^{f*} &= A^*(a^*)^\alpha(n^\alpha)^{1-\alpha} = 0.7619, && (130) \\ c^{f*} &= (1-FI)a_p^f B^f v_c^{f*} q^{f*} = 0.0546, && (131) \\ i^* &= i^*/a^* \cdot a^* = 0.2662, && (132) \\ a_b^f &= B^f a_p^f = 0.0393. && (133) \end{aligned}$$

Now I can calculate ω^f, λ^i and Ω_i using the value of c^{f*}:

$$\begin{aligned} \omega^{f*} &\equiv E \cdot U'(c^*) = 7.4215, && (134) \\ \lambda^i &\equiv F \cdot U'(c^*) = 0.4760, && (135) \\ \Omega_i^* &\equiv G \cdot U'(c^*) = 4.4572. && (136) \end{aligned}$$

Using the seventh target, which is that the shopping time of the population is 11.17% of the working time and the working time is 30% of agents discretionary time, I can calculate the buyer's search intensity in the finished goods market s^{f*}. Once s^{f*} is known, z^f and z_1^f can be determined as follows:

$$\begin{aligned} s^{f*} &= 0.1117 * 0.3(a_p^f(1+n) + a_p^i)/a_b^f = 0.5162, && (137) \\ z^f &= z^f(s^f)^\alpha/((1-FI)(s^{f*})^\xi), && (138) \\ z_1^f &= z^f * (B^f)^{1-\xi} = 3.0524. && (139) \end{aligned}$$

Since I assume the intermediate goods market and the finished goods market are symmetric, a_b^i, s^{i*}, and z_i^i can be determined in a similar way:

$$a_b^i = B^i a_p^i = 0.0393, \tag{140}$$

$$s^{i*} = 0.1117 * 0.3(a_p^f(1+n) + a_p^i)/a_b^i = 1.7207, \tag{141}$$

$$z^i = z^i(s^i)^\xi/(s^i)^\xi = v_c^i/(s^i)^\xi, \tag{142}$$

$$z_1^i = z^i(B^i)^{1-\xi} = 0.0934. \tag{143}$$

Now the quantity of intermediate goods per trade (q^i), the constant in the disutility function of producing intermediate goods (b) and the constant in the disutility of posting vacancies (K_0) can be calculated by using equation (104), the function of $\varphi^i(q^i)$ and the last target ($K = 3.72 * 10^{-4}$):

$$q^{i*} = \frac{a^* - i^*}{v_c^{i*} a_b^i} = 6.5096, \tag{144}$$

$$\begin{aligned} b &= \varphi^{i\prime}(q^{i*})/q^{i*}, \\ &= \omega^{i*}/q^{i*} = 0.6706, \end{aligned} \tag{145}$$

$$K_0 = K/v^{*2} = 5.9501e - 004. \tag{146}$$

Finally, the parameters $(\varphi^i, \varphi_0^i, \varphi^f, \varphi_0^f)$ can be determined by using the steady state relations:

$$\varphi^i = \frac{b}{2}(q^i)^2 = 14.2091, \tag{147}$$

$$\varphi^f = [\beta(1-\alpha)\sigma B^f v_c^{f*}\omega^{f*}q^{f*}/((1-FI)n^*) - [1-\beta(1-\delta_n)]\Omega_n]/(\beta\sigma), \tag{148}$$

$$= 1.6025,$$

$$\varphi_0^i = \left\{\frac{z^i(s^{i*})^{\xi-1}\big[B^f a_p^f v_c^{f*}\lambda_i^*/(1-FI) + (1-B^f a_p^f v_c^{f*}/(1-FI))[(1-\delta_i^*)\Omega_i^* - bq^{i*}]\big]q^{i*}}{\varphi^i(1+1/\epsilon_i)(s^{i*})^{1/\epsilon_i}}\right\}^{\frac{\epsilon_i}{1+\epsilon_i}}$$

$$= 0.1104, \tag{149}$$

$$\varphi_0^f = \Big(\frac{z^f(s^{f*})^{\xi-1}((1-FI)(c^{f*})^{-\eta} - \omega^{f*})q^{f*}}{(\varphi^f(1+1/\epsilon_f)(s^{f*})^{1/\epsilon_f})}\Big)^{\epsilon_f/(1+\epsilon_f)} \tag{150}$$

$$= 1.8043.$$

Program codes

Structural VAR

```
clear all
set more off
set mem 800m

gen year = substr(date,1,4)
gen quarter = substr(date,6,1)
destring year quarter, replace
gen date_new = yq(year, quarter)
drop date year quarter
rename date_new date
format date %tq
tsset date , q

* generate intermediate inputs/final sales ratio
rename vinputs Vinputs
rename vfs VFS
rename fs FS
rename niir NIIR
rename isr ISR
egen IPS_mean = mean(Vinputs/VFS)
di IPS_mean

scalar a_b_f = 0.0295*10000
scalar a_p_f =  0.2017*10000
scalar alpha = 0.4060
scalar B_f =  0.1463
scalar FI = 0.269
gen ln_A = log(FS) - log(a_b_f*m2v*B_f) - alpha*log(Vinputs/ppi_i) +
alpha*log(a_b_f*m2v*B_f/(1-FI)) - (1-alpha)*log(empl/a_p_f)

gen ln_M2V = log(m2v)
gen ln_FS = log(FS)
```

```
gen ln_empl = log(empl)
gen ln_inputs = log(Vinputs)
gen ln_ppi_i = log(ppi_i)
gen ln_gamma = log(gamma)

********detrending data via the Hodrick-Prescott filter********
hprescott ln_A, stub(hp)
drop hp_ln_A_sm_1
rename hp_ln_A_1 hp_ln_A

hprescott gamma, stub(hp)
drop hp_gamma_sm_1
rename hp_gamma_1 hp_gamma

hprescott NIIR, stub(hp)
drop hp_NIIR_sm_1
rename hp_NIIR_1 hp_NIIR

hprescott ln_empl, stub(hp)
drop hp_ln_empl_sm_1
rename hp_ln_empl_1 hp_ln_empl

hprescott ISR, stub(hp)
drop hp_ISR_sm_1
rename hp_ISR_1 hp_ISR


********Check unit roots by Dickey Fuller (ADF) tests********
dfuller   hp_gamma, lags(2)
dfuller   hp_ln_A, lags(2)
dfuller   hp_ln_empl, lags(2)
dfuller   hp_NIIR, lags(2)
dfuller   hp_ISR, lags(2)

*Estimate the shocks
var hp_gamma hp_ln_A,lag(1) noc
matrix sigma=e(Sigma)
matrix list sigma

varsoc  hp_gamma hp_ln_A  hp_ln_empl hp_NIIR hp_ISR

mat A =  (1,0,0,0,0\.,1,0,0,0\.,.,1,0,0\.,.,.,1,0\.,.,.,.,1)
mat B =  (.,0,0,0,0\0,.,0,0,0\0,0,.,0,0\0,0,0,.,0\0,0,0,0,.)

svar hp_gamma hp_ln_A  hp_ln_empl hp_ISR hp_NIIR, lag(1/2)  aeq(A) beq(B)
vargranger
irf drop order1
irf create order1, step(15) set(myirf1)
irf graph sirf, impulse(hp_ln_A) response(hp_gamma hp_ln_A  hp_ln_empl hp_NIIR hp_ISR)
irf graph sirf, impulse(hp_gamma) response(hp_gamma hp_ln_A  hp_ln_empl hp_NIIR hp_ISR)
```

```
irf table sfevd, impulse(hp_gamma) response(hp_gamma)
irf table sfevd, impulse(hp_ln_A) response(hp_gamma)
irf table sfevd, impulse(hp_ln_empl) response(hp_gamma)
irf table sfevd, impulse(hp_ISR) response(hp_gamma)
irf table sfevd, impulse(hp_NIIR) response(hp_gamma)

irf table sfevd, impulse(hp_gamma) response(hp_ln_A)
irf table sfevd, impulse(hp_ln_A) response(hp_ln_A)
irf table sfevd, impulse(hp_ln_empl) response(hp_ln_A)
irf table sfevd, impulse(hp_ISR) response(hp_ln_A)
irf table sfevd, impulse(hp_NIIR) response(hp_ln_A)

irf table sfevd, impulse(hp_gamma) response(hp_ln_empl)
irf table sfevd, impulse(hp_ln_A) response(hp_ln_empl)
irf table sfevd, impulse(hp_ln_empl) response(hp_ln_empl)
irf table sfevd, impulse(hp_ISR) response(hp_ln_empl)
irf table sfevd, impulse(hp_NIIR) response(hp_ln_empl)

irf table sfevd, impulse(hp_gamma) response(hp_NIIR)
irf table sfevd, impulse(hp_ln_A) response(hp_NIIR)
irf table sfevd, impulse(hp_ln_empl) response(hp_NIIR)
irf table sfevd, impulse(hp_NIIR) response(hp_NIIR)
irf table sfevd, impulse(hp_ISR) response(hp_NIIR)

irf table sfevd, impulse(hp_gamma) response(hp_ISR)
irf table sfevd, impulse(hp_ln_A) response(hp_ISR)
irf table sfevd, impulse(hp_ln_empl) response(hp_ISR)
irf table sfevd, impulse(hp_NIIR) response(hp_ISR)
irf table sfevd, impulse(hp_ISR) response(hp_ISR)
```

Calibration

```
clc;
clear all;

beta =0.995;
delta_n = 0.105; %quarterly job separation rate
LP = 0.6445;        % labor participation rate = u+(1+n)a_p_f+a_p_i
UR =  0.061;    % unemployment rate = u/LP
v_c_f = 1.8236;   %velocity of M2 money stock
v_c_i =0.2;
n=1;               %normalize n to 1

phi =0.28;       %elasticity of labor market matching function
xi = 0.7;            %elasticity in the goods matching function
FI = 0.269 ;      %FI/output ratio
eta = 0.2;      %risk aversion
sigma = phi;    %workers' bargaining weight
epsilon_i = 1000;   % search elasticity in the disutility function of producing intermediate goods
epsilon_f = 0.01;
M_bar = 0.364;
x = 1;

u = LP*UR ;     %measure of unemployment
a_p_f=(LP-u)/(1+n+x);
a_p_i = x*a_p_f;
v = (delta_n*n/(M_bar*(a_p_f/u)^(phi-1)))^(1/phi);
MU = M_bar*(a_p_f*v/u)^(phi-1);

gamma_star =  1.0167;

%delta_i = NII_GDP / i_GDP, where NII_GDP = delta_i*a_p_f*i/GDP and i_GDP =
a_p_f*i/GDP
delta_i = .0038;

A = 1;    %TFP
markup = 0.7;

ISR = 0.984; %inventory/sales ratio
IPS = 0.549; %intermediate inputs/final sales ratio

B_f = (1-delta_i)/((ISR/(IPS*(1+markup))+(1-delta_i))*v_c_f/(1-FI));

a_q_f = (ISR +IPS*(1-delta_i)*(1+markup))*B_f*v_c_f/(1-delta_i);

E = beta*v_c_f/(gamma_star-beta+beta*v_c_f/(1-FI));
F = (gamma_star-beta)*E/(beta*v_c_i*(1+markup));
G = beta*B_f*a_p_f*v_c_f/((1-FI)*(1-beta*(1-B_f*v_c_f*a_p_f/(1-FI))*(1-
delta_i)))*(E/(1+markup)+F);
H = (a_p_f*(E/(1+markup)+F)+(1-a_p_f)*(1-delta_i)*G)/E;
```

```
alpha = H*a_q_f;
a = (A*n^(1-alpha)/a_q_f)^(1/(alpha-1));
q_f = A*a^alpha*n^(1-alpha);
c_f = a_p_f*B_f*v_c_f*q_f;
% To compute B_f

i_a = ISR*B_f*v_c_f/((1-FI)*IPS*(1+markup));
i = i_a*a;
B_i = B_f;
a_b_i = a_p_i*B_i;

q_i = (a-i)*a_p_f/(v_c_i*a_b_i);

% %To compute alpha, a_star, c_f*

a_b_f = B_f*a_p_f;

%Determine K_0, omega, Omega_n, varphi_f, Omega_i, lembda_i

K_0 =0.000372/v^2;
Omega_n = 2*K_0*v/MU;

omega_f = E*c_f^(-eta);
lembda_i = F*c_f^(-eta);
Omega_i = G*c_f^(-eta);
omega_i = omega_f/(1+markup);

varphi_f = (beta*(1-alpha)*sigma*B_f*v_c_f*omega_f*q_f/((1-FI)*n) - (1-beta*(1-
delta_n))*Omega_n)/(beta*sigma);

%To compute s_f, z_f, z_f_1, q_f, phi_f_0, a_i
s_f = 0.1117*0.3*(a_p_f*(1+n)+a_p_i)/a_b_f;
z_f = v_c_f/((s_f^xi)*(1-FI));
z_f_1 = z_f*B_f^(1-xi);

varphi_f_0 = (z_f*s_f^(xi-1)*((1-FI)*c_f^(-eta)-
omega_f)*q_f/(varphi_f*(1+1/epsilon_f)*s_f^(1/epsilon_f)))^(epsilon_f/(1+epsilon_f));


b = omega_i/ q_i;
varphi_i = b*q_i^2/2;

s_i = 0.1117*(a_p_f*(1+n)+a_p_i)/a_b_i;
z_i = v_c_i / (s_i^xi);
z_i_1 = z_i*B_i^(1-xi);

varphi_i_0 = (z_i*(s_i)^(xi-1)*(B_f*v_c_f*a_p_f*lembda_i/(1-FI)+(1-B_f*v_c_f*a_p_f/(1-
FI))*((1-delta_i)*Omega_i-
omega_i))*q_i/(varphi_i*(1+1/epsilon_i)*(s_i)^(1/epsilon_i)))^(epsilon_i/(1+epsilon_i));
```

```
%calculate SS
inv_target = a_p_f*i/c_f;
Delta = q_i*omega_i*a_b_i/(q_i*omega_i*a_b_i+q_f*omega_f*a_b_f) ;
NII = (1-delta_i)*(z_i*s_i^xi*a_b_i*q_i-z_f*B_f*s_f^xi*a_p_f*a);
GDP = c_f + NII;
velocity =  v_c_f;
welfare = (1/(1-beta))*((c_f^(1-eta)-1)/(1-eta)-z_i*B_i*s_i^xi*a_p_i*b*q_i^2/2-
a_p_f*n*varphi_f-a_b_i*varphi_i*(varphi_i_0*s_i)^(1+1/epsilon_i)-
a_b_f*varphi_f*(varphi_f_0*s_f)^(1+1/epsilon_f)-a_p_f*K_0*v^2 );
inv = a_p_f*i;
c_i = v_c_i*a_b_i*q_i;


revenue = omega_f*B_f*z_f*s_f^xi*q_f;
velocity_i = v-c_i;
matches_i = z_i_1*(a_b_i*s_i)^xi*a_p_i^(1-xi);
matches_f = z_f_1*(a_b_f*s_f)^xi*a_p_f^(1-xi);
wagecost = a_p_f*n*((1-sigma)*z_f*B_f*s_f^xi*(omega_f-a_p_f*(b*q_i+lembda_i-(1-
delta_i)*Omega_i)-(1-delta_i)*Omega_i)+sigma*varphi_f);
searchcost_f = a_b_f*varphi_f*(varphi_f_0)^(1+1/epsilon_f)*(1+1/epsilon_f)*(s_f)^(1/epsilon_f);
profitability = beta*[sigma*(1-alpha)*z_f*B_f*s_f^xi*q_f/n*omega_f  - sigma*varphi_f];
D_NII =NII/ GDP;


parameters = [FI ;
delta_n ;
a_b_i ;
a_b_f ;
a_p_i ;
a_p_f ;
xi ;
xi ;
beta ;
delta_i ;
sigma ;
eta ;
u ;
phi ;
b ;
K_0 ;
varphi_i ;
varphi_f ;
varphi_i_0 ;
varphi_f_0 ;
epsilon_i ;
epsilon_f;
z_i_1 ;
z_f_1 ;
M_bar;
z_i;
z_f;
```

```
B_i;
B_f ;
A;
alpha]

log_linearize=[
log(inv);
log(GDP) ;
log(s_i) ;
log(s_f) ;
log(Delta);
log(c_i);
log(c_f);
log(n) ;
log(q_f);
log(NII);
log(revenue);
log(v) ;
log(Omega_n);
log(q_i);
log(omega_f) ;
log(c_f);
log(Omega_i);
log(lembda_i) ;
log(i) ;
log(1.01669);
log(a)
log(inv_target) ;
log(velocity);
log(velocity_i);
log(matches_i);
log(matches_f);
log(wagecost);
log(searchcost_f);
log(profitability);
log(D_NII)]

%case 1: money growth shock only, AR(1)
rho_gamma = .5692752;
sigma_gg = .00789;
save parameterfile.mat rho_gamma markup  A FI delta_n a_p_f a_b_i xi a_p_i a_b_f beta delta_i
sigma varphi_f eta u phi b K_0 varphi_i varphi_i_0 epsilon_i varphi_f_0 epsilon_f z_i_1 z_f_1
sigma_gg z_i z_f B_i B_f M_bar A alpha;


%case 2: money growth rate and technology shock, VAR(1)
rho_gg = .2740597 ;
rho_gA = -.0773366 ;
rho_AA = .8445634 ;
rho_Ag =  .9422566   ;
```

```
sigma_gg =.00004157^(1/2);
sigma_AA =  .000191^(1/2);
sigma_gA = .00001986^(1/2);


save parameterfile.mat rho_gg rho_gA rho_AA rho_Ag markup sigma_gA FI delta_n a_p_f
a_b_i xi a_p_i a_b_f beta delta_i sigma varphi_f eta u phi b K_0 varphi_i varphi_i_0 epsilon_i
varphi_f_0 epsilon_f z_i_1 z_f_1 sigma_gg sigma_AA z_i z_f B_i B_f M_bar A alpha;
```

Dynare (case 2)

```
var  s_i s_f Delta  n  q_f   v Omega_n  q_i omega_f c Omega_i lembda_i i gamma a A  velocity_f
NII  GDP inv_target  D_NII sales_f sales_i revenue;

varexo e_gamma e_A;

parameters  rho_gg rho_gA rho_AA rho_Ag markup sigma_gA FI delta_n a_p_f a_b_i xi a_p_i
a_b_f beta delta_i sigma varphi_f eta u phi b K_0 varphi_i varphi_i_0 epsilon_i varphi_f_0
epsilon_f z_i_1 z_f_1 sigma_gg sigma_AA  z_i z_f B_i B_f M_bar alpha;

load parameterfile;
set_param_value('FI', FI)
set_param_value('delta_n', delta_n)
set_param_value('a_b_i', a_b_i )
set_param_value('a_b_f', a_b_f)
set_param_value('a_p_i', a_p_i)
set_param_value('a_p_f',a_p_f )
set_param_value('xi', xi)
set_param_value('xi', xi)
set_param_value('beta', beta )
set_param_value('delta_i', delta_i)
set_param_value('sigma', sigma )
set_param_value('eta', eta)
set_param_value('u', u)
set_param_value('phi',phi)
set_param_value('b', b)
set_param_value('K_0', K_0)
set_param_value('varphi_i', varphi_i)
set_param_value('varphi_f', varphi_f)
set_param_value('varphi_i_0', varphi_i_0)
set_param_value('varphi_f_0', varphi_f_0)
set_param_value('epsilon_i', epsilon_i)
set_param_value('epsilon_f', epsilon_f)
set_param_value('z_i_1', z_i_1)
set_param_value('z_f_1', z_f_1)
set_param_value('markup', markup)
set_param_value('sigma_gg', sigma_gg)
set_param_value('sigma_AA', sigma_AA)
set_param_value('sigma_gA', sigma_gA)
set_param_value('rho_gg', rho_gg)
set_param_value('rho_gA', rho_gA)
set_param_value('rho_AA', rho_AA)
set_param_value('rho_Ag', rho_Ag)
set_param_value('M_bar', M_bar)
set_param_value('z_i', z_i)
set_param_value('z_f', z_f)
set_param_value('B_i', B_i)
set_param_value('B_f', B_f)
set_param_value('alpha', alpha)
```

```
Model;
exp(n) = (1-delta_n)*exp(n(-1)) + exp(v)*M_bar*(a_p_f*exp(v)/u)^(phi-1);

a_p_f*exp(i) = (1-delta_i)*(a_p_f*exp(i(-1))+z_i*exp(s_i)^xi*a_b_i*exp(q_i)-
z_f*B_f*exp(s_f)^xi*a_p_f*exp(a));

exp(i(-1)) = exp(a) - z_i*exp(s_i)^xi*a_b_i*exp(q_i)/a_p_f;

exp(q_f) = exp(A)*(exp(a))^alpha*exp(n(-1))^(1-alpha);

exp(q_i) = exp(omega_f)/(1+markup)/b;

(1-exp(Delta))*exp(gamma)*exp(omega_f)*exp(q_f)/((1-exp(Delta(-1)))*exp(q_f(+1))) =
beta*(exp(omega_f(+1))+z_f*exp((s_f(+1)))^xi*((1-FI)*exp(c(+1))^(-eta)-exp(omega_f(+1))));

(1-exp(Delta))*a_b_i/(exp(Delta)*a_b_f) = exp(omega_f(+1))*exp(q_f(+1))/(b*exp(q_i(+1))^2);

exp(Omega_i) = beta*((1-delta_i)*exp(Omega_i(+1)) +
z_f*B_f*(exp(s_f(+1)))^xi*a_p_f*(exp(lembda_i(+1))+b*exp(q_i(+1))-(1-
delta_i)*exp(Omega_i(+1))));

exp(Omega_n) = beta*((1-delta_n)*exp(Omega_n(+1))
+sigma*z_f*B_f*(exp(s_f(+1)))^xi*exp(A(+1))*exp(a(+1))^alpha*exp(n)^(-
alpha)*exp(omega_f(+1))*(1-alpha)- sigma*varphi_f);

exp(Omega_n) = 2*K_0*exp(v)*(a_p_f*exp(v)/u)^(1-phi)/M_bar;

varphi_i*(varphi_i_0)^(1+1/epsilon_i)*(1+1/epsilon_i)*(exp(s_i))^(1/epsilon_i) =
z_i*(exp(s_i))^(xi-1)*(z_f*B_f*exp(s_f)^xi*a_p_f*exp(lembda_i)+(1-
z_f*B_f*exp(s_f)^xi*a_p_f)*((1-delta_i)*exp(Omega_i)-b*exp(q_i)))*exp(q_i);

varphi_f*(varphi_f_0)^(1+1/epsilon_f)*(1+1/epsilon_f)*(exp(s_f))^(1/epsilon_f) =
z_f*(exp(s_f))^(xi-1)*((1-FI)* (exp(c))^(-eta)-exp(omega_f))*exp(q_f);

exp(A)*alpha*(exp(a))^(alpha-1)*exp(n(-1))^(1-alpha)*exp(omega_f) =
a_p_f*(b*exp(q_i)+exp(lembda_i)) + (1-a_p_f)*(1-delta_i)*exp(Omega_i);

exp(c) = (1-FI)*a_p_f*B_f*z_f*exp(s_f)^xi*exp(q_f);

exp(gamma) =  (1-rho_gg)*1.01669 + rho_gg*exp(gamma(-1)) + rho_gA*A(-1) + e_gamma;

A =  -rho_Ag*1.01669 + rho_AA*A(-1)+ rho_Ag*exp(gamma(-1)) + e_A;

inv_target = a_p_f*exp(i)/exp(c);

exp(GDP) =  NII+exp(c);

NII = (1-delta_i)*(z_i*exp(s_i)^xi*a_b_i*exp(q_i)-z_f*B_f*exp(s_f)^xi*a_p_f*exp(a));

exp(velocity_f) =  z_f*exp(s_f)^xi;
```

```
exp(sales_i) = z_i*exp(s_i)^xi*a_b_i*exp(q_i);

exp(revenue) = exp(omega_f)*B_f*z_f*exp(s_f)^xi*exp(A)*exp(a)^alpha*exp(n(-1))^(1-alpha);

exp(sales_f) = a_p_f*B_f*z_f*exp(s_f)^xi*exp(q_f);

D_NII = NII/(NII+exp(c));
end;

initval;
 GDP        = -2.9043;
 s_i         = 0.5427;
 s_f         = -0.6612;
 Delta       = -0.1815;
 sales_f    = -2.9080;
 n           = 0;
 q_f         = -0.2720;
 NII         = -8.4969;
 v           = -0.2348;
 Omega_n     = -4.9497;
 q_i          = 1.8733;
 omega_f      = 2.0044;
 c             = -2.9080;
 Omega_i     = 1.4945;
 lembda_i    = -0.7422;
 i             = -1.3233;
 gamma       = 0.0166;
 a             = -0.6543;
 A             = 0;
 inv_target  = -0.0161;
 velocity_f  = 0.6008;
 D_NII        = -5.5926;
 sales_i      =  -2.9730;
 revenue     = 1.0105;
end;

steady(solve_algo=1);

check;

shocks;
var e_gamma = sigma_gg^2;
var e_A = sigma_AA^2;
var e_gamma, e_A = sigma_gA^2;
end;

stoch_simul(order=1, irf=15,  periods=1000, drop=300);

csvwrite('simuldata.csv', oo_.endo_simul') ;
```

The long run effects of money growth rate

```
clear all;
clc;
global FI delta_n a_b_i a_b_f a_p_i a_p_f xi beta delta_i sigma eta u A phi b K_0 varphi_i
varphi_f varphi_i_0 varphi_f_0 epsilon_i epsilon_f z_i_1 z_f_1 M_bar z_i z_f B_i B_f alpha
gamma
FI = 0.269;
delta_n =  0.1050;
a_b_i =  0.0393;
a_b_f =  0.0393;
a_p_i = 0.2017;
a_p_f =  0.2017;
xi = 0.8;
beta = 0.9950;
delta_i =  0.0038;
sigma = 0.28;
eta = 0.8;
u = 0.0393;
phi= 0.28;
b =  0.6706;
K_0 =0.0006;
varphi_i = 14.2091;
varphi_f = 1.6025;
varphi_i_0 = 0.1104;
varphi_f_0 =1.8043;
epsilon_i = 0.4;
epsilon_f = 0.01;
z_i_1 =  0.0934;
z_f_1 = 3.0524;
M_bar = 0.3640;
z_i = 0.1296;
z_f = 4.234;
B_i = 0.1947;
B_f =0.1947;
A = 1;
alpha = 0.4156;
gamma = 0.998;

step=0.0001;

options = optimset('MaxFunEvals',10000, 'MaxIter', 10000,  'TolFun', 1e-13);

v=0.7907;
s_i =  1.7207;
s_f = 0.5162;
lembda_i = 0.4760;
a =  0.5198;
for ii=1:1000
```

```
[theta, fval] = fsolve(@Dai2011_steadystate7, [0.7907      1.7207    0.5162    0.4760   0.5198  ],
options);

v = theta(1);
s_i = theta(2);
s_f = theta(3);
lembda_i = theta(4);
a =theta(5);

   n = v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n;

   q_i=beta*z_i*s_i^xi*lembda_i/((gamma-beta)*b);

   i=a-z_i*s_i^xi*a_b_i*q_i/a_p_f;

   q_f = A*a^alpha*(v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n)^(1-alpha);

   omega_f = beta*z_f*(s_f)^xi*(1-FI)*((1-
   FI)*a_p_f*B_f*z_f*s_f^xi*A*a^alpha*(v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n)^(1-alpha))^(-
   eta)/(gamma-beta+beta*z_f*(s_f)^xi);

    c = (1-FI)*a_p_f*B_f*z_f*s_f^xi*A*a^alpha*(v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n)^(1-
alpha);
   ISR = a_p_f*i/c;

   Omega_i=beta*z_f*B_f*(s_f)^xi*a_p_f*(lembda_i+b*q_i)/(1-beta*(1-
   z_f*B_f*(s_f)^xi*a_p_f)*(1-delta_i));
   NII = (1-delta_i)*(z_i*s_i^xi*a_b_i*q_i-z_f*B_f*s_f^xi*a_p_f*a);

   GDP = c + NII;

   D_NII =NII/ GDP;

    results(:,ii)= [gamma; theta'; ISR; GDP; NII; q_f];

gamma = gamma + step;
end

figure(1)
subplot(2,2,1);
plot(results(1,:),results(8,:)),xlabel('money growth rate'),ylabel('GDP')
subplot(2,2,2);
plot(results(1,:),results(9,:)),xlabel('money growth rate'),ylabel('NII')
subplot(2,2,3);
plot(results(1,:),results(7,:)),xlabel('money growth rate'),ylabel('ISR')
subplot(2,2,4);
plot(results(1,:),results(10,:)),xlabel('money growth rate'),ylabel('q^f')

[GDP_star,I_1]=max(results(8,:));
gamma_star=results(1,I_1)
```

```
[NII_star,I_2]=max(results(9,:));
gamma_star_ISR=results(1,I_2)

[q_f_star,I_2]=max(results(10,:));
gamma_star_ISR=results(1,I_2)

[ISR_star,I_2]=max(results(7,:));
gamma_star_ISR=results(1,I_2)
```

Solving steady state

```
function f=Dai2011_steadystate(theta)
global FI delta_n a_b_i a_b_f a_p_i a_p_f xi beta delta_i sigma eta u A phi b K_0 varphi_i
varphi_f varphi_i_0 varphi_f_0 epsilon_i epsilon_f z_i_1 z_f_1 M_bar z_i z_f B_i B_f  alpha
gamma
v = theta(1);
s_i = theta(2);
s_f = theta(3);
lembda_i = theta(4);
a =theta(5);

f=[ delta_i*a_p_f*(a-z_i*s_i^xi*a_b_i*(beta*z_i*s_i^xi*lembda_i/((gamma-beta)*b))/a_p_f)-(1-
delta_i)*(z_i*s_i^xi*a_b_i*(beta*z_i*s_i^xi*lembda_i/((gamma-beta)*b))-
z_f*B_f*s_f^xi*a_p_f*a);

2*K_0*v*(a_p_f*v/u)^(1-phi)/M_bar-beta*((1-delta_n)*2*K_0*v*(a_p_f*v/u)^(1-phi)/M_bar
+sigma*z_f*B_f*(s_f)^xi*A*a^alpha*(v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n)^(-
alpha)*(beta*z_f*(s_f)^xi*(1-FI)*((1-
FI)*a_p_f*B_f*z_f*s_f^xi*A*a^alpha*(v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n)^(1-alpha))^(-
eta)/(gamma-beta+beta*z_f*(s_f)^xi))*(1-alpha)- sigma*varphi_f);

varphi_i*(varphi_i_0)^(1+1/epsilon_i)*(1+1/epsilon_i)*(s_i)^(1/epsilon_i)-z_i*(s_i)^(xi-
1)*(z_f*B_f*s_f^xi*a_p_f*lembda_i+(1-z_f*B_f*s_f^xi*a_p_f)*((1-
delta_i)*(beta*z_f*B_f*(s_f)^xi*a_p_f*(lembda_i+b*(beta*z_i*s_i^xi*lembda_i/((gamma-
beta)*b)))/(1-beta*(1-z_f*B_f*(s_f)^xi*a_p_f)*(1-delta_i)))-
b*(beta*z_i*s_i^xi*lembda_i/((gamma-beta)*b))))*(beta*z_i*s_i^xi*lembda_i/((gamma-
beta)*b));

varphi_f*(varphi_f_0)^(1+1/epsilon_f)*(1+1/epsilon_f)*(s_f)^(1/epsilon_f)-z_f*(s_f)^(xi-1)*((1-
FI)*((1-FI)*a_p_f*B_f*z_f*s_f^xi*A*a^alpha*(v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n)^(1-
alpha))^(-eta)-(beta*z_f*(s_f)^xi*(1-FI)*((1-
FI)*a_p_f*B_f*z_f*s_f^xi*A*a^alpha*(v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n)^(1-alpha))^(-
eta)/(gamma-beta+beta*z_f*(s_f)^xi)))*A*a^alpha*(v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n)^(1-
alpha);

A*alpha*a^(alpha-1)*(v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n)^(1-alpha)*(beta*z_f*(s_f)^xi*(1-
FI)*((1-FI)*a_p_f*B_f*z_f*s_f^xi*A*a^alpha*(v*M_bar*(a_p_f*v/u)^(phi-1)/delta_n)^(1-
alpha))^(-eta)/(gamma-beta+beta*z_f*(s_f)^xi)) -
a_p_f*(b*(beta*z_i*s_i^xi*lembda_i/((gamma-beta)*b))+lembda_i)-(1-a_p_f)*(1-
delta_i)*(beta*z_f*B_f*(s_f)^xi*a_p_f*(lembda_i+b*(beta*z_i*s_i^xi*lembda_i/((gamma-
beta)*b)))/(1-beta*(1-z_f*B_f*(s_f)^xi*a_p_f)*(1-delta_i)));

];

end
```

The role of inventories

```
clear all;
clc;
ISR=[1.0402, 0.9499];
ii=1;
periods=[0:14];


for ISR =ISR
beta =0.995;
delta_n = 0.105;
LP = 0.6445;
UR =  0.061;
v_c_f = 1.8236;
v_c_i =0.2;
n=1;

phi =0.28;
xi = 0.8;
FI = 0.269 ;
eta = 0.8;
sigma = phi;
epsilon_i = 0.4;
epsilon_f = 0.01;
M_bar = 0.364;
x = 1;

u = LP*UR ;
a_p_f=(LP-u)/(1+n+x);
a_p_i = x*a_p_f;
v = (delta_n*n/(M_bar*(a_p_f/u)^(phi-1)))^(1/phi);
MU = M_bar*(a_p_f*v/u)^(phi-1);

gamma_star =  1.0167;

delta_i = 0.0038;

A = 1;

markup = 0.7;

IPS = 0.549

B_f = (1-delta_i)/((ISR/(IPS*(1+markup))+(1-delta_i))*v_c_f/(1-FI));

a_q_f = (ISR +IPS*(1-delta_i)*(1+markup))*B_f*v_c_f/(1-delta_i);

E = beta*v_c_f/(gamma_star-beta+beta*v_c_f/(1-FI));
F = (gamma_star-beta)*E/(beta*v_c_i*(1+markup));
```

```
G = beta*B_f*a_p_f*v_c_f/((1-FI)*(1-beta*(1-B_f*v_c_f*a_p_f/(1-FI))*(1-
delta_i)))*(E/(1+markup)+F);
H = (a_p_f*(E/(1+markup)+F)+(1-a_p_f)*(1-delta_i)*G)/E;

alpha = H*a_q_f;
a = (A*n^(1-alpha)/a_q_f)^(1/(alpha-1));
q_f = A*a^alpha*n^(1-alpha);
c_f = a_p_f*B_f*v_c_f*q_f;
% To compute B_f

i_a = ISR*B_f*v_c_f/((1-FI)*IPS*(1+markup));
i = i_a*a;
B_i = B_f;
a_b_i = a_p_i*B_i;

q_i = (a-i)*a_p_f/(v_c_i*a_b_i);

% %To compute alpha, a_star, c_f*

a_b_f = B_f*a_p_f;

%Determine K_0, omega, Omega_n, varphi_f, Omega_i, lembda_i

K_0 =0.000372/v^2;
Omega_n = 2*K_0*v/MU

omega_f = E*c_f^(-eta);
lembda_i = F*c_f^(-eta);
Omega_i = G*c_f^(-eta);
omega_i = omega_f/(1+markup);

varphi_f = (beta*(1-alpha)*sigma*B_f*v_c_f*omega_f*q_f/((1-FI)*n) - (1-beta*(1-
delta_n))*Omega_n)/(beta*sigma);

%To compute s_f, z_f, z_f_1, q_f, phi_f_0, a_i
s_f = 0.1117*0.3*(a_p_f*(1+n)+a_p_i)/a_b_f;
z_f = v_c_f/((s_f^xi)*(1-FI));
z_f_1 = z_f*B_f^(1-xi);

varphi_f_0 = (z_f*s_f^(xi-1)*((1-FI)*c_f^(-eta)-
omega_f)*q_f/(varphi_f*(1+1/epsilon_f)*s_f^(1/epsilon_f)))^(epsilon_f/(1+epsilon_f));


b = omega_i/ q_i;
varphi_i = b*q_i^2/2;

s_i = 0.1117*(a_p_f*(1+n)+a_p_i)/a_b_i;
z_i = v_c_i / (s_i^xi);
z_i_1 = z_i*B_i^(1-xi);
```

```
varphi_i_0 = (z_i*(s_i)^(xi-1)*(B_f*v_c_f*a_p_f*lembda_i/(1-FI)+(1-B_f*v_c_f*a_p_f/(1-FI))*((1-delta_i)*Omega_i-omega_i))*q_i/(varphi_i*(1+1/epsilon_i)*(s_i)^(1/epsilon_i)))^(epsilon_i/(1+epsilon_i));

rho_gg = .2740597 ;
rho_gA = -.0773366 ;
rho_AA = .8445634 ;
rho_Ag =  .9422566   ;

sigma_gg =.00004157^(1/2);
sigma_AA =  .000191^(1/2);
sigma_gA = .00001986^(1/2);

save parameterfile.mat rho_gg rho_gA rho_AA rho_Ag markup sigma_gA FI delta_n a_p_f a_b_i xi a_p_i a_b_f beta delta_i sigma varphi_f eta u phi b K_0 varphi_i varphi_i_0 epsilon_i varphi_f_0 epsilon_f z_i_1 z_f_1 sigma_gg sigma_AA z_i z_f B_i B_f M_bar A alpha;

  dynare Tiantian2011_test14 noclearall

  irf_GDP(ii,:)=[oo_.irfs.GDP_e_gamma];
  irf_sales_f(ii,:)=[oo_.irfs.sales_f_e_gamma];
  irf_i(ii,:)=[oo_.irfs.i_e_gamma];
  irf_q_f(ii,:)=[oo_.irfs.q_f_e_gamma];
  irf_q_i(ii,:)=[oo_.irfs.q_i_e_gamma];
  irf_s_f(ii,:)=[oo_.irfs.s_f_e_gamma];
  irf_s_i(ii,:)=[oo_.irfs.s_i_e_gamma];
  irf_omega_f(ii,:)=[oo_.irfs.omega_f_e_gamma];
  irf_Delta(ii,:)=[oo_.irfs.Delta_e_gamma];
  irf_Omega_i(ii,:)=[oo_.irfs.Omega_i_e_gamma];
  irf_lembda_i(ii,:)=[oo_.irfs.lembda_i_e_gamma];
  irf_c(ii,:)=[oo_.irfs.c_e_gamma];
  irf_n(ii,:)=[oo_.irfs.n_e_gamma];
  irf_inv_target(ii,:)=[oo_.irfs.inv_target_e_gamma];
  irf_D_NII(ii,:)=[oo_.irfs.D_NII_e_gamma];
  irf_velocity_f(ii,:)=[oo_.irfs.velocity_f_e_gamma];
  irf_a(ii,:)=[oo_.irfs.a_e_gamma];


  irf_GDP_A(ii,:)=[oo_.irfs.GDP_e_A];
  irf_sales_f_A(ii,:)=[oo_.irfs.sales_f_e_A];
  irf_i_A(ii,:)=[oo_.irfs.i_e_A];
  irf_q_f_A(ii,:)=[oo_.irfs.q_f_e_A];
  irf_q_i_A(ii,:)=[oo_.irfs.q_i_e_A];
  irf_s_f_A(ii,:)=[oo_.irfs.s_f_e_A];
  irf_s_i_A(ii,:)=[oo_.irfs.s_i_e_A];
  irf_omega_f_A(ii,:)=[oo_.irfs.omega_f_e_A];
  irf_Delta_A(ii,:)=[oo_.irfs.Delta_e_A];
  irf_Omega_i_A(ii,:)=[oo_.irfs.Omega_i_e_A];
  irf_lembda_i_A(ii,:)=[oo_.irfs.lembda_i_e_A];
  irf_c_A(ii,:)=[oo_.irfs.c_e_A];
  irf_n_A(ii,:)=[oo_.irfs.n_e_A];
```

```
    irf_inv_target_A(ii,:)=[oo_.irfs.inv_target_e_A];
    irf_D_NII_A(ii,:)=[oo_.irfs.D_NII_e_A];
    irf_velocity_f_A(ii,:)=[oo_.irfs.velocity_f_e_A];
    irf_a_A(ii,:)=[oo_.irfs.a_e_A];

    ii = ii+1;
    delta_i
end

ss=zeros(15);

figure(1)
subplot(3,2,1);
plot(periods, irf_GDP(1,:), periods, irf_GDP(2,:),'--', periods,
ss),xlabel('periods'),ylabel('percentage deviation'), title('GDP'),legend('ISR=1.0402','ISR=0.9499')
subplot(3,2,2);
plot(periods, irf_D_NII(1,:), periods, irf_D_NII(2,:),'--', periods,
ss),xlabel('periods'),ylabel('percentage deviation'), title('NII')
subplot(3,2,3);
plot(periods, irf_i(1,:), periods, irf_i(2,:),'--', periods, ss),xlabel('periods'),ylabel('percentage
deviation'), title('Inventories')
subplot(3,2,4);
plot(periods, irf_sales_f(1,:), periods, irf_sales_f(2,:),'--', periods,
ss),xlabel('periods'),ylabel('percentage deviation'), title('Sales_f')
subplot(3,2,5);
plot(periods, irf_n(1,:), periods, irf_n(2,:),'--', periods, ss),xlabel('periods'),ylabel('percentage
deviation'), title('n(+1)')
subplot(3,2,6);
plot(periods, irf_a(1,:), periods, irf_a(2,:),'--', periods, ss),xlabel('periods'),ylabel('percentage
deviation'), title('a')


figure(2)
subplot(3,2,1);
plot(periods, irf_GDP_A(1,:), periods, irf_GDP_A(2,:),'--', periods,
ss),xlabel('periods'),ylabel('percentage deviation'), title('GDP'),legend('ISR=1.0402','ISR=0.9499')
subplot(3,2,2);
plot(periods, irf_D_NII_A(1,:), periods, irf_D_NII_A(2,:),'--', periods,
ss),xlabel('periods'),ylabel('percentage deviation'), title('NII')
subplot(3,2,3);
plot(periods, irf_i_A(1,:), periods, irf_i_A(2,:),'--', periods, ss),xlabel('periods'),ylabel('percentage
deviation'), title('Inventories')
subplot(3,2,4);
plot(periods, irf_sales_f_A(1,:), periods, irf_sales_f_A(2,:),'--', periods,
ss),xlabel('periods'),ylabel('percentage deviation'), title('Sales_f')
subplot(3,2,5);
plot(periods, irf_n_A(1,:), periods, irf_n_A(2,:),'--', periods,
ss),xlabel('periods'),ylabel('percentage deviation'), title('n(+1)')
subplot(3,2,6);
plot(periods, irf_a_A(1,:), periods, irf_a_A(2,:),'--', periods,
ss),xlabel('periods'),ylabel('percentage deviation'), title('a')
```