

THREE ESSAYS ON ASSET BUBBLES AND CONTAGION
OVER FINANCIAL NETWORKS

by

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Abstract

This thesis studies financial market stability by exploring asset bubbles and contagions over financial markets. First I construct a model where bubbles arise from a lack of common knowledge about the asset value among traders with private information, and I evaluate the effects of capital gain tax and transaction costs on bubbles. I find that capital gains tax has no effect on the size of the bubble when there is a perfect tax credit for capital losses, and the size of the bubble decreases in the tax when there is no tax credit. Therefore dealing with bubbles with capital gains tax not only requires imposing the tax, but also tightening the policies on tax credits. In a simplified bubble model, it can be shown that the model is equivalent to an auction, and bubbles arise for the same reason that bidding prices fail to reveal the true value in that auction. Several experiments on taxes and subsidies are devised to reduce or eliminate bubbles. Then I study the contagion of bankruptcy through downward price pressure among investors with overlapping portfolios. I calculate the probability of an extensive contagion and the expected bankruptcy rate during such a contagion. System-wide contagion happens only when the diversification of portfolios is in a certain range and, in the upper area of that range, the probability of a crisis may be small, but the spread of contagion can be extremely extensive.

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I take full responsibility for any remaining errors

Dedication

To my family for their unconditional love, support and understanding throughout my life.

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Chapter 1

Introduction

Financial crises in the past decades have raised serious concerns about the applicability of the equilibrium concept to financial markets, or at least the extent to which financial markets can be regarded as stable. In this thesis I explore the non-stable side of financial markets from two perspectives: asset bubbles and the contagion.

Financial crises often feature dramatic price fluctuations. One literature interprets these fluctuations as prices significantly deviating from intrinsic asset values and refers to them as *bubbles*. These models study bubbles from the perspective of information asymmetry and generate bubbles in three different ways. One approach adheres to the rationality of economic agents and creates bubbles from a lack of common knowledge (e.g. Allen et al. (1993) and Allen and Gorton (1993)). A different approach allows investors to have heterogeneous prior beliefs and acknowledge this difference (“agree to disagree”) to induce bubbles (e.g. Harrison and Kreps (1978) and Scheinkman and Xiong (2003)) due to psychological biases. In particular Scheinkman and Xiong (2003) evaluate the effects of a transaction tax on bubbles and show that the tax has very limited impact on the size of bubbles. Another branch generates bubbles from the interaction between rational, sophisticated traders and behavioral market

participants (e.g. De Long et al. (1990) and Abreu and Brunnermeier (2003)).

My thesis (the first and third essay) studies the source of asset bubbles and suggests measures to deflate them. In Chapter 2 (the first essay) I construct a model where bubbles arise from the interaction between rational traders and behavioral agents. This model incorporates purchases into the framework of Abreu and Brunnermeier (2003) and evaluates the effects of capital gains tax and transaction costs on bubbles. We find that the capital gains tax has no effect on the size of the bubble when there is a perfect capital loss tax credit, and the bubble size decreases in the capital gain tax when there is no tax credit. Therefore dealing with bubbles with capital gains tax not only requires imposing the tax, but also tightening the policies on tax credits. In addition, the size of the bubble decreases in the transaction cost, and when returns from the outside option deteriorate the bubble becomes larger.

In Chapter 4 (the third essay) I focus on the comparative analysis between bubbles and auctions. A simplified version of the model in Chapter 2 can be shown to be equivalent to a common value auction, and bubbles arise due to the same reason that the price fails to converge to the true value of the object in a first-price auction. By modifying the payoff structure with taxes and subsidies and thus turning the model from a first-price into a second-price auction, bubbles are eliminated. Based on this finding, several experiments to reduce bubbles are devised and discussed.

A different literature that studies the stability of financial markets focuses on the phenomenon of *contagion*. Modern financial systems have become much more complex and interconnected, and financial crises in past decades have demonstrated this fragility. Preventing a local market disturbance from being transmitted to other sectors and regions and leading to a systemic collapse has become one of the top

priorities of financial authorities. The pioneering work of Allen and Gale (2000) first modeled banks interconnected by loans as networks and showed the effects of network structures and risk diversification on the contagion of default on interbank loans. More recent literature (e.g. Gai and Kapadia (2010)) uses techniques developed in physics and biology to study the contagion of default on loans over more complex networks.

In Chapter 3 (the second essay) I study the contagion of asset liquidation among investors with overlapping portfolios. When a shareholder goes bankrupt and liquidates, the downward pressure on asset price could bring down other shareholders and trigger liquidations on other assets. The interconnected portfolios are modeled as a complex network. I characterize the extent of contagion and show that systemic contagion happens only when the network connectivity (represented by the average number of assets in an investor's portfolio) is within a certain window. The probability of a crisis first increases in the connectivity and then decreases. The system exhibits a robust-yet-fragile tendency: when the probability starts to decrease, the spread of the contagion, if it happens, continues to increase and becomes extremely extensive. This extreme consequence is related to the gradual erosion to investors by multiple rounds of downward price impact, which implies the importance of early government interventions after the initial outbreak of a contagion.

In Chapter 5 I conclude.

Chapter 2

The Impact of Capital Gains Taxes and Transaction Costs on Asset Bubbles

2.1 Introduction

Since the 2008 US subprime mortgage crisis, housing prices in certain major cities outside the United States have been steadily increasing at speeds higher than their historical norms, including those in London (UK), Vancouver, Toronto, Beijing, Shanghai, Hong Kong¹, and most capital cities in Australia. For example, house prices in London rose by 18% in 2013 alone. The upsurging prices have raised serious concerns that bubbles are developing in these cities. Although the ongoing discussion on macroprudential policies might help regulate domestic financial practitioners, these policies seem to be ineffective when international “hot money” and private investors contribute greatly to the bubbles, and there is no consensus on the implementation of these policy tools. A different choice would be a tax. There is a large literature on the effects of financial transaction taxes on price volatilities, but to our knowledge the effectiveness of taxes on asset bubbles, especially capital gains tax, has not been

¹Housing prices in hundreds of smaller cities in China are also inflating rapidly.

studied in the existing literature.

With or without economic theories, governments and lawmakers are ready to intervene, or have already done so. Chancellor George Osborne of the British Parliament said that as of April 2015 he would introduce a capital gains tax on future gains made by non-residents who sell residential properties in the United Kingdom². In 2013 the Chinese government introduced a 20% tax on capital gains from selling residential properties if a family owns multiple properties. There is an urgent need to assess the effectiveness of these tax policies on reducing bubbles.

We evaluate the effects of capital gains tax and transaction costs on asset bubbles. Our model incorporates purchases into the framework of Abreu and Brunnermeier (2003) (henceforth AB2003). When the asset fundamental rises, privately informed rational traders purchase the asset from behavioral agents, which continuously drives up the price. Each rational trader has incomplete knowledge about the new fundamental value and does not know how many others have a higher (or lower) belief. This belief dispersion is such that there is no common knowledge about the emergence of a bubble when the price rises above the fundamental value. As the price continues to rise, traders with the lowest belief stop purchasing and simply hold while others are buying. As the price is further driven up, low-belief traders start to sell, intermediate-belief traders stop buying and hold, and only high-belief traders are still buying. When traders with the highest belief stop buying, the bubble bursts.

In the unique equilibrium a bubble exists, and traders ride the bubble and try to make a profit from buying and selling. Upon the bursting, low-belief traders have already exited the market while others are caught in the crash. Each trader uses a

²See Chancellor George Osborne's Autumn Statement 2013 speech on 5 December 2013 at the UK Parliament.

trigger strategy that consists of two price thresholds: a selling price and a stop-buy price. The optimal selling strategy trades off the marginal price appreciation with the marginal risk of being caught in the crash. The stop-buy strategy dictates that a trader should not buy the asset if the expected after-tax profit is negative.

A trader with a profit is subject to a capital gains tax, while a trader with a loss is entitled to a tax credit.³ The most favorable treatment of tax credits is refunding the trader an amount of the loss multiplied by the tax rate, which we call a perfect tax credit.⁴ But in most countries the credits can only be used to offset past or future gains, instead of an immediate refund, and there are restrictions on how gains can be offset, such as time restrictions, ceilings and inclusion rates.⁵ We use a single parameter (the inclusion rate) to summarize these restrictions and allow it to vary from no credit at all to a perfect tax credit.

Our paper presents two main results. The first is that the tax credit can offset the deflating effect of capital gains tax on bubbles and when the tax credit is perfect, the tax has no effect on bubbles at all. The capital gains tax can reduce bubbles in our model because, intuitively, it widens the relative payoff difference between fleeing and being caught. This increased difference makes traders behave more cautiously by selling early to secure their gains. The lowered selling strategy then squeezes the

³The capital loss tax credit is also called capital loss deduction or tax carryover. It aims to reduce the tax burden when tax payers incur capital losses and help smooth fluctuations in their incomes so that the tax is levied on a long-term average base.

⁴This is also called a symmetric tax treatment or full loss refundability in the tax literature. Symmetry means an investor pays the tax whenever a profit occurs and receives a refunding whenever a loss occurs.

⁵In the United States, an individual who incurs capital losses can use the credit to offset any taxable income up to \$3000 each year, and carry the unused credit forward indefinitely. A corporation can only offset capital gains (not other profit) with capital losses and carry it 3 years backward and 5 years forward, without a dollar limit. In Canada the tax code is similar, except that there is no \$3000 limit on individuals, but only 50% of the capital gains/losses are included in the tax base for both individuals and corporations, and the tax credit can be carried 3 years backward and forward indefinitely.

stop-buy strategy downwards, which bursts the bubble early. The credit, on the other hand, serves as a compensation to a trader's loss such that the loss is also "taxed" and becomes smaller. It thus reduces the payoff difference and traders become less concerned about being caught. They behave aggressively by selling at high prices, which in turn encourages buying at higher prices and the bubble is inflated. Under a perfect tax credit, this compensatory effect completely neutralizes the effect of the tax, even when we raise the tax to 100%! This result suggests that to deflate a bubble with capital gains tax, a tax authority should not only impose or raise the tax, but also examine its tax credit policies and refrain from granting overly favorable tax credits on capital losses.

The second result is that, while it may not be surprising that a transaction cost or outside option (returns from alternative investment opportunities) can reduce bubbles, their marginal effects on bubbles are very large when they are small. When the cost or the outside option rises slightly from zero, a small gap between buying and selling is required (the profit margin). But since the bubble could burst in between this gap, a larger gap (profit margin) is needed to compensate the risk. A larger gap in turn means a larger probability of bursting in between, which in turn requires an even larger gap. Hence a small cost or outside option can significantly push down the stop-buy strategy and deflate bubbles. This result backs the practice in some countries to reserve financial transaction taxes at very low rates. If we interpret the outside option as the interest on treasury bonds, for instance, then conversely it implies that lowering interest rates when they are already low has a significant inflating effect on bubbles. This argument is supported by the timing of the Federal Reserve's low interest rate policies and the housing bubble in the United States between 2001

and 2008 and their potential causality. It also warns that the Bank of Canada's recent move of lowering the interest rate from 1% to 0.75% (a counter measure to the oil price plunge) will further inflate the Canadian housing market.

Besides the two main results, our model also generates other empirically testable predictions. Some of the implications are consistent with existing empirical evidences. For example, an increase in capital gains tax not only reduces demand from rational traders, but also delays their selling and reduces supply (and hence decreases the trading volume). These effects are the "capitalization effect" and "lock-in effect", respectively, in empirical literature and are documented in empirical studies of tax incidences such as the Revenue Act of 1978, the Tax Reform Act of 1986 and the Taxpayer Relief Act of 1997 in the United States. We will explore empirical evidences and discuss these implications in detail in Section 2.6.

The framework of AB2003 is shown to be consistent with recent empirical studies of stock market data. Temin and Voth (2004) show that a major investor in the South Sea Bubble knew that a bubble was in progress and nonetheless invested in the stock and hence was profiting from riding the bubble. Brunnermeier and Nagel (2004) and Griffin et al. (2011) both study the tech bubble in the late 1990s. They show that instead of correcting the price bubble, hedge funds turned out to be the most aggressive investors. They profited during the upturn and unloaded their positions before the downturn.

Our model is also related to a large literature that studies the effects of taxes on asset prices. Constantinides (1983) shows that investors have incentives to sell assets with losses immediately and secure tax credits while deferring the selling of assets with gains to put off tax payment. His research focuses on trading under stochastic shocks

whereas we focus on trading with private beliefs facing asset bubbles and crashes. In terms of the effects of the transaction tax on the volatility of asset prices, Westerhoff and Dieci (2006) show that the transaction tax can stabilize prices, whereas others suggest that the tax actually amplifies volatility (e.g. Lanne and Vesala (2010)). Empirical evidences show that the ability of the tax to reduce volatility is very limited (e.g. Umlauf (1993) and Hu (1998)).

The research on the effects of taxes on bubbles, on the other hand, is scarce. Scheinkman and Xiong (2003) show that the financial transaction tax can substantially reduce speculative trading volume, but has only a limited impact on the size of the bubble. Our paper explicitly models an asset bubble and evaluates the effects of capital gains tax on trading behavior and the size of bubble, and is complementary to the above literature in understanding the effects of taxes on financial markets.

From a modeling perspective, our paper is related to a quickly growing literature on bubbles. For surveys, refer to Brunnermeier (2009), Brunnermeier (2001), Brunnermeier and Oehmke (2012) and Xiong (2013). AB2003 relies on the asynchronous timing of awareness to generate bubbles, whereas we transform the uncertainty from time to value/price and remove the sequential awareness assumption. Our model allows for any continuous and strictly increasing price path instead of a exogenous exponential price path. We also add purchases to their framework such that the price keeps increasing exactly because rational traders are buying. The bubble bursts in our model when no one wants to buy, which is in contrast to AB2003, where a threshold in accumulated sales triggers the bursting.

Doblas-Madrid (2012) removes behavioral agents from the framework of AB2003 and instead uses idiosyncratic liquidity shocks to force rational traders to sell to

generate trades. The price is determined in equilibrium every period and agents also update their belief when facing the noisy price. This feature is difficult to apply to our setting because we do not have any noise in the price⁶.

The remainder of this paper is organized as follows. Section 2.2 introduces the model. Section 2.3 shows that a trader's strategy space can be reduced, which gives us a simple game to solve. In Section 2.4 we solve a trader's problem, characterize the equilibrium and discuss several policy implications associated with the results. Section 2.5 discusses the downward price overshooting in recession which is the reverse process of bubble and crash. Section 2.6 presents other empirical predictions. Section 2.7 concludes.

2.2 The model

There is one asset (henceforth the *asset*) with a total supply Q . Time t is continuous and at $t = 0$ the asset's fundamental value jumps up from p_0 to θ and does not change henceforth. θ is uniformly distributed on $[p_0, \infty)$ ⁷ and is unobservable. Without loss of generality, we assume $p_0 = 0$. There are two types of agents: risk neutral rational traders (henceforth *traders*) and a large passive behavioral agent (or a pool of behavioral agents), though the latter is not our primary interest. All shares of the asset are held by the passive agent at the beginning. Rational traders have an outside investment option. The outside option provides a constant profit $R \geq 0$, which is common knowledge and uncorrelated with θ . A trader cannot hold the asset

⁶To conceal the very first strategic sale in our model, one must assume that there is an extra amount of money injected to balance the strategic sale at that moment, which can be difficult to justify.

⁷The improper uniform distribution on $[0, \infty)$ has a well defined posterior belief when we specify how signals are distributed. The uniform prior distribution gives tractable solutions and is adapted from Li and Milne (2014).

and the outside option at the same time and cannot switch to the other if she has bought one. Capital gains from both the asset and the outside option are subject to a capital gains tax, and a trader will choose the asset only when its expected profit is strictly higher than the outside option. At this moment we restrict that $R = 0$. In Section 2.4.6 we discuss the implication of strictly positive R . Each trader is infinitesimal and, without loss of generality, a trader's asset position is restricted and normalized to $[0, 1]$.⁸

The passive agent has an inverse asset supply function

$$p = \alpha(D_r) \tag{2.1}$$

where p is the price that is publicly observable, D_r is the total shares held by all traders (the aggregate position of all traders) and $\alpha(\cdot)$ is a continuous, strictly increasing function. When the fundamental value jumps up, traders start to buy the asset from the passive agent. This drives up the price continuously because, as shares are sold to traders, the passive agent keeps raising the price. This behavior can be interpreted as portfolio diversification requirements that make the risky asset more valuable to the passive agent when its weight decreases in her portfolio. Or it can be interpreted as an adverse selection problem where, when traders with private information keep buying the asset, it is natural for the uninformed passive agent to respond by raising the price, as in Kyle (1985). Similar behavioral asset supplies have also been adopted by De Long et al. (1990), where passive investors supply the asset at an increasing price when rational speculators are buying, and by Brunnermeier and Pedersen (2005b) and Carlin et al. (2007b), where long-term investors sell the asset when strategic

⁸Limited short selling of the asset is allowed for traders.

traders' buy-back pushes up the price.

When the price rises above the fundamental value, we say a bubble emerges. When the price is driven so high that no one wants to buy any more, the price stops rising. Denote this random stopping price p_T . If $p_T > \theta$ at this moment, it will be clear shortly that the existence of the bubble becomes common knowledge and the bubble bursts at p_T *endogenously*.⁹ We also assume that the price can be arbitrarily high when traders' aggregate position approaches Q , i.e. $\lim_{D_r \rightarrow Q} \alpha(D_r) = \infty$. This rules out the possibility that the asset price cannot catch up to its fundamental value simply because shares are running out.

After entering the market, a trader can buy and sell the asset at any time. A trader's purchases and sales cannot be observed by others. Since each trader is infinitesimal, her transaction is executed instantly at the spot price. After the bursting, the price is fixed at θ and the passive agent is willing to buy the asset only at this price. All traders who still hold the asset have to liquidate at this price. In the end, no trader holds the asset and all shares go back to the passive agent.

Each trader receives a private signal v that is uniformly distributed on $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$. We call a trader with signal v a type v trader or trader v . There is enough mass of traders (potentially infinite) for each type v .¹⁰ Let $\Phi(\theta|v)$ denote the cumulative distribution function of the belief of a trader v conditioned on v , and $\phi(\theta|v)$ the corresponding probability density function. Given a signal v , the posterior belief of a trader is that θ is uniformly distributed on $[v - \frac{\eta}{2}, v + \frac{\eta}{2}]$. Therefore $\phi(\theta|v) = \frac{1}{\eta}$

⁹In a nutshell, given the strategy profile of all types of traders, p_T is a function of θ . Thus everyone can perfectly infer θ from the stopping price p_T .

¹⁰This is to rule out the possibility that the price cannot catch up to its fundamental value simply because we are running out of buyers.

and $\Phi(\theta|v) = \frac{\theta - (v - \frac{\eta}{2})}{\eta}$.¹¹ Figure 4.2 depicts the posterior belief about θ for traders v , v' and v'' . These different posterior beliefs reflect different opinions about the asset

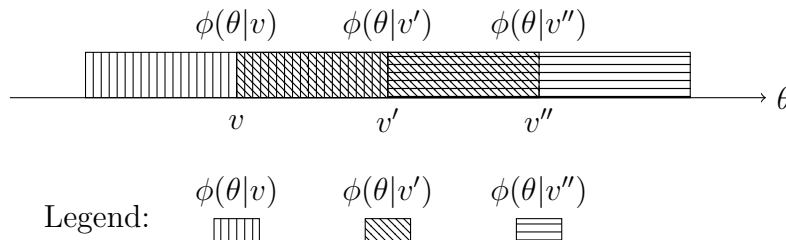


Figure 2.1: Posterior beliefs

fundamental value. A trader is not sure about her position among the population $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$, i.e., a trader does not know how many others' signals are lower or higher than hers. This is an important element in the model because the lack of common knowledge about “what all others are thinking about” prevents traders from perfectly coordinating with each other. In contrast, in the standard literature with a common posterior belief, perfect coordination rules out the existence of the bubble by backward induction.

Traders enter the market gradually and steadily due to an exogenous friction that is not modeled here. This friction is simply in order to achieve a gradual upward price path before the crash. Without this friction the bubble still exists, but the run-up becomes instantaneous: all traders rush into the market all at once at the beginning and push the price up to the peak infinitely fast, the bubble bursts and the price then plummets back. Such a friction could arise because traders need time to sell their other assets first or to wait for the maturity of other investments to buy this one,

¹¹When $\theta < \eta$, traders with $v < \frac{\eta}{2}$ have a truncated belief support because θ cannot be below zero. This causes these traders to have different strategies. As explained in Section 2.4 these traders are not important and thus we ignore these traders. When $\theta < \frac{\eta}{2}$, some traders will receive negative signals, but this is perfectly compatible with the assumption that $\theta \geq 0$.

or simply because they did not notice this asset earlier. With this friction, traders enter the market in a continuous stream. We assume that the newcomers are always confident enough about the asset value and in particular the highest type, $\theta + \frac{\eta}{2}$, is entering at every instant.¹² If we let $u(t, v)$ denote the mass of traders entering the market at time t with signal v , then $u(t, \theta + \frac{\eta}{2}) > 0, \forall t > 0$. Thus the price is smoothly increasing¹³ and there will be no ambiguity when the price stops rising at the peak when all traders cease buying. We can thus simplify the bursting of the bubble as a vertical drop right at the peak.¹⁴ But beyond that $u(t, \theta + \frac{\eta}{2}) > 0$, traders (and the behavioral agent) have no knowledge about $u(t, v)$. Thus traders anticipate a smooth and strictly increasing price path before the crash, and are unaware when some of them start to sell strategically since they do not know the shape of the path.¹⁵ This unawareness parallels and simplifies traders' uncertainty in a real market about other traders' behavior when facing a noisy price, and complex belief updating (if traders have some partial knowledge) is avoided, so that we can focus on our main targets.

A trader incurs a fixed transaction cost c each time she changes her position, irrespective of the price or the volume of the transaction. A trader's profit before tax is thus determined by her purchase prices and sale prices, minus the transaction costs.

There is a capital gains tax of rate τ , which is levied when a trader has a *realized*

¹²This guarantees that the bubble will not burst accidentally and prematurely. It is actually simpler to imagine that the newcomers always have a full support $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$, though this is not necessary.

¹³Our model allows an arbitrary price path, as long as it is continuous and strictly increasing.

¹⁴As we can observe in real markets, there are usually gradual but short downturns after the peaks but before the largest crashes. Facing noisy prices, some traders are uncertain about the coming of the final downturns within this period.

¹⁵We assume that $u(t, v)$ is large enough so that it outweighs the strategic exiting. Alternatively we can assume that there is no market entry friction but all traders finance their purchase by borrowing and there is friction on the availability of loans. The loans are available in a stream. By allowing transferring of rational sellers' repayment directly to rational buyers who are borrowing, the stream of loans can also achieve a smoothly increasing price path that is irresponsive to strategic sales.

profit (after deducting the fixed transaction cost). If the trader incurs a loss, then she is entitled to a tax credit. We summarize complex tax credit policies (such as ceilings and expirations) into a single parameter τ_c , which can vary between 0 and τ . When a trader has a realized capital loss L , she gets a refund $\tau_c L$.¹⁶ A more lenient tax credit policy, such as a longer period within which the credits can be used, corresponds to a higher τ_c . The capital loss tax credit is said to be perfect when $\tau_c = \tau$.

To rule out the possibility that some types of traders never stop buying so that the bubble grows forever, we assume that the size of the bubble has an upper bound \bar{B} . Once $p - \theta > \bar{B}$, the bubble bursts exogenously. We are only interested in the endogenous bursting, so \bar{B} is large enough.¹⁷ Lastly, a technical assumption is that all traders selling exactly at the instant when the bubble bursts receive the full pre-crash price.

Figure 2.2 shows a simple example of the dynamics. In each panel, traders' types (signals) are continuously distributed between $\theta - \frac{\eta}{2}$ and $\theta + \frac{\eta}{2}$ along the horizontal axis, given θ , while the vertical axis is the mass density for each type of traders. The shaded area is the mass of traders who hold the asset. If we assume that every buyer holds the same amount of shares (which will turn out to be the case in equilibrium), the shaded area is proportional to D_r . In panel (a) at $t = 0$, no one has entered the market yet. In panel (b), some traders have entered and bought. In panel (c), the price is high enough such that low types are no longer buying but simply hold (the shaded area of "hold" freezes and does not rise any more), while the rest are entering and buying. In panel (d), the price is even higher such that not only does "no-buy" spread to higher types of traders, low types now start to sell. The highest types,

¹⁶Constantinides (1983) models the tax credit in a similar way.

¹⁷We will show that in the equilibrium with endogenous bursting the size of the bubble size is finite and constant so that \bar{B} is never binding.

however, are still buying. In panel (f), the “no-buy” finally reaches the highest type, $\theta + \frac{\eta}{2}$, who stops buying. As a result, no one buys any more and the price stops rising and the bubble bursts. Note that in Figure 2.2, we have assumed that all traders use a trigger strategy, where a trader will not restart buying once she has stopped buying and she will never re-enter the market once she has sold. This strategy will turn out to be the equilibrium strategy.

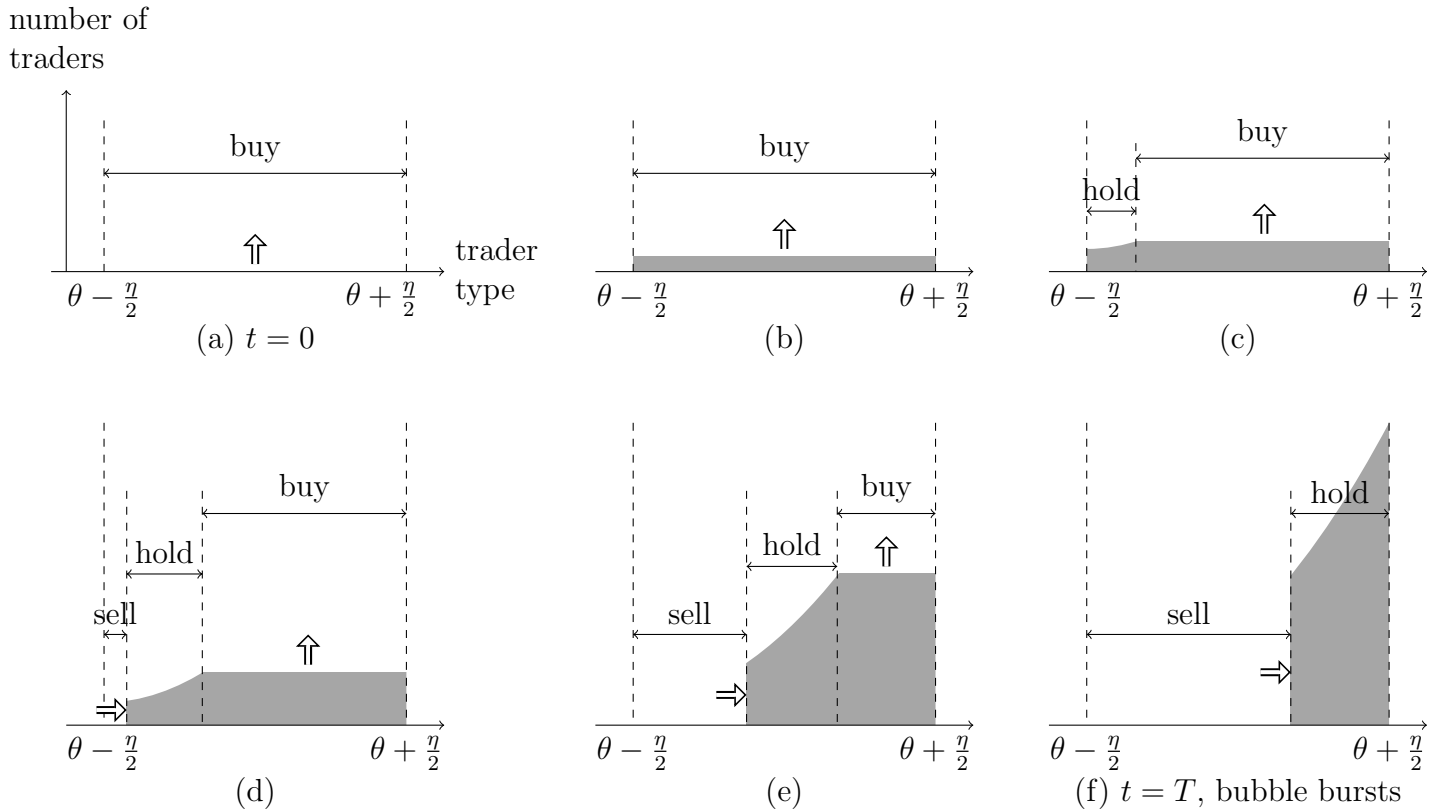


Figure 2.2: Dynamics of the mass of shareholders

2.3 Preliminary analysis

In this section, we define the equilibrium, impose two technical assumptions on traders' strategies and show that the dynamic game can be simplified to a static-like game by reducing traders' strategy space. We will establish that in equilibrium traders use trigger strategies (Proposition 2.3.1) and their decisions are strictly increasing (Lemma 2.3.1). Readers who are not interested in these details can jump to Section 2.4.

Definition 2.3.1. *A trading equilibrium is a Perfect Bayesian Nash equilibrium in which traders hold the (correct) belief: whenever a trader v is not buying (temporarily or permanently), she (correctly) believes that all traders with signal equal to or smaller than v are not buying.*

This definition imposes a natural assumption on traders' equilibrium beliefs, without which it will be difficult to characterize an equilibrium.

With the positive fixed transaction cost c , a trader will trade only a finite number of times. Due to the linearity of the problem and the cost c , it is optimal for traders to either hold the maximum long position or not hold any asset at all. At any given price, a trader's asset value is linear in her position. With the transaction cost, if buying is profitable, then it is optimal to buy to the maximum long position; conversely, if selling is profitable, then it is optimal to sell all shares¹⁸. Hence, the space of a trader's asset position in equilibrium reduces to $\{0, 1\}$.

Because we assumed a simple linear tax rate, a trader's profit or losses realized after sales (whether in the current tax year or previous years) and realized tax payments and benefits are all sunk and will not affect her future decisions. The trading

¹⁸See Appendix A and proof of Lemma 1 in AB2003 for details.

history affects her decisions only when she is currently holding the asset and whether she can sell before the crash is uncertain, i.e. her most recent purchase price may affect her selling decision. At any given price, a trader who has entered the market has three options: buy to the maximum long position (*buy*), not change her current position (*hold*), and sell all her shares (*sell*). Let $A(p, v, h, P_p)$ denote the strategy of an in-market trader v at price p with position $h \in \{0, 1\}$ and the most recent purchase price P_p . P_p is relevant only when $h = 1$. Then A is defined on $[0, \infty) \times [-\frac{\eta}{2}, \infty) \times \{0, 1\} \times [0, \infty) \rightarrow \{\textit{buy}, \textit{hold}, \textit{sell}\}$.

Definition 2.3.1 immediately implies the following corollary, which states that when trader v is not buying, then all types weakly lower than v are not buying, and when she is buying, all types weakly higher than her are still buying, irrespective of trading histories or current positions.¹⁹ This is because the belief in Definition 2.3.1 must be correct for all types of traders.

Corollary 2.3.1. $A(p, v, h, P_p) \neq \textit{buy} \implies A(p, v', h', P'_p) \neq \textit{buy}, \forall v' \leq v$ and $\forall p, h, P_p, h', P'_p$;

$A(p, v, h, P_p) = \textit{buy} \implies A(p, v', h', P'_p) = \textit{buy}, \forall v' \geq v$ and $\forall p, h, P_p, h', P'_p$.

Corollary 2.3.1 implies that traders' strategies are symmetric in the sense that traders with the same signal are either all buying or all not buying. Now we can formally define the bursting price p_T :

$$p_T = \inf\{p | A(v, p, h, P_p) \neq \textit{buy}, \forall v \in [\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}], \forall h \text{ and } \forall P_p\}$$

That is, the bubble bursts when no one wants to buy.

¹⁹For the ease of description, we allow a trader with asset position 1 to use action *buy*, though she is not able to further increase her holding due to the limit.

Let $P_b^*(v)$ denote the price at which the action of trader v is not *buy* for the first time. Let $P_s^*(v, P_p)$ denote the price at which trader v sells her shares for the first time, given her purchase price P_p (since her position is either 0 or 1 and this is the first-time sale, P_p is well defined.). By definition, $P_p < P_b^*(v) \leq P_s^*(v, P_p), \forall v$ and $\forall P_p$.

To derive an equilibrium, we need two technical assumptions:

Assumption 2.3.1. $P_b^*(v)$ is continuous in v , $\forall v$ and differentiable $\forall v > \frac{\eta}{2}$, and $P_s^*(v, P_p)$ is continuous in both v and P_p .

Lemma 2.3.1. $P_b^*(v)$ is strictly increasing in v , $\forall v \geq \frac{\eta}{2}$.

Then the inverse function $P_b^{*-1}(v), \forall v \geq \frac{\eta}{2}$, is well defined.

Assumption 2.3.2. A trader cannot switch actions more than once in one instant.

Then we have the following proposition.

Proposition 2.3.1. (*Trigger-strategy*): In equilibrium, if an in-market trader's action is not "buy", she will never restart buying again. If a trader has sold her shares, she will never return to the market.

Proposition 2.3.1 implies that when the highest type $\theta + \frac{\eta}{2}$ decides to stop buying for the first time at $P_b^*(\theta + \frac{\eta}{2})$, all other types have already done so. Since no one is buying at this moment, the bubble bursts at $p_T = P_b^*(\theta + \frac{\eta}{2})$. Let $\Theta(p_T) \equiv P_b^{*-1}(p_T) - \frac{\eta}{2}$. Then if the bubble bursts at p_T , the realized θ must be $\Theta(p_T)$. This is how everyone can perfectly infer θ from the bursting price p_T .

Proposition 2.3.1 further reduces a trader's strategy space to $\{P_b^*(v), P_s^*(v, P_p)\}$. After $t = 0$, traders keep entering the market and buying the asset. If a trader v

has bought the asset at price $P_p (< P_b^*(v))$, she holds and waits until the price rises to $P_s^*(v, P_p)$ then she sells all her shares and will never restart buying again. If she has not entered the market, then at price $P_b^*(v)$ she will no longer try to enter the market. Readers can review the price dynamics under this strategy profile in Figure 2.2 and a sketch of traders' strategies and where they end up in Figure 2.3.²⁰

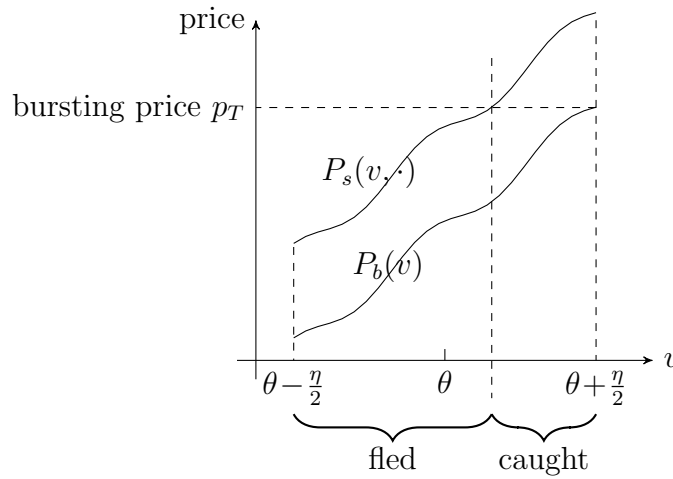


Figure 2.3: Traders' strategies and outcomes

2.4 Reduced-form game

From Lemma 2.3.1 and Proposition 2.3.1 we know that for an arbitrary θ and the signal profile generated by θ , the bursting price is fully determined by the stop-buy strategy $P_b^*(\cdot)$, if all other traders follow this strategy.²¹ Therefore, given $P_b^*(\cdot)$, a trader needs to best respond to it. This is the reduced-form game we discuss in this section.

²⁰As this is a sketch, the effect of P_p on P_s is not depicted here.

²¹Since $p_T = P_b^*(\theta + \frac{\eta}{2})$

If a trader v sells before the crash, she gets the selling price. Otherwise, she gets the post-crash price θ . Now we solve the individual trader's problem backwards, starting with her sale decision. Consider a trader v who has bought at price P_p and plans to sell at P_s . Let $\omega(P_p, P_s)$ denote her expected profit. Then, given that all other traders follow a strategy $P_b^*(\cdot)$, trader v would like to maximize $\omega(P_p, P_s)$ by choosing an optimal selling price P_s ,

$$\max_{P_s} \omega(P_p, P_s) \quad (2.2)$$

This gives her optimal selling strategy $P_s^*(v, P_p)$.

Knowing her selling plan, trader v should buy the asset until the price reaches a level of P_b such that

$$\omega(P_b, P_s^*(v, P_b)) = (1 - \tau)R = 0 \quad (2.3)$$

i.e. she should stop buying at price P_b .²² Then as long as $\omega(P_p, P_s^*(v, P_p)) > 0$ under the current price P_p , trader v should keep buying. In an equilibrium it must be that $P_b(\cdot) = P_b^*(\cdot)$.

All traders are ready to stop buying at $P_b^*(\cdot)$ if they have not bought yet. But once a trader has bought (of course below $P_b^*(\cdot)$), then she is no longer a concern to others because all that matters is the stop-buy decisions of those who have not bought yet (and are entering or waiting to enter the market). Recall that the crash is triggered by stop-buy decisions (of the highest type of traders). Call a trader v a *break-even trader* if she has not bought yet and finds that buying at the current price P_p and

²²The bubble will not burst below $P_b(v)$ for trader v according to Corollary 2.3.1. P_b is unique because in equilibrium $\omega(\cdot, \cdot)$ decreases in its first variable, and the second variable, P_s^* at its optimum, does not affect ω due to the envelope theorem.

selling optimally at $P_s^*(v, P_p)$ gives a zero expected profit so that she decides to stop buying at P_p . Only break-even traders are important to identify the bursting price and the size of the bubble. So the equilibrium is defined by equations (2.2) and (2.3). Now we focus on the strategies of break-even traders. The price at which a break-even trader stops, P_b , is just her stop-buy strategy $P_b^*(v)$. We can write a break-even trader's (planned) selling strategy as $P_s^*(v, P_b^*(v))$, though she never actually buys (so her selling strategy never gets implemented).

To better understand $\omega(P_p, P_s)$, we write it semantically as follows (not considering any tax yet).

$$\omega(P_p, P_s) = \int_{v - \frac{\eta}{2}}^{\Theta(P_s)} \left[\text{post-crash profit} \right] \phi(\theta|v) d\theta + \left[\text{pre-crash profit} \right] \times \text{Prob} \left[\text{sell before the crash} \right]$$

The first term is trader v 's *expected* post-crash profit, in which case the bubble bursts before she sells. The probability of selling before the crash is the subjective probability that $\theta > \Theta(P_s)$.²³ Since a trader can only sell at price θ after the crash, her post-crash profit in the brackets depends on θ . The second term is her expected pre-crash profit, in which case θ is higher than $\Theta(P_s)$ so that trader v will be able to sell before the crash.

To write $\omega(P_p, P_s)$ formally, let $G_{pre} = P_s - (P_p + 2c)$ denote a trader's pre-crash sale profit (before tax), and $G_{post} = \theta - (P_p + 2c)$ denote the trader's post-crash sale profit (before tax, negative in case of loss). A trader is taxed only if $G_{pre} > 0$ ²⁴

²³The upper bound of the integral is $\Theta(P_s)$ because the trader will be caught in the crash if $P_s > p_T = P_b^*(\theta + \frac{\eta}{2})$. If we inverse $P_b^*(\cdot)$ and rearrange, then equivalently, she will be caught in the crash if $\Theta(P_s) > \theta$. Intuitively, if the realized θ is smaller than $\Theta(P_s)$, then the majority of the population have signals lower than v and v is too high.

²⁴In equilibrium, pre-crash sales always give positive profits. A pre-crash sale is a scheduled sale. If a trader buys at P_p and sells at planned P_s and incurs a loss, then he will not buy at P_p .

or $G_{post} > 0$ and not taxed otherwise. She also receives a tax benefit $-\tau_c G_{post}$ if $G_{post} < 0$.

Since the purchase price P_p affects the taxability of G_{post} , $\omega(P_p, P_s)$ has two different possible forms depending on P_p . When P_p is high enough such that $\Theta(P_s) - (P_p + 2c) < 0$, G_{post} is always negative and is non-taxable.²⁵ In this case, given that all other traders follow strategy $P_b^*(\cdot)$, trader v 's expected profit is

$$NT \text{ trader: } \omega(P_p, P_s) = (1 - \tau_c) \int_{v - \frac{\eta}{2}}^{\Theta(P_s)} G_{post} \phi(\theta|v) d\theta + (1 - \tau) G_{pre} [1 - \Phi(\Theta(P_s)|v)] \quad (2.4)$$

We call a trader who faces such an expected profit an *NT* trader. When a trader's purchase price P_p is low enough such that $P_p + 2c < \Theta(P_s)$, it is possible that the post-crash sale is profitable and taxable.²⁶ In this case the expected payoff is²⁷

$$T \text{ trader: } \omega(P_p, P_s) = (1 - \tau_c) \int_{v - \frac{\eta}{2}}^{P_p + 2c} G_{post} \phi(\theta|v) d\theta + (1 - \tau) \left[\int_{\theta = P_p + 2c}^{\Theta(P_s)} G_{post} \phi(\theta|v) d\theta + G_{pre} [1 - \Phi(\Theta(P_s)|v)] \right] \quad (2.5)$$

We call a trader who faces such an expected profit a *T* trader. A trader must be either a *T* trader or an *NT* trader.

²⁵Given $P_b^*(\cdot)$, the highest G_{post} a trader can have is when $\theta = \Theta(P_s)$, i.e. the bubble bursts right before she sells. If this G_{post} is still negative, G_{post} is always negative.

²⁶This happens when $\theta > P_p + 2c$.

²⁷In the first integral the post-crash sale incurs a loss and is not taxable, and there is a tax benefit; in the second integral the post-crash sale is profitable and taxable. The third term is the expected payoff from selling before the crash, which is also taxable.

Because the bursting is triggered by break-even traders, we say that the bubble is an NT (T) bubble if break-even traders are NT (T) traders.²⁸ It turns out that both types of bubbles exist but each belongs to a different zone in the parameter space (see Figure 2.8). Since all historical bubbles we explore are NT bubbles, the parameters (τ and c) associated with the T bubble are less likely to be observed in practice and the characterization of the two types of bubbles are similar, we relegate the T bubble to Appendix A.4.

By conjecturing that $P_s^*(v, P_p)$ and $P_b^*(v)$ are linear in v , we can solve for the unique equilibrium strategies of break-even traders in an NT bubble from equations (2.2), (2.3) and (2.4), and have the following proposition:²⁹

Proposition 2.4.1. (*Equilibrium with NT bubble*) When $\frac{c}{\eta} < \frac{3-2\sqrt{2}}{2}$ and

$$\begin{cases} \tau < \tau^{TNT1} \text{ and } 0 \leq \tau_c \leq \tau \text{ or} \\ \tau > \tau^{TNT1} \text{ and } \tau_c^1 \leq \tau_c \leq \tau \end{cases},$$
 there exists a unique trading equilibrium in which the bubble size is $B = \frac{\eta}{2} + D_{NT} > 0$, the bubble bursts at $\theta + B$ and all the break-even

traders are NT traders with

$$\begin{cases} P_b^*(v) = v + D_{NT} \\ P_s^*(v, P_b^*(v)) = v + \frac{\eta}{1-\tau_c} \left[h_{NT} + \frac{2(\tau-\tau_c)}{1-2\tau+\tau_c} d_{NT} \right] \end{cases}$$

where $D_{NT} \equiv \frac{\eta}{1-\tau_c}(h_{NT}-d_{NT})$, $h_{NT} \equiv \frac{1}{2}[1-2\tau+\tau_c]-(\tau-\tau_c)\frac{2c}{\eta}$, $d_{NT} \equiv \sqrt{\frac{4c}{\eta}(1-2\tau+\tau_c)(1-\tau)}$, $\tau^{TNT1} \equiv \frac{1}{2} - \frac{\frac{4c}{\eta}}{(1-\frac{2c}{\eta})^2}$ and $\tau_c^1 \equiv 2\tau - 1 + 8(1-\tau)\frac{\frac{2c}{\eta}}{(1+\frac{2c}{\eta})^2}$.

Non-break-even traders' strategies will be characterized in Section 2.4.4. For traders with $v \leq \frac{\eta}{2}$, their strategies do not affect, but are rather determined by, strategies of those with $v > \frac{\eta}{2}$ and are more complicated while less important, so we omit the characterization.

²⁸In equilibrium all break-even traders are either all NT or all T traders.

²⁹This corresponds to an interior solution of equation (2.2).

The equilibrium strategies in an NT bubble are depicted in Figure 2.4. The two solid lines, $P_s^*(v, P_b^*(v))$ and $P_b^*(v)$, are break-even traders' strategies,³⁰ which is a special case of Figure 2.3. The selling and stop-buy strategies of all traders (break-even and non-break-even) always increase in her signal. Low-belief traders (signals in the lower range of $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$) always flee the market before the crash, while the rest are caught.

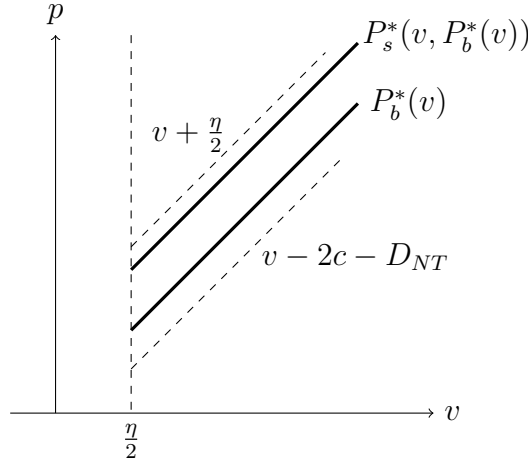


Figure 2.4: Equilibrium strategies with an NT bubble

The bubble size is $B = \frac{\eta}{2} + D_{NT}$, which is a constant and does not depend on the realization of θ . It can be verified that B increases in the belief dispersion η and the tax credit rate τ_c and decreases in the tax rate τ and the fixed transaction cost c . The first relationship, $\frac{\partial B}{\partial \eta} > 0$, is consistent with AB2003. Now we depict the latter three relationships: $\frac{\partial B}{\partial \tau_c} > 0$, $\frac{\partial B}{\partial \tau} < 0$ and $\frac{\partial B}{\partial c} < 0$ in Figure 2.5 and 2.6 (measured in normalized bubble size) and discuss their implications. In both figures, the dark (red) surface in the upper area represents NT bubbles and the light (blue) surface in the lower area represents T bubbles. In particular the two main results of our paper

³⁰Dashed lines are about non-break-even traders' strategies. See Section 2.4.4.

are related to the third and the fourth relationship and are explained in Section 2.4.2 to 2.4.3.

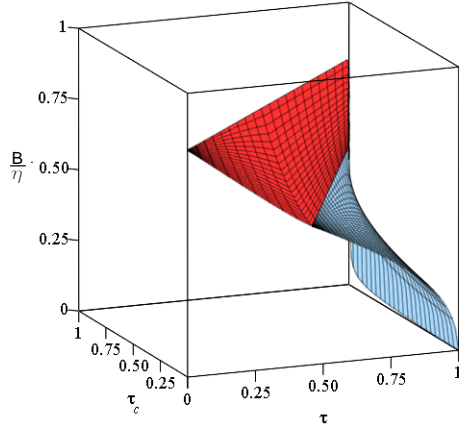


Figure 2.5: Normalized bubble size ($c = 0.01, R = 0$)

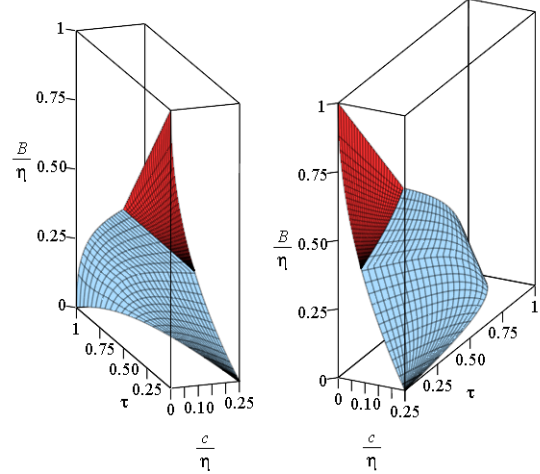


Figure 2.6: Normalized bubble size ($\tau_c = 0, R = 0$)

2.4.1 $\frac{\partial B}{\partial \tau} < 0$

The intuition of how tax affects rational traders' decisions is as follows. The capital gains tax is distortionary in that the tax is effectively imposed only on pre-crash transactions for NT traders and selling strategies are distorted downward. This can be seen from the first-order condition of (2.4) w.r.t. P_s ,

$$1 - \tau = [(1 - \tau)G_{pre} - (1 - \tau_c)G_{post}^\Theta] \frac{1}{P_b^{*'}(P_b^{*-1}(P_s))} \frac{\phi(\Theta(P_s)|v)}{1 - \Phi(\Theta(P_s)|v)} \quad (2.6)$$

where $G_{post}^\Theta \equiv \Theta(P_s) - P_p - 2c$. The first-order condition can be interpreted in terms of marginal benefit and cost. Consider a trader who wants to sell at $P_s + \delta$ instead of P_s . On the left-hand side is the marginal benefit $(1 - \tau)\delta$ (the after-tax price

appreciation). On the right-hand side is the marginal cost of being caught in the crash, which equals the profit difference³¹ multiplied by the probability or hazard rate of bursting between P_s and $P_s + \delta$. Dividing both sides by δ and letting $\delta \rightarrow 0$, we have equation (2.6). If we normalize this first-order condition by dividing $1 - \tau$ on both sides, we have $1 = [G_{pre} - \frac{1-\tau_c}{1-\tau} G_{post}^\Theta] \frac{1}{P_s^*} \frac{\phi}{1-\Phi}$. We call

$$G_{pre} - \frac{1 - \tau_c}{1 - \tau} G_{post}^\Theta \tag{2.7}$$

the normalized payoff difference between pre- and post-crash. See Figure 2.7 for the equilibrium selling strategy P_{s1}^* determined by hazard rate h_1 and normalized payoff difference $1/npd_1$ under tax rate τ_1 .³² If τ increases from τ_1 to τ_2 , the normalized payoff

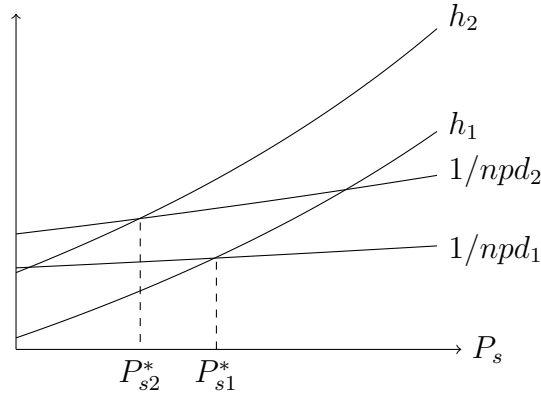


Figure 2.7: Equilibrium P_s^* determined by the payoff difference and hazard rate under different tax rates

difference tends to increase. Then a break-even trader has to lower her P_s ($G_{pre} =$

³¹The first term in the brackets is the pre-crash profit, which is always positive and taxable, and the second term is post-crash payoff, which is negative for an *NT* trader.

³²Our equilibrium is different than the endogenous equilibrium in AB2003 in that the hazard rate in AB2003 is a constant because their bubble is triggered by a fixed fraction of sale, while in our model the fraction of traders caught in the crash varies with the gap between P_s^* and P_b^* so that the hazard rate is not a constant.

$P_s - P_p - 2c$ falls) so that the first-order condition still holds and her P_s is optimal.³³ Intuitively, facing a larger payoff difference between fleeing and being caught due to a rise in the tax, break-even traders behave more cautiously and conservatively by selling early and securing their gains sooner.³⁴ See Figure 2.7 for the equilibrium selling strategy P_{s2}^* under a higher tax rate τ_2 .

The change in a break-even trader's selling decision then transmits to her stop-buy decision. Since a break-even trader's expected profit (2.4) is zero, a rise in tax, combined with a fall in P_s , will force P_p to decrease.³⁵ Intuitively, a rise in tax distorts a break-even trader's selling decision P_s downwards, and a lower P_s reduces the profit margin and in turn squeezes the stop-buy decision P_b downwards, which bursts the bubble early.

When evaluating the effects of taxes on asset prices empirically, usually two forces that move the prices in opposite directions are considered: the lock-in effect (a rise in tax causes traders to defer selling and thus tends to reduce supply and raise the price) and the capitalization effect (a rise in tax discourages buying and thus tends to reduce demand and lower the price). In our model the behavior agent is part of the supply side and does not respond to the tax incidence and thus there is no lock-in effect on them. However, the absence of lock-in effect from the behavioral agent is not why the tax can deflate the bubble in our model. The driving force of asset bubbles is rational traders in our model as well as in the real world (as demonstrated by Griffin

³³If we do not take into account the equilibrium response of $P_b^*(\cdot)$, then clearly a rise in τ increases the normalized payoff difference. Recall that $G_{post}^\Theta < 0$. In equilibrium, while a break-even trader's P_s indeed decreases, the final effects of a rise in τ are that the hazard rate rises while the normalized payoff difference decreases and their product remains unity.

³⁴This does not conflict with the lock-in effect in that the lock-in effect applies to traders who have already bought with a given purchase cost while break-even traders are those who have not bought whose stop-buy strategies vary with the tax.

³⁵In addition, the probability of being caught is also higher in equilibrium.

et al. (2011) and Brunnermeier and Nagel (2004)). It is their responses to the tax incidence that deflate bubbles. In particular, that they stop buying early when facing a higher tax bursts the bubble prematurely.³⁶

2.4.2 $\frac{\partial B}{\partial \tau_c} > 0$ and the ineffectiveness of the tax under perfect tax credit

The tax credit rate, τ_c , works in the opposite direction to the tax rate τ and serves as a compensation to the loss from being caught in the crash. In particular, the post-crash loss G_{post} in the first-order condition (2.6) is scaled by a factor $1 - \tau_c$. So the tax credit is a “tax” on G_{post} , which reduces the loss. If τ_c increases, the normalized payoff difference decreases. With a smaller payoff difference, traders care less about being caught and behave more aggressively by selling at higher prices. A higher P_s then enlarges the profit margin and encourages buying at higher prices, which allows the bubble to grow larger. This suggests that weakening tax credits has an effect similar to raising the tax rate in deflating bubbles. This result can be useful when raising the tax is difficult or infeasible for economic or political reasons.

A special case is that the tax credit is perfect, i.e. $\tau_c = \tau$. In this case the equilibrium is simplified to $P_b^*(v) = v + \frac{\eta}{2} - \sqrt{4c\eta}$ and $P_s^*(v) = v + \frac{\eta}{2}$, with a bubble $B = \eta - \sqrt{4c\eta}$.³⁷ Notice that now the bubble stays at its tax-free level and is unaffected by the capital gains tax, even if we impose a 100% capital gains tax! This is because the first-order condition becomes $1 - \tau = \left[(1 - \tau)G_{pre} - (1 - \tau)G_{post} \right] \frac{1}{P_b^*} \frac{\phi}{1 - \Phi}$,

³⁶If we allow the behavioral agent to respond to a tax rise by deferring or even suspending selling and if we allow the price to rise, the price will still be driven to the same level at the peak of the bubble and then crashes, because the crash is triggered by rational traders and their stop-buy strategies are not affected by the behavioral agent.

³⁷The parameter requirement is also simplified to $\frac{2c}{\eta} < \frac{1}{2}$. The difference between NT and T traders disappears, since equation (2.4) is equivalent to (2.5) when $\tau_c = \tau$. Break-even and non-break-even traders both sell at $P_s^*(v) = v + \frac{\eta}{2}$.

which, after normalization, is exactly the same as without tax. Hence the perfect tax credit provides a perfect compensation to the loss in crash, reduces the payoff difference between pre- and post-crash and restores it to its tax-free level. It induces high selling prices and encourages aggressively high purchase prices. Traders behave as if there is no tax and the tax credit entirely neutralizes the deflating effect of the tax. In Figure 2.5 this corresponds to the horizontal top edge of the dark (red) surface, where $\tau_c = \tau$ and the bubble is not responsive to the increasing tax rate.

A 28% capital gains tax was introduced in the United Kingdom on April 6, 2015, on gains from residential property by overseas investors that, most analysts believe, targets the potential housing bubble in London that has been growing since 2009. To take the advantage of this policy change to its full extent, our paper suggests that a tax authority should examine its policies on tax credit before imposing or increasing the tax rate, because an overly favorable tax credit can entirely offset the deflating effects of the tax and leave the bubble unaffected, no matter how much the tax rate is raised.³⁸ Measures to limit the tax credits include reducing the tax credit rate τ_c , setting or tightening the dollar limit on the tax credit and shortening the duration within which tax credits can be carried forward or backward.

³⁸One clarification is that this suggestion should be viewed as an ad hoc tax policy instead of a long-run, regular optimal tax policy. For those who believe that capital gains taxes should be calculated on the basis of long-run average (net) capital gains, weakening the tax credit may seem unfair because, without the tax credit, investors are taxed whenever they have gains and are left alone when they incur losses, and hence the tax is not based on long-run average gains. So our suggestion is to apply this policy only when there is strong concern that a bubble may be in progress in a specific market. In this paper we do not intend to find an optimal tax policy that can automatically deflate bubbles when they arise and otherwise does not interfere with the market.

2.4.3 $\frac{\partial B}{\partial c} < 0$ and the large marginal effect of a small c

An increase in c directly erodes the profit margin between P_s and P_b and a break-even trader has to lower P_b to make a profit.³⁹ This lowered stop-buy strategy therefore deflates the bubble.

One implication arising from our model is that the transaction cost can have an arbitrarily large marginal effect on the bubble when the cost approaches zero: $\lim_{c \rightarrow 0} \frac{\partial B}{\partial c} = -\infty$ (see Figure 2.6 View 2). Notice that this result does not depend on τ or τ_c . In addition, it does not even depend on the assumptions about belief and signal distributions in our model. The reason is as follows. When $c = 0$, the selling price and stop-buy price of a break-even trader coincide, i.e. $P_b^*(v) = P_s^*(v, P_b^*(v))$. In this case a break-even trader does not need a margin at all between her P_s and P_b .⁴⁰ When c increases slightly from zero, $2c$ is the minimal gap between purchase and selling prices for a break-even trader, if no risk of crash is considered. But since it is always possible that the bubble bursts in between this gap, a trader needs a larger profit margin/gap to compensate for the risk. This leads to a lower P_b . A larger gap means a higher probability of bursting in between, which requires an even larger gap. Hence the stop-buy strategy is lowered dramatically by a small transaction cost, which deflates the bubble significantly.⁴¹

The large marginal effect of the transaction cost on bubbles justifies the practice of implementing financial transaction taxes at very low rates. Although we use a fixed

³⁹Under an imperfect tax credit, selling price P_s actually rises in response to a rise in c . This can be seen from the fact that the change in normalized payoff difference (2.7) will be widened since $\frac{1-\tau_c}{1-\tau} > 1$. The net effect of c on P_b is nevertheless negative. Under a perfect tax credit P_s is invariant in c .

⁴⁰With a zero profit margin, the tax has no effect.

⁴¹But P_b will not be shifted downwards infinitely, since the bubble size is also decreasing as $P_b^*(\cdot)$ decreases and the loss of being caught in the crash diminishes. There is a stop-buy price where the expected profit equals zero. So this marginal effect decreases in c .

transaction cost instead of a proportional tax, every trader holds the same amount of the asset in our model and the effect of a fixed cost is quite close to that of a proportional transaction tax. Since the 1970s many countries have experimented with financial transaction taxes on trades of shares, bonds, currencies and derivatives. As mentioned earlier, empirical evidences show that these taxes tend to reduce trading volumes but not price volatilities, and some countries have decided to abolish the tax. Other countries⁴² nevertheless still maintain these taxes at low rates. For example, the United States currently levies a 0.0034% tax on stock transactions, the United Kingdom levies a 0.5% tax on share purchases and Japan has a 0.1 – 0.3% tax on stock transactions, and these countries are among those which have experienced the most notorious bubbles in history.

2.4.4 Non-break-even traders

In fact, all traders who have already bought the asset are non-break-even traders. An NT bubble actually includes both NT and T traders: those whose purchase prices are not too low are NT traders, and those who entered the market very early and bought the asset at very low prices (so it would be possible to make a profit even if caught in the crash) are T traders. $v - 2c - D_{NT}$ is the dividing line between the purchase prices of T and NT traders. Non-break-even traders' selling strategies are

$$P_s^*(v, P_p) = \begin{cases} \frac{1}{1-2\tau+\tau_c} \left[(1-\tau)v - (\tau-\tau_c)P_p + \frac{\eta}{1-\tau_c} [(1-\tau)h_{NT} + (\tau-\tau_c)d_{NT}] \right], & \text{if } P_p > v - 2c - D_{NT} \text{ (} NT \text{ trader)} \\ v + \frac{\eta}{2}, & \text{if } P_p < v - 2c - D_{NT} \text{ (} T \text{ trader)} \end{cases} \quad (2.8)$$

⁴²There were about 40 countries in 2011 that imposed financial transaction taxes.

Non-break-even NT traders' selling prices span between $v + \frac{\eta}{2}$ and $P_s^*(v, P_b^*(v))$, while all T traders sell exactly at $v + \frac{\eta}{2}$. See Figure 2.4.

2.4.5 NT bubble, T bubble and no bubble

The NT bubble emerges when τ_c is high while τ and $\frac{c}{\eta}$ are low. When τ_c , τ and $\frac{c}{\eta}$ are moderate, the unique equilibrium involves a T bubble.⁴³ There also exists a unique trading equilibrium without a bubble when τ_c is low while τ and $\frac{c}{\eta}$ are high. See Appendix A.4 and A.5, respectively. Figure 2.8 depicts the parameter space partitioned by the existence and type of bubble. The space of NT bubble is enclosed by dark (red) surfaces and the space of T bubble between light (blue) and dark (red) surfaces. Outside light (blue) surfaces no bubble exists.

2.4.6 The outside option R and the interest rate policy

The effect of R is similar to that of c . When $R > 0$, the main change to Proposition 2.4.1 is that $d_{NT} \equiv \sqrt{\frac{4c+2R}{\eta}(1-2\tau+\tau_c)(1-\tau)}$. See other changes in Appendix A.6.

In an NT bubble, it can be verified that the size of the bubble also decreases in the outside option, i.e. $\frac{\partial B}{\partial R} < 0$. This result means that high returns from outside investment opportunities help deflate a bubble. Conversely a deterioration of these returns reduce the opportunity costs of riding the bubble and thus can induce a larger bubble. The potential housing bubbles that we have observed since 2009 in major cities outside the United States are partially the result of arbitrageurs switching their

⁴³It may feel counterintuitive that when τ and c are very low break-even traders always incur losses, while when τ and c are moderate break-even traders can possibly make a profit. This is because, when τ and c are very low, only a small gap between buy and sale prices is required for break-even and the stop-buy price is close to the selling price. With such small profit margin, however, a break-even trader must incur a loss if caught in the crash, even if θ turns out to be high ($\theta = v + \frac{\eta}{2}$), which makes her an NT trader.

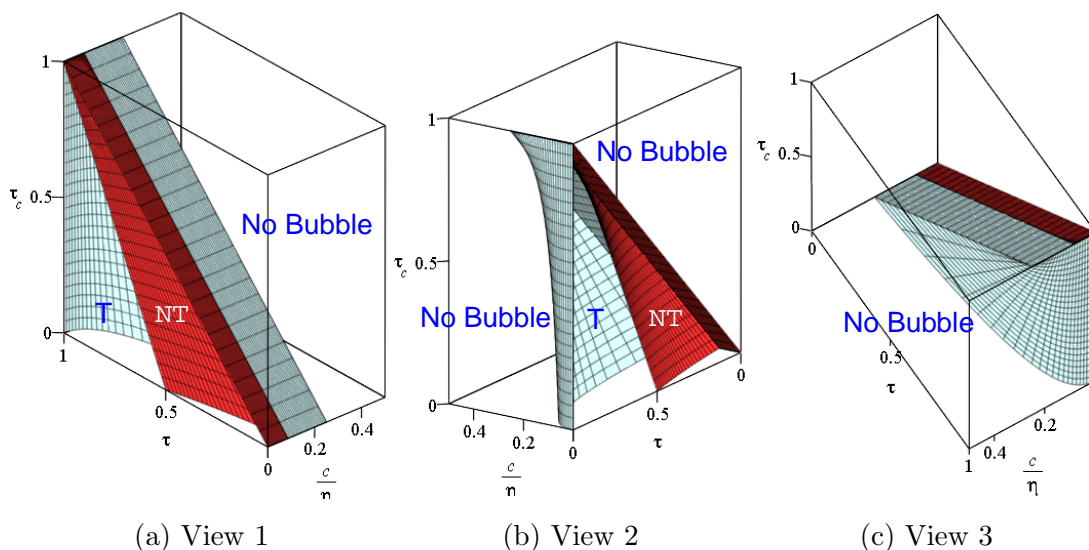


Figure 2.8: In the $\frac{c}{n}$ - τ - τ_c space ($R = 0$)

investments from US real estate markets plagued by the subprime mortgage crisis.

Similar to the transaction cost c , the outside option R also has a large marginal effect on the bubble when R and c are small: $\lim_{R \rightarrow 0, c \rightarrow 0} \frac{\partial B}{\partial R} = -\infty$, though this marginal effect decreases in both R and c . If we interpret R as an interest rate (on treasury bill, for instance), then when a central bank sets a very low interest rate, this could inflate bubbles significantly in certain markets, as suggested by the Federal Reserve's low interest rate policy after 2001 and the rise of the housing bubble in the United States until 2008.⁴⁴ Thus central banks often face a dilemma: low interest rates help fight unemployment and recessions (in the case of the United States, it was the recession after the tech bubble), but also sow the seeds for the next round of recession. The bank of Canada lowered the rates after the 2008 financial crisis and has maintained an overnight rate at 1% since 2010. In January 2015, the Bank surprisingly further

⁴⁴Although there is no agreement that Federal Reserve's low interest rate policy during 2001-2005 indeed caused the housing bubble that burst in 2008, the timing of the housing price run-up and bursting roughly coincide with the declining and rising of the federal funds rate.

lowered the rate to 0.75%, partially to fight the adverse effect of the oil price plunge in 2014. Our model warns that further lowering the interest rate, when it is at an already very low level, will have a disproportionately inflating effect on housing prices. This could be especially dangerous when there is strong suspicion that a large housing bubble already exists in Canada at this time.

2.5 Discussion: downward price overshooting in recession

To some extent, a recession can be regarded as the reverse process of the rise of a bubble. When an asset's fundamental value deteriorates, observing that price is decreasing, rational investors will not buy the asset until they believe that the price has touched the bottom⁴⁵. By doing so, they collectively overact to the recession and there will be a downward price overshooting, which is the reverse of the bubble. This may help explain why some recessions have had a surprisingly huge impact and last for decades.

Consider a model that is same as the bubble model, with the following differences. At $t = 0$, the asset's fundamental value jumps down from θ_0 to an unobservable value θ . θ has a distribution over $[\underline{\theta}, \theta_0]$ with density $\phi(\theta)$. The two types of agents are the same as those in the bubble model, and each rational trader receives the same signal v , which has a continuous distribution over a finite support. At $t = 0$, all of the asset is held by rational traders and initial price $p(0) = \theta_0$. When the fundamental of the asset jumps down, all traders want to sell the asset. But since the passive agent has a limit absorbing rate, at each instant/price only some of the traders successfully sell the asset. As the sale goes on, the price is continuously depressed. Observing that

⁴⁵Assume that the transaction cost is small and traders' signals about the deteriorated asset fundamental value, or economy as a whole, are sufficiently dispersed

the price keeps falling, a trader has incentive to sell the asset as soon as possible while the price is still high; let the asset depreciate further; and buy back at lower price, ideally at the bottom. Although traders' positions are still restricted in $[0, 1]$, they have their own funding and do not rely on loans to buy back. That is, they have to wait to sell their shares, but they can buy at any price and are not subject to loan availability. When the price is low enough, a trader v believes it is the right price and then buys back. Because of the transaction cost, at an earlier moment, a trader with same signal v who has not been able to sell the asset will no longer seek to sell. When the price is so low that no traders want to sell any more, the price stops decreasing. The fundamental value θ is thus perfectly revealed by the fact of no sale, and the price jumps up to θ .

With this process, it is possible to have V-shaped price dynamics, where the price falls to a low level that is lower than θ , and then bounces back to θ . This arises from the fact that when the price is lower than θ , some low types of traders are still selling due to their uncertainty about θ . Also, when the price is lower than θ , some low types of traders who already sold, are reluctant to buy back and are still waiting for a even lower price, which aggravates the recession and postpones the possible recovery.

If we allow $\underline{\theta}$ (the lower bound of support of θ) to be $-\infty$, and signal support $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$, and hence for the price to be negative, then the analysis in the bubble model goes through in this recession model, and we have a symmetric result.

2.6 Empirical discussion

Our model also generates other testable results. First, the model suggests that the average margin between traders' buying and selling prices are positively correlated

with $\frac{c}{\eta}$, $\frac{R}{\eta}$ and τ and negatively with τ_c . These correlations may be tested by comparing average margins before and after policy changes affecting brokers' commission rate, bid-ask spread, interest rate, tax rates and capital loss carryovers, or may be tested across different bubble events.

Second, our model indicates that traders' selling prices are negatively correlated with their purchase prices, i.e. the earlier a trader bought, the later she sells. This can be seen from (2.8) where a non-break-even NT traders' selling strategy P_s^* decreases in her purchase price P_p .⁴⁶ While an NT bubble also involves T traders, since their purchase prices are even lower and selling prices are the highest (uncorrelated with their purchase prices), the overall correlation among the population is negative. In addition, across different bubble events, the higher the tax rate (and the weaker the tax credit policy), the stronger this correlation will be.

2.7 Conclusion

In this paper we study the effects of capital gains tax and the transaction cost on asset bubbles. Our model incorporates purchases into the framework of Abreu and Brunnermeier (2003). In the unique equilibrium, we find that the capital gains tax helps deflate bubbles but the capital loss credit tends to offset this deflating effect. Under a perfect tax credit, the bubble becomes immune to the tax. Therefore dealing with bubbles with the capital gains tax also requires tightening the policies on tax

⁴⁶This is because the marginal purchase cost of the pre- and post-crash sales in the first-order condition are scaled by different factors: when purchase cost $P_p + 2c$ increases by δ , the net purchase cost in the pre-crash sale increases by $(1 - \tau)\delta$, whereas the purchase cost of the post-crash sale increases by $(1 - \tau_c)\delta$ due to the tax credit, with the latter larger than the former. This implies that the tax lowers the pre-crash purchase cost relative to the post-crash cost and makes the former relatively more attractive. In particular the break-even traders who have the highest purchase prices (if they have purchased) sell the lowest (earliest) at $P_s^*(v, P_b^*(v))$.

credit. We also find that a small transaction cost and outside option have very large marginal deflating effects on bubbles. This implies that the low interest rate policies in the United States between 2001 and 2008 may have contributed considerably to the housing bubble, and that the Bank of Canada's recent move to a lower interest rate from 1% to 0.75% may induce a larger housing bubble in Canada. The model also has an appealing dynamic in which the stop-buy decision and selling decision spread smoothly and continuously from low-belief traders to high-belief traders and high types are still buying when low types have already sold.

To demonstrate how to empirically test these results, we explore several historical bubbles and show that we can use the belief dispersion to normalize bubbles. By this normalization, we eliminate the fundamental variation in bubbles so that we can compare the effects of policy variables. We also show how to infer belief dispersions from the actual index/price histories when there is no explicit data on beliefs. It would be desirable to include more bubble events into the data set so that empirical tests could generate statistically significant results on the effects of the capital gains tax, tax credit, interest rates and transaction costs.

Our model can be extended, for example, by interpreting the compensation factor, τ_c , as government bailout to show that past government assistance may increase the expectation for future bailout and hence induce larger bubbles.

Chapter 3

Contagion of Liquidation on the Asset-Trader Network

3.1 Introduction

When the distressed financial institutions face bankruptcy, margin calls or regulatory constraints, they may have to sell their assets quickly. A large volume of sales in a short period of time may depress the price. This downward price pressure can adversely affect other shareholders, cause more bankruptcies and induce a further round of liquidations in other asset markets. In modern financial systems, institutions are interlinked by overlapping portfolios across assets and the liquidations and rapid price declines may become contagious and spread extensively across markets, as demonstrated by the recent crises. In the United States during the financial crisis of 2008, when the housing bubble burst and the house prices declined, home buyers started to default. Various mortgage-backed securities held by institutional and private investors also became essentially worthless. These distressed investors (e.g. Bear Stearns) were forced to liquidate other assets in their portfolios. The Dow Jones Industrial Average declined by 18% as of October 10, 2008. The price effects were

felt almost instantly by Europe. In the same week, the FTSE100 declined by 20%. In other financial crises, such as the 1987 stock markets crash, this knock-on price effect also played an important role in transmitting the distress.

Modern financial systems have become much more complex and interconnected over the past decades. This increasing complexity of interdependence makes the system more susceptible to systemic collapse and less transparent for investors and policy makers to assess the consequences. The complexity and the fragility of the financial system have fostered a fast growing literature that studies the contagion over financial systems using techniques developed in physics and biology. This literature models the interconnected agents and institutions as networks. The existing literature on financial networks mostly focuses on the contagion via the channel of direct credit exposures/interbank loans, and very few treat the asset price and the price effect as the primary cause of a contagion.

In this paper we explore how a shock to an individual investor can give rise to a systemic crisis of a contagion of liquidations through the price effect and examine how a greater complexity affects the probability and the extent of the contagion and the declines in asset prices.

The most well know study of contagion on a simple financial network is Allen and Gale (2000). With four banks linked by interbank loans they show that the contagion depends crucially on the connecting pattern of the banks - when banks only have exposures to a few others, the counterparty risk is not well diversified and the system is vulnerable; when every bank has exposures to all others, the risk is diversified and contagion is less likely. Although the insights from simple network structures are seminal, their applicability to the real world financial systems is doubtful. As indicated

by Cifuentes et al. (2005), in a more complicated network, there is a non-monotonic relationship between the connectivity and the financial stability: the contagion is limited when the network is sparse or very well connected, but is extensive when the network are moderately connected.

Gai and Kapadia (2010) study more complex networks. The banks linked by inter-bank loans are modeled as random graph-based networks,¹ which can accommodate arbitrary and complex networks. They introduce the techniques of generating function from the literature on complex networks (Strogatz (2001)) and derive a condition for *phase transition*. The phase transition is a threshold for the connectivity, above which a system-wide contagion becomes possible. In addition to the non-monotonic relationship between the network connectivity and the probability of contagion, they also find a robust-yet-fragile tendency of the financial networks: when portfolios are relatively well diversified, the probability of contagion may be low, but once it happens, it can be extremely widespread. While both Cifuentes et al. (2005) and Gai and Kapadia (2010) incorporate the asset price effect, there is only one generic asset in their models and the contagion still spreads through credit channel per se - without default, there will be no contagion.

We construct a theoretical model to investigate the contagion of bankruptcy through the asset price effect and identify the relationship between the portfolio diversification and the probability of contagion. In particular, the financial system is modeled as a bipartite random network in which assets and traders are explicitly represented by two groups of nodes. We call this type of networks the asset-trader networks. A simple asset-trader network is shown in Figure 3.1. This is in contrast

¹A random network is a graph generated by some random process. In particular, the number of links each node has is determined by a given probability distribution, and who is connected to who is also determined by the random process that implement this distribution.

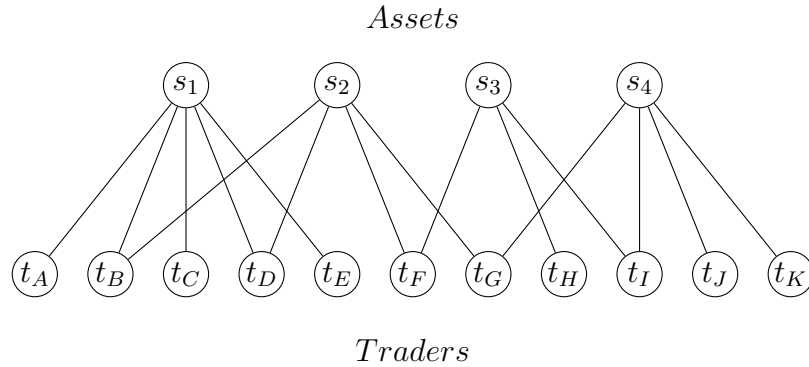


Figure 3.1: An assets-traders network

to most previous models on financial networks, where the networks do not involve assets, only agents or banks. This representation allows us to clearly identify the group of shareholders affected when the price of a specific asset falls and the group of assets affected when a specific investor liquidates her portfolio. We can also calculate the price of an asset as a function of the number of the shareholders and calculate the market-wide price declines in an extensive contagion. To highlight the price effect, we abstract away the direct credit exposures between two traders.

Unlike the generic contagion models such as Watts (2002), we explicitly define the balance sheet of an investor and specify her solvency condition. Following Gai and Kapadia (2010), we use the techniques introduced from the literature on complex networks to derive the condition under which the contagion of liquidation becomes widespread. Gai and Kapadia (2010) conjecture a non-monotonic relationship between the extent of the contagion and the network connectivity from their results, they do not derive a solution for this relationship. We use the method introduced in Newman et al. (2001) to explicitly characterize the size of the contagion and show a non-monotonic relationship between the size and the network connectivity. As the average portfolio becomes more diversified and the network become better connected,

the size of the contagion first increases and then decreases. The intuition is that when there are only a few links in the network, the contagion is contained by the limited connectivity; when the network gets better connected but asset ownerships are not well diversified, one shareholder's liquidation has a significant price impact on others and bankruptcies can easily spread from the shareholders of one asset to those of another; but when the network is very well connected, the price impact diminishes and eventually the contagion disappears. The results from the simulations confirm this non-monotonic relationship and also show a robust-yet-fragile tendency similar to that in Gai and Kapadia (2010). With our explicit result we find that the robust-yet-fragile tendency is caused by the repeated, multiple rounds of adverse price impacts to investors that arrive at different times from different routes rather than the first wave alone, which is in turn rooted in the existence of closed loops in the network. From a policy perspective, this implies that early government interventions and bailouts may be crucial in containing the crisis before it cascades system-wide.

I then discuss the effects of front running (predatory trading, see Brunnermeier and Pedersen (2005a) and Carlin et al. (2007a)) on the contagion of liquidation. Based on a simple model, the simulation results suggest that with front running, both the probability of contagion and the extent of the contagion are larger than the benchmark model.

Besides Allen and Gale (2000), Cifuentes et al. (2005) and Gai and Kapadia (2010), our paper is related to a growing literature that studies the contagion of credit risks on banking networks. May and Arinaminpathy (2010) also model the complex banking network with banks holding several different assets and use mean-field approximations to provide intuitive explanations. However, their assets do not play a crucial role in

the contagion. Gleeson et al. (2011) provide a mean-field approximation method by which they can numerically calculate the extent of contagion without Monte Carlo simulations. Their results are fairly accurate compared with simulations and their methods are complementary to ours. Geertsema (2014) constructs a model of fire sale of assets with overlapping portfolios and uses an approximation method to calculate the final equilibrium price after the downward price spirals triggered by an initial shock. Though not explicitly represented as a network, his model is close to ours but he focus on the equilibrium price while we focus on the number of failed investors.

The shareholdings and the portfolios in our model form randomly and exogenously, and we do not address the issue of endogenous network formation in equilibrium or the design of optimal network topology.² Given that there is very limited empirical evidence on the shareholding structures of stock markets and the lack of established theory of optimal portfolio design against the contagion of liquidation via price effects, it does not seem necessary to restrict our attention to specific network structures. In addition, our network structures are entirely random and thus accommodate all possible network structures. This implies that the results of our model are compatible with all endogenously formed or optimally designed networks.

The rest of the paper are organized as follows: in Section 3.2.1 we introduce the model and the contagion process. In Section 3.2.2 we explain how to characterize the contagion process by using the techniques of generating functions. Section 3.2.4 and 3.2.3 characterize the extent of a contagion with and without a systemic cascading,

²See Allen and Babus (2008) for a survey on this topic. Leitner (2005) studies a model where agents form a network and linkages can cascade liquidations but also serve as an insurance, because agents may be willing to bail out other agents to prevent the collapse of the whole network. Acemoglu et al. (2014) study a model with exogenous network structures but endogenous investment level on an underlying asset. They characterize the equilibrium and socially optimal investment levels as a function of the structure of the network

respectively, and explain why the extent of a contagion is non-monotonic in the network connectivity. In Section 3.2.5 we calculate the probability of a contagion. In Section 3.3 we simulate the contagion process to obtain the probability and the extent of the contagion as well as the asset price decline. We compare the simulated results with analytical results and explain why the robust-yet-fragile phenomenon arises. In Section 3.4, we discuss the effects of the front running on contagion. A final section concludes.

3.2 The Model

3.2.1 Setting

The complex financial system is represented by a network with S assets and T traders. Each trader is linked to a random number of assets, meaning that those assets are in this trader's portfolio. Each asset is linked to a random number of traders who are its shareholders. These assets and traders are, in the language of graph theory, *nodes* in a network and the exposure of a trader to an asset is the *link* in between them. Links are undirected. Since we are primarily interested in financial impact through the mark-to-market price effect, we ignore the direct credit exposure between traders and therefore there is no direct linkage between any two traders. There is no direct linkage between any two assets. See Figure 3.1 for a simple example of asset-trader network.

In the network literature, a network as described above is called a bipartite network. The number of links of a node is called its *degree*. In our asset-trader network, an asset s 's degree, denoted d_s , is the number of the shareholders of this asset, and

a trader t 's degree, denoted d_t , is the number of assets in her portfolio. The degrees of assets and traders are random numbers and are governed by their respective degree distributions. Let $\{p_i\}$, $i = 0, 1, 2, \dots$ be the assets' degree distribution and $\{q_j\}$, $j = 0, 1, 2, \dots$ the traders' degree distribution, which are exogenously given. The network here is a random network and include all possible realizations of networks under the two degree distributions. The two distributions are arbitrary, though we will assume specific forms for them in the numerical simulations.

All assets in the network are illiquid and traders in the network are large shareholders in the sense that each trader's liquidation can affect the price and bring down the asset price considerably. In a reduced form, the price of an asset s is a strictly increasing function:

$$\rho(x_s) \tag{3.1}$$

where x_s is the fraction of shares held by traders in the network. When a trader in the network liquidates, her shares are absorbed by behavioral passive investors outside the network and x_s drops. These behavioral investors outside the network are not in our primary interest and only passively absorb in-network liquidations.³ $\rho(\cdot)$ captures the selling pressure that drives the asset price down. In the rest of this paper we will only discuss traders in the network and ignore the behavioral investors. To simplify

³The behavioral investors outside the network can be interpreted as follows: for the long-term fundamental investors to be willing to buy the liquidated asset, they must be compensated by lower prices. Or their behavior can be justified by an adverse selection problem. When traders have private information about the asset fundamental value and keep buying the asset, it is natural for the uninformed passive agent to respond by raising the price continuously, as in Kyle (1985). Similar behavioral asset supplies have also been adopted by De Long et al. (1990), where passive investors supply the asset at an increasing price when rational speculators are buying, and by Brunnermeier and Pedersen (2005b) and Carlin et al. (2007b), where long-term traders buy when strategic traders' liquidation pushes down the price and sell when strategic traders' buy-back pushes up the price.

the contagion process, we assume that traders do not actively or strategically change their asset positions and in particular, they do not purchase, only sell. They sell only when their losses from the asset prices declines exceed their capital buffers so that they are bankrupt (traders are leveraged). Short sale is not allowed.

A trader t 's total asset, A_t , is the market value of her portfolio in the network. When asset prices change, A_t changes accordingly. Her total liability is her fixed, exogenous debt D_t , plus capital K_t . The trader remains solvent when $A_t \geq D_t$. When K_t is exhausted by the declines in asset prices, i.e. $A_t < D_t$, the trader becomes insolvent. An insolvent shareholder has to exit the market by liquidating all her assets and is not allowed to re-enter the market.

Let B_s denote the total supply of asset s in the number of shares, and $x_{s,t}$ the fraction of shares held by trader t , with $\sum_t x_{s,t} = x_s$.⁴ Hence trader t holds $x_{s,t}B_s$ shares of asset s , which does not change unless she is forced to liquidate. When another shareholder t' liquidates her shares, the in-network fraction of shares drops by $x_{s,t'}$ and the price drops from $\rho(x_s)$ to $\rho(x_s - x_{s,t'})$. Trader t will suffer a mark-to-market loss $L_{s,t,t'} \equiv x_{s,t}B_s [\rho(x_s) - \rho(x_s - x_{s,t'})]$. Trader t will become insolvent upon the first liquidation of asset s made by any other trader t' if $L_{s,t,t'} > K_t$.

In what follows, we assume that the number of shares held in the network, $x_s B_s$, is evenly distributed over each link of asset s . Each shareholder of asset s thus has $\frac{x_s B_s}{d_s}$ shares, where d_s is the initial asset degree and x_s is the initial in-network fraction of shares before anyone liquidates. In addition we assume that x_s and B_s are identical across all assets and the capital buffer are identical across all traders.⁵

⁴If trader t does not hold asset s , then $x_{s,t} = 0$.

⁵Alternatively, we can assume x_s , B_s and K_t random variables, so that assets and traders becomes heterogenous. But we will lose the tractability of the model and the analytical results will not be available.

Although these assumptions are stylized, they provide a benchmark to demonstrate that contagion can happen even when assets holding is diversified. They also simplify the characterization of the contagion. In Section 3.2.6 we will discuss the implications of relaxing these assumptions.

Under the above assumptions if we drop some of the subscripts such that each shareholder of asset s initially has $\frac{xB}{d_s}$ shares and a shareholder's mark-to-market loss upon the first liquidation of s (by another shareholder), $L(d_s)$, is

$$L(d_s) \equiv \frac{xB}{d_s} \left[\rho(x) - \rho\left(x - \frac{x}{d_s}\right) \right] \quad (3.2)$$

Recall that the solvency condition is $L(d_s) \leq K$. Since $\rho(\cdot)$ is a strictly increasing function, $L(d_s)$ is strictly decreasing in d_s and the equation $L(d_s) = K$ has exactly one solution. Denote this unique solution d^* . If an asset's degree d_s is greater than or equal to d^* , we have $L(d_s) \leq K$. In this case if one of the shareholder is insolvent and liquidates, the price decline will be small and all other shareholders will survive this first round of impact. Conversely, if an asset's degree is smaller than d^* , we have $L(d_s) > K$. In this case, if any of the shareholders liquidates, the mark-to-market loss for the rest of the shareholders will exceed their capital buffers, and all of them will go bankrupt and be forced to liquidate all their portfolios. In this sense, we define the vulnerability of an asset:

Definition 3.2.1. *An asset s is vulnerable if its degree $d_s < d^*$.*

At this time we assume that there is no closed loop or cycle in the network, so that each trader will be hit by the contagion of liquidation at most once.⁶ If a trader

⁶The use of probability generating functions requires a tree-like network, i.e. there cannot be any loop in the network.

survives the first wave of contagion, she remains alive till the contagion ends. If loops are allowed, traders will suffer multiple hits and this will make a systemic contagion more likely and more extensively. This restriction will be relaxed later and we will discuss its implications and compare the simulation results under both the single hit (no loop) assumption and the multiple hits (loops allowed) assumption.

The links of a vulnerable asset are said to be *infectious* (excluding the incoming link that triggered the liquidations). We assume that $d^* \geq 2$. One particular simple form of $\rho(\cdot)$ is $\rho(x) = \gamma x$, where $\gamma > 0$ is a constant. The price is then linear in x . When a shareholder liquidates, the price drops from γx to $\gamma(x - \frac{x}{d_s})$ and the market-to-market loss of each remaining shareholder is $\frac{xB}{d_s}\gamma\frac{x}{d_s}$. If this loss is greater than the capital buffer K , then this asset is vulnerable. So $d^* = x\sqrt{\gamma BK}$ in this example and the assumption that $d^* \geq 2$ requires that $x\sqrt{\gamma BK} \geq 2$. We will adopt this simple linear form of $\rho(\cdot)$ in simulations. Define $V \equiv x^2 B \gamma$ and V is then the initial market value of the shares of an asset held in the network under the linear form of $\rho(\cdot)$.

Define an indicator function $v(d_s)$,

$$v(d_s) = \begin{cases} 1 & \text{if } 2 \leq d_s < d^*; \\ 0 & \text{if } d_s < 2 \text{ or } d_s \geq d^*. \end{cases}$$

3.2.2 Contagion and Generating Functions

In this section we use the techniques of generating functions to calculate the size of the contagion after a randomly chosen trader fails and liquidates. Randomly choosing an asset s , the probability that s has i shareholders is p_i ; randomly choosing a trader t , the probability that t holds j assets is q_j . Let $\mu = \sum_i i p_i$ be the average asset degree, and $\nu = \sum_j j q_j$ be the average asset degree. Since every link connects a

trader and an asset, the total number of links in the network calculated from asset degrees and from trader degrees must be the same, i.e. $S\mu = T\nu$.

Time is discrete and starts with period 0. In period 0 a randomly chosen trader is hit by a shock and forced to liquidate all her portfolio. For the contagion of liquidation spreads to other traders, at least one asset liquidated by the initial chosen trader needs to be vulnerable. When this vulnerable asset is liquidated by the initial trader in period 0, all other shareholders of this asset become insolvent and liquidate all their portfolios in period 1.

The probability that a randomly chosen trader has degree j (and hence holds j assets) is q_j and the distribution of this probability can be represented by a generating function

$$g_0(x) = \sum_{j=0}^{\infty} q_j x^j \quad (3.3)$$

A generating function as such contains all the information of the degree distributions $\{q_j\}$ and are convenient when we generate the distribution of the number of assets or traders we can reach in period 2, 3 Appendix B.1 lists some basic properties of generating functions. Note that

$$g_0(1) = \sum_{j=0}^{\infty} q_j = 1$$

Let us forget the initial chosen trader for a moment and start with links instead. Following a link from the initial trader to an asset, we want to know how likely it is vulnerable and how many shareholders it has. We know that randomly choosing an asset, the probability that it has degree i is p_i . However, it is important to note that the degree distribution of an asset reached by following a randomly chosen link

is different from the degree distribution of a randomly chosen asset. This is because the higher the degree an asset has, the more likely it will be reached by following a randomly chosen link. Therefore the degree distribution of an asset reached as such should be proportional to its degree, i.e. $\propto ip_i$.⁷ After the normalization, this degree distribution should be

$$P_i \equiv \frac{ip_i}{\sum_{i=0}^{\infty} ip_i} = \frac{ip_i}{\mu}$$

Note that P_0 is always zero because we will never reach a node with zero link. Among the i links of a vulnerable asset, one of them is the link that we followed onto that asset, so there are only $i - 1$ links that are infectious. Define

$$\begin{aligned} f_{1v}(x) &\equiv \sum_{i=1}^{\infty} P_i v(i) x^{i-1} \\ &= \sum_{2 \leq i < d^*} P_i v(i) x^{i-1} \end{aligned}$$

Hence, randomly choosing a link and follow it to an asset, the probability that this asset is vulnerable is $f_{1v}(1)$. The second line comes from the fact that $v(i)$ equals 0 for $i = 1$ and $i \geq d^*$, and equals 1 otherwise. $f_{1v}(x)$ does not capture the the probability that this asset is safe (non-vulnerable),⁸ The probability of being safe is $P_1 + \sum_{i \geq d^*} P_i = 1 - f_{1v}(1)$.⁹ Therefore the probability distribution of infectious links of an asset reached by following a randomly chosen link is generated by

$$f_1(x) = 1 - f_{1v}(1) + f_{1v}(x) \tag{3.4}$$

Follow a link of a vulnerable asset, we reach a trader of degree j with a probability

⁷See Newman (2003) and Feld (1991) for detailed explanation.

⁸This can be seen from the fact that $f_{1v}(1) < 1$.

⁹An asset with degree 1 is always safe.

that is proportional to j . So the probability that this trader has $j - 1$ outgoing links (excluding the incoming link) is

$$Q_j \equiv \frac{j q_j}{\sum_{j=1}^{\infty} j q_j}$$

and the generating function for this probability is

$$g_1(x) = \sum_{j=1}^{\infty} Q_j x^{j-1} \quad (3.5)$$

Combine $g_1(\cdot)$ and $f_1(\cdot)$, if a trader t (reached by following a random link) liquidates, we can calculate how many t 's first-neighbors (traders) will be infected and forced to liquidate via the vulnerable assets they share with trader t . This probability distribution can be generated by

$$\begin{aligned} G_1(x) &= g_1(f_1(x)) \\ &= Q_1(f_1(x))^0 + Q_2(f_1(x))^1 + Q_3(f_1(x))^2 + \dots \end{aligned} \quad (3.6)$$

The various possibilities contained in $G_1(x)$ is depicted in Figure 3.2. In Figure 3.2 each small tree unit represents a possible transmission episode. It is assumed that $v(\cdot)$ is such that only assets of degree 2 and 3 are vulnerable. In the first row trader t (reached by following a link) has degree 1, which is just the link leading t and there is no link leading out to another asset. In the second row trader t leads to exactly one asset (other than the asset leading onto t), and this asset leads to 0, 1, 2 ... traders (other than t), respectively. In the third row trader t leads out to exactly two other assets and the various combinations are depicted. There are of course other

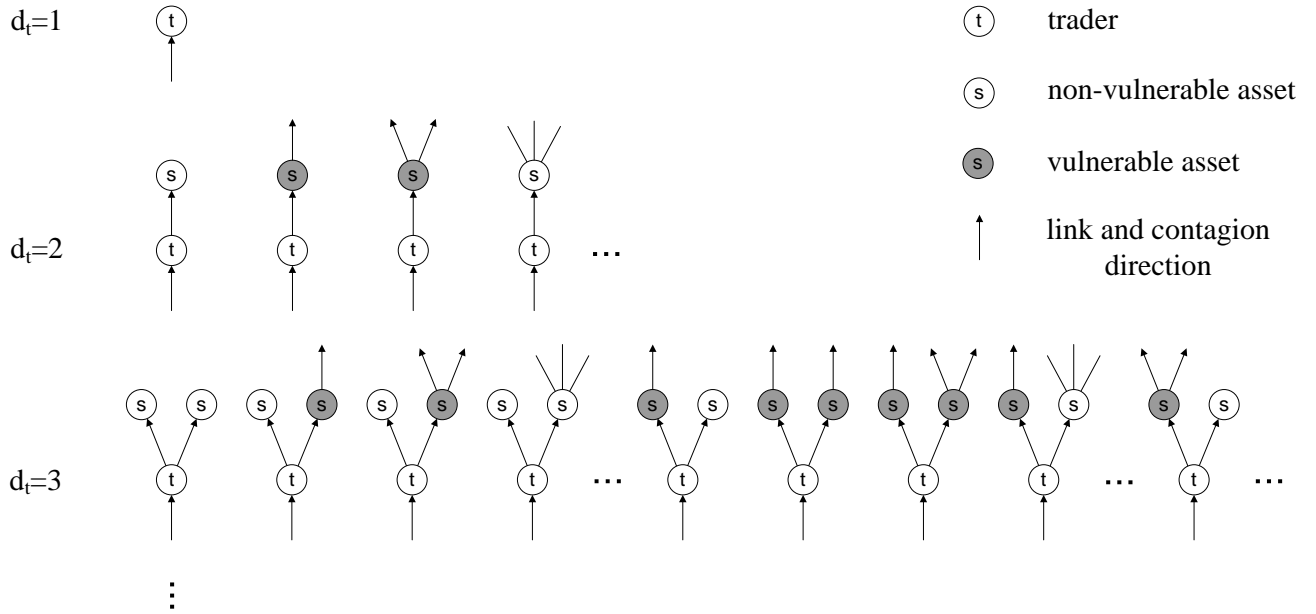


Figure 3.2: Schematic representation of the $G_1(x)$

possibilities which are not depicted here.

To better understand $G_1(x)$, recall that $G_1(x)$ is a polynomial of x . Like all generating functions, the exponents of x in equation (3.6) indicates the number of infected neighbors and the coefficients (after the polynomial expanded) attached to each of the x^i , $i = 0, 1, 2, \dots$, are the corresponding probabilities. Also in equation (3.6) we use the characteristic that if a generating function generates the probability distribution of some property of an object, then the sum of that property over i independent such objects is distributed according to the i th power of the generating function.

Each unit depicted in Figure 3.2, where trader t reached by following a link reaches out to a number of assets and then further reaches out a number of links (to other traders), is the smallest but a complete episode in the contagion process. It is like a (biological) cell that is the smallest but fully functional unit that can replicate

itself. In this sense we call it the *contagion unit* and $G_1(\cdot)$ describes the likelihood of different forms of contagion units we can reach by following a link.

If the liquidation spreads beyond first-neighbors and to the second-neighbors, what happens is exactly the same as depicted in Figure 3.2 and we can still use $G_1(\cdot)$ to describe the various possibilities that ensue. If we start with a randomly chosen link and follow it to t , the probability distribution of the number t 's first- and second-neighbors that can be infected by t is

$$G_2(x) = G_1(xG_1(x))$$

The x between the two layers of G_1 accounts for the first-neighbors and the inner x accounts for the second-neighbors. Similarly, the probabilities of contagion to the third, fourth ... neighbors can be described by more nesting layers of $G_1(\cdot)$.

Now we follow a link to a trader t , and then to every trader that can be ultimately infected by t , directly and indirectly, and call this set of traders (connected by vulnerable assets) the *vulnerable cluster*. Let $H_1(x)$ be the generating function of the probability distribution of the size (number of infected traders) of this vulnerable cluster reached by following a link to a trader. Then

$$H_1(x) = xG_1(xG_1(xG_1(\cdots xG_1(x)\cdots)))$$

Traders that share a vulnerable asset must be in the same vulnerable cluster. There can be many disjoint vulnerable clusters in a network and they are segregated by safe assets (or no link between them at all). The vulnerable cluster plays an important role in our analysis. If any trader in a vulnerable cluster liquidates, all traders in

that cluster will be forced to liquidate eventually. Therefore the sizes of the vulnerable clusters essentially determines the extent of a contagion at the system level.

Such a tree-like vulnerable cluster we can reach as stated above can take many different forms, which are illustrated in Figure 3.3. Each circle in Figure 3.3 is a

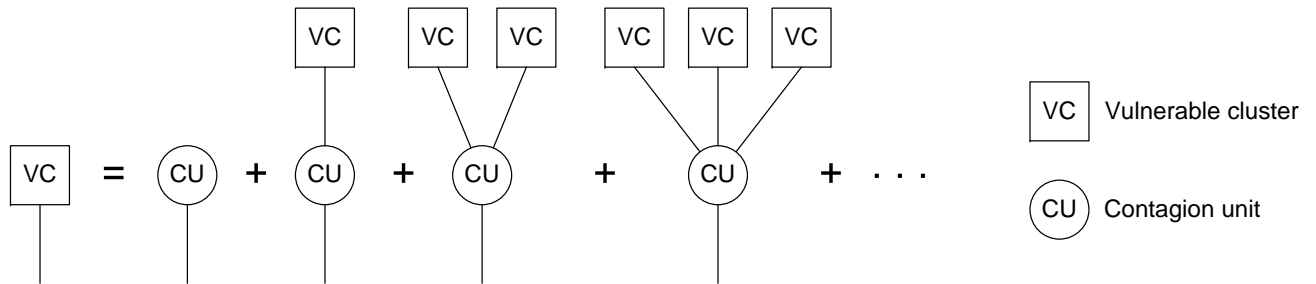


Figure 3.3: Schematic representation of the vulnerable cluster

contagion unit and each square is a vulnerable cluster. On the right hand side of the “=”, the initial contagion unit reached can emanate 0, 1, 2, 3, ... links. Each link leads to a vulnerable cluster and each cluster can also take many different forms, just like the original vulnerable cluster on the left hand side. Each vulnerable cluster on the RHS has the same size distribution as the original vulnerable cluster on the LHS. Therefore $H_1(x)$ must satisfy the following self-consistency condition:¹⁰

$$H_1(x) = xG_1(H_1(x)) \tag{3.7}$$

The vulnerable cluster described in H_1 starts with a randomly chose link. But we are interested in a contagion that starts with a randomly chosen trader. If a randomly chosen trader is hit by an exogenous shock and forced to liquidate all her assets, the distribution of the size of the vulnerable cluster (the total number of traders that can

¹⁰That $H_1(x)$ satisfy this recursive form of condition does not necessarily mean that the size of vulnerable cluster goes to infinity. It means that $H_1(x)$ includes all possible size of the cluster, including the infinite large one. See Newman et al. (2001).

be ultimately infected starting from this trader, directly or indirectly) is generated by

$$H_0(x) = xg_0(f_1(H_1(x))) \quad (3.8)$$

The leading x on right hand side accounts for the initial randomly chosen trader.

3.2.3 The Extent of Contagion Outside The Window

Unfortunately, a closed-form solution for $H_0(x)$ does not exist in general. However we can derive the average size of the vulnerable cluster from (3.7) and (3.8), which is enough for us to infer the stability of the system. We can calculate the expected extent of contagion $\langle s \rangle$ (in the number of infected traders) by

$$\langle s \rangle = H_0'(1)$$

Since $H_1(x)$ is a standard generating function and it includes all possible realizations, all the coefficients in this polynomial sum up to 1, i.e. $H_1(1) = 1$. For the same reason, $f_1(1) = 1$ and $g_0(1) = 1$. From equation (3.5) we know $g_0'(1) = \nu$. It follows from equation (3.8) that

$$\begin{aligned} H_0'(1) &= g_0(f_1(H_1(1))) + g_0'(f_1(H_1(1)))f_1'(H_1(1))H_1'(1) \\ &= 1 + \nu f_1'(1)H_1'(1) \end{aligned} \quad (3.9)$$

From equation (3.7) we have

$$H'_1(1) = \frac{1}{1 - G'_1(1)} \quad (3.10)$$

Substitute equation (3.10) into (3.9), we have

$$\langle s \rangle = 1 + \frac{\nu f'_1(1)}{1 - G'_1(1)} \quad (3.11)$$

From equation (3.4) we know that $f'_1(1)$ must be finite.¹¹ Then the expected size of the contagion diverges when

$$G'_1(1) = 1 \quad (3.12)$$

which signifies the phase transition. Recall that $G'_1(1)$ is the average number of the links reaching out of a contagion unit (excluding the incoming link). So if this number is smaller than 1, then the contagion is expected to die out early. If it is larger than 1, then a giant vulnerable cluster emerges, which occupies a finite fraction of the trader population.¹² In this case, if the initial trader hit by the exogenous shock is in the giant vulnerable cluster, then a finite fraction of the network will be infected.

The condition $G'_1(1) = 1$ can be rewritten as $g'_1(1)f'_1(1) = 1$ and then in turn as

$$\sum_i \sum_j ij(i-1)(j-1)p_i q_j v(i) = \nu\mu \quad (3.13)$$

Suppose that the network starts with very low average degrees, e.g. an empty network, and then let the average degrees ν and μ increase.¹³ As ν and μ increase, the mass

¹¹Since there are only finite terms containing x in $f_1(x)$.

¹²Statistically there is only one giant cluster in a network. The probabilities of two segregated giant clusters present in a network are very small and are usually ignored.

¹³Recall that $S\mu = T\nu$.

of the degree distributions moves to p_i and q_j of higher i and j . In particular, the mass of p_i moves into the vulnerable range where $v(i) = 1$, so more assets become vulnerable and connect to more traders. This could be the first phase transition point where $G'_1(1) = 1$. As the average degrees continue to increase, although the network becomes physically better connected, the mass of p_i moves out of the vulnerable range and the diversification starts to dominate. When more vulnerable assets become safe, the giant vulnerable cluster gets disconnected and isolated by these safe assets. This is the second phase transition that could arise, which marks the disappearance of the giant vulnerable cluster and $G'_1(1)$ drops from above to below unity. The emergence of the second phase transition implies that the giant vulnerable cluster exists only in between the two transitions. Therefore when the average degree is either very low or very high, there is no systemic contagion because no giant vulnerable cluster exists in the network; but when the average degree is in an intermediate range, the contagion will be extensive.

3.2.4 The Extent of The Contagion Between The Window

The systemic contagion over a network can be better seen if we can calculate the average size of vulnerable cluster within the contagion window, where the network is above the phase transition. Since the vulnerable cluster now occupies a finite fraction of the traders in the network, its size is in scale with the size of the network. When $T \rightarrow \infty$, the size of the vulnerable cluster also diverges. In this case we can calculate the size of vulnerable cluster as a fraction of the population. But the problem is that the existence of the closed loops in the giant vulnerable cluster can no longer be ignored, so we cannot treat it as a “tree” structure any more. As the “no-loop”

assumption is not applicable, we cannot use the method in Section 3.2.3 to calculate the size of the giant vulnerable cluster. As Newman et al. (2001) shows, however, this issue can be circumvented by calculating the size distribution of the components excluding the giant component, because the non-giant components can still be treated as tree-structures.

Within the contagion window, all traders must belong to either the giant vulnerable cluster or non-giant vulnerable clusters.¹⁴ Randomly choosing a trader, the distribution of the size of the non-giant vulnerable cluster to which this trader belongs is generated by $H_0(x)$. Then the average fraction of traders who are in non-giant components must be $H_0(1)$, and the average fraction of traders who are in the giant component, $\langle s \rangle$, is

$$\langle s \rangle = 1 - H_0(1)$$

From equation (3.7) we know $H_0(1) = g_0(f_1(H_1(1)))$. Let $u = H_1(1)$, then u is the probability that a randomly chosen link leads to a trader that belongs to a non-giant component. From equation (3.8), $\langle s \rangle$ can be calculated by solving

$$\langle s \rangle = 1 - g_0(f_1(u))$$

$$\text{and} \tag{3.14}$$

$$u = G_1(u) \tag{3.15}$$

Now we show that, as the average degree increases, the average size of the giant

¹⁴Even a trader without any link at all, or a trader whose portfolio consists of safe assets only, can be regarded as a vulnerable cluster of size 1.

vulnerable cluster $\langle s \rangle$ first increases and then decreases.

When the network has just entered the contagion window and the average degree starts to increase from the lower bound of the window, we show that both $f_1(u)$ and $g_0(f_1(u))$ decrease. Then it follows that $\langle s \rangle$ increases. From (3.15) we know

$$u = g_1(f_1(u)) \quad (3.16)$$

$f_1(u)$ is the probability that a randomly chosen link leads to an asset that is not connected to the giant component, i.e. either this asset is safe or it is vulnerable but connected to a non-giant component. Since u is the probability that a trader (reached by following a randomly chosen link) belongs to a non-giant component, equation (3.16) says that u must equal the probability that all her assets, if any, are either safe or their contagious links only connect to non-giant components. If any of the contagious links of her assets connects to the giant component, then this case is excluded from (3.16).

When the average degree increases, the mass in the degree distribution P_i moves to higher degree terms (higher i), more assets become vulnerable and function $f_1(\cdot)$ shifts down (since $u < 1$ within the contagion window).¹⁵ The intuition is that, since each link of a vulnerable asset has the positive probability $1 - u$ to connect to (a trader that belongs to) the giant component, then when each vulnerable asset has more links, the chance that its holders are connected to the giant component is higher. Similarly, $g_1(\cdot)$ also shifts down, because each link of this trader has the positive probability $1 - f_1(u)$ to connect to the giant component, then the more

¹⁵When the probability mass moves to higher degrees, i.e. the average degree increases, a generating function $f(x)$ shifts down in the range $x < 1$. This property also applies to other generating functions in our paper.

assets she has, the higher the chance that she is connected to the giant component. Therefore, if u remains unchanged, the downward shift of $f_1(\cdot)$ and $g_1(\cdot)$ makes a contagion unit less likely to connect to a non-giant component, i.e. $g_1(f_1(u)) < u$. Recall that for a non-giant vulnerable cluster, $G'_1(1) = g'_1(1)f'_1(1) < 1$ (even within the contagion window), otherwise this cannot be a non-giant vulnerable cluster. Since $u < 1$, then $G'_1(u) < G'_1(1) < 1$. For (3.16) to hold, u cannot stay unchanged, but has to decrease. Then $f_1(u)$ must decrease as well when the average degree increases.

Now let us check the response of $g_0(f_1(u))$ to changes in the average degree. Similar to $g_1(\cdot)$, when the average degree increases, the mass in degree distribution q_j moves to higher degree terms (higher j). Since $f_1(u) < 1$, we know that function $g_0(\cdot)$ shifts down, i.e. it is less likely for a randomly chosen trader not to connect to the giant component. In addition, as shown above, each link of this trader is more likely to connect to the giant component, we conclude that the fraction of traders who are not in the giant component is declining, i.e. the giant vulnerable cluster is growing disproportionately compared to the network.

As the average degree continues to increase, each asset tends to have more shareholders. The mass in distribution P_i moves out of the vulnerable range and vulnerable assets turn safe. Now $f_1(\cdot)$ shifts up and eventually u and $f_1(u)$ start to increase. This means that a randomly chosen link is now more likely to lead to an asset that is not connected to the giant vulnerable cluster. $g_0(\cdot)$ is still shifting down, i.e. a random chosen trader tends to have more and more links, but this is dominated by the decreasing probability of each of her links to connect to the giant component. Therefore a trader is less likely to connect to the giant component, which implies that the giant vulnerable cluster is shrinking. As the average degree continues to

increase and the network moves out of the contagion window, most of the assets are safe now. The giant component disappears and there will be no systemic contagion.

The average size of the giant vulnerable cluster within the contagion window as a function of the average trader degree is shown in Figure 3.4.¹⁶ The size is calculated from equation (3.14) and (3.15) and is measured as a fraction of the trader population. The average size of vulnerable clusters outside the window is shown in Figure 3.5, which is calculated from equation (3.11) and is measured in the number of traders. In Figure 3.5 the average size of the vulnerable clusters diverges when the average trader degree is between 0.708 and 8.22, which corresponds to the contagion window in Figure 3.4 where the giant vulnerable cluster emerges and occupies a finite fraction of the trader population. The lower and upper bound correspond to the two phase transitions where $G'_1(1) = 1$. In Section 3.3 we will examine whether these theoretical results are consistent with those from simulations.

3.2.5 The Probability of Systemic Contagion

The size of the giant vulnerable cluster closely relates to the probability of the systemic contagion, because randomly picking trader t , if t is in the giant vulnerable cluster, then the probability of a systemic contagion is one; otherwise, it is zero. So theoretically the size of the giant vulnerable cluster as a fraction of the network should coincide the probability of the systemic contagion triggered by the liquidation of a random trader.

But the numerical results from simulations where multiple hits are allowed (Figure 3.6) show that the size and the probability coincide only when the network connectivity is in the lower range of the contagion window. In the higher range the probability

¹⁶Assuming Poisson degree distributions, $\rho(x) = x$ and $K/V = 0.01$

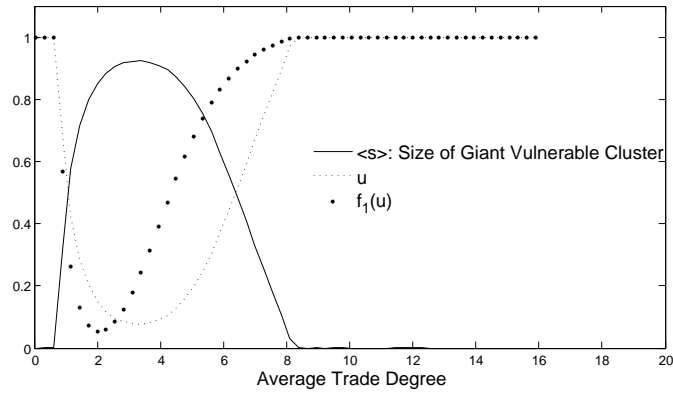


Figure 3.4: Average size of giant vulnerable cluster above phase transition (as a fraction of trader population)

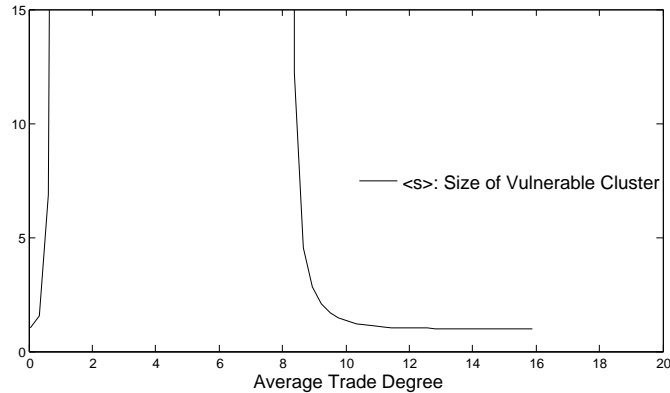


Figure 3.5: Average size of vulnerable cluster below phase transition (in number of traders)

starts to drop whereas the size of the contagion (conditional on contagion breaking out and at least 5% traders infected) continues to increase and almost occupies the entire network. As such, the financial networks exhibit a “robust-yet-fragile” tendency, as Gai and Kapadia (2010) put it, in the higher range. The the portfolio diversification starts to reduce the probability of an extensive contagion, but given that a contagion has already broken out, there will be a substantial portion of the population will be infected. The reason for this divergence is that the methods of generating functions, including Newman’s method, do not take into account of the issue of multiple hits of

liquidations, which will eventually infect a trader that is not connected to the giant vulnerable cluster. We will discuss this issue in detail in Section 3.3.2.

3.2.6 Implications of Relaxing Some Assumptions

So far we have assumed that the initial market value of each asset held in the network is independent of the degree of asset and the shares are evenly distributed over its shareholders. In reality it might be reasonable to allow the market value of an asset to increase with the number of its shareholders. Given a constant capital buffer, this would increase a trader's exposure to the price impacts and dilute the benefit from portfolio diversification. On the other hand, we might also expect that a trader's capital buffer to increase with the number of assets in her portfolio. Since we assumed a fixed ratio between the total number of the assets and the total number of the traders in the network, the ratio between μ and ν does not change as μ and ν increase and the network gets better connected. Therefore, as long as the rate at which the initial in-network value of an asset increases with the asset degree is less than the rate at which the capital buffer increases with the trader degree, the portfolio diversification still benefits and our result still holds qualitatively and equation (3.12) and (3.13) continue to have two solutions, though the contagion window might be wider. Conversely, if the former rate exceeds the latter, having more assets will be unambiguously riskier; the better connected a network, the more likely and more extensive the contagion.

An uneven distribution of asset shares among shareholders will not change our results qualitatively, though we would expect that the contagion window to be wider because some of the safe assets may become vulnerable to those who hold more shares.

3.3 Numerical Simulations

3.3.1 Methodology

To verify our theoretical results, we simulate the contagion process numerically. We assume a random graph in which each possible link between a trader and a asset is present with an independent and identical probability p (binomial distribution). Such a random graphs is called a Poisson random graph.¹⁷ The networks generated as such allow closed loops and accommodate all possible network structures. The Poisson random graph is chosen for simplicity.

We are interested in the expected extent and the probability of a contagion and will examine networks with different connectivity. Even when loops are present, in simulations we still examine both cases where multiple impacts of liquidation are allowed and not allowed. This is to verify that the divergence between the extent and the probability of contagions is caused by the multiple impacts. When calculating the extent of contagion we only count episodes with more than 5% of the traders infected, which is a reasonable signal that the initial exogenous shock has hit within the giant vulnerable cluster. Otherwise, we consider the initial shock has missed the giant vulnerable cluster and a small contagion, if any, is irrelevant in the calculation of the sizes of the giant vulnerable clusters.

We generate networks with 1000 assets and 2000 traders. The choice of 1000 assets and 2000 traders seems to be reasonable if we consider large investment banks, hedge funds, insurance companies, etc in the global financial system. Higher numbers of

¹⁷The networks with this degree distribution are generated by Configuration Model. See Jackson (2008), Section 4.1.4

assets and traders in the simulations do not change our results qualitatively.¹⁸

For simplicity, we assume that the price is proportional to the fraction of the asset held in network, i.e. asset price equals x ($\gamma = 1$). Recall that we denoted the initial market value of the portion of an asset held in the network, $x_0^2 B$, as V , where x_0 is the initial fraction of shares held in the network. From equation (3.2) it follows that the loss of a trader each time an asset she holds is liquidated by another trader is $B \frac{x_0^2}{d_s^2} = \frac{V}{d_s^2}$, where d_s is the initial degree of that asset. In the case where multiple impacts are allowed, if the accumulated loss of a trader has not exceeded her capital buffer K , then she is solvent. In the case where multiple impacts are not allowed, a trader becomes immune after the first impact. We also examine the effect of the capital buffer (and hence the leverage) on contagions by varying the ratio between K and V .

In the simulations we draw 500 realizations of the random networks for each average trader degree ν . In each of these draws, a trader is randomly chosen and forced to liquidate all her portfolio in period 0. Any neighboring traders whose (accumulated) mark-to-market losses are larger than K are assumed to be bankrupt and must also liquidate all their assets. This process iterates until no trader fails any more.

3.3.2 Simulation Results

The simulation results are shown in Figure 3.6. By varying the average degree of traders, ν , we have networks with different average degrees. The average degree of a

¹⁸The use of generating functions requires an infinite network (infinite number of nodes). Watts (2002) shows that the infinite network can be well approximated by a network with 10,000 nodes, and Gai and Kapadia (2010) shows that the results from networks with 1,000 nodes and those from networks with 10,000 nodes agree quite well.

network is an indicator of how well a network is connected. When multiple liquidation impacts are allowed, we can see that if the average degree is either very low or very high, system-wide contagion is not likely to happen. Whereas within a certain window where the average degree is moderate, an extensive contagion is more likely and the extent of the contagion is non-monotonic in the average degree. This confirms the intuition in Section 3.2.3 and 3.2.4.

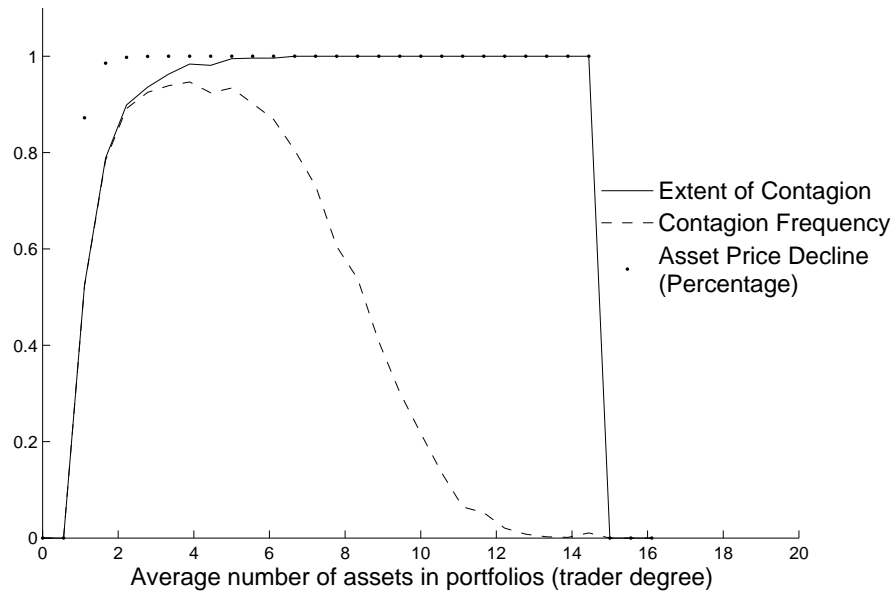


Figure 3.6: Contagion frequency, conditional extent of contagion and price decline ($K/V = 0.01$, multiple impacts allowed)

Figure 3.6 also shows the average percentage decline of the asset prices conditional on a widespread contagion. In calculating the average decline all assets are equally weighted. The percentage price declines are roughly consistent with the extent of contagions, with the price declining larger than the extent at the lower end of the contagion window. The average price decline result can be viewed as what could happen to the stock market indices in a financial crisis and it suggests that even when investors are relatively well diversified, the collapse of asset prices could be

significant and that a moderate diversification does not mitigate the consequences of a financial crisis once it spreads beyond its initial neighborhood.

In Figure 3.6 the extent of the contagions (conditional on more than 5%), measured as a fraction of population, is approximately the same as the frequencies of contagions in the lower range of the contagion window. In the higher range of the window, they diverge: though an extensive contagion is rare, once it happens, almost all traders will fail. So the financial system exhibits a robust-yet-fragile feature. Above the upper bond of the contagion window, the extent of the contagion suddenly drops to zero and the contagion disappears.

If we compare the theoretical sizes of the giant vulnerable clusters in Figure 3.4 with the simulated sizes of contagions in Figure 3.6, we find that in the upper range of the contagion window the theoretical size falls gradually, whereas the simulated extent of contagion converges to one (when multiple impacts allowed). This is shown in Figure 3.7. The main difference between the theoretical model and the model used in the simulations is whether closed loops are allowed in the network. This implies that the robust-yet-fragile tendency in the simulations, in particular the extreme contagion extent, is primarily caused by the multiple price impacts to a trader, rather than by the first single impact alone. When the initial contagion starts out, it is very likely that the contagion dies out at an early stage because well diversified investors are subject to only a minor mark-to-market loss at the beginning. This corresponds to the low frequency. But once the contagion somehow spreads beyond its initial neighborhood via vulnerable assets, it starts to backfire over the loops and a trader may hit multiple times by the price impacts simultaneously and sequentially. As the waves of liquidation bounce back and forth on the intertwined financial network, the

losses accumulate over time and can finally bring down well diversified investors.

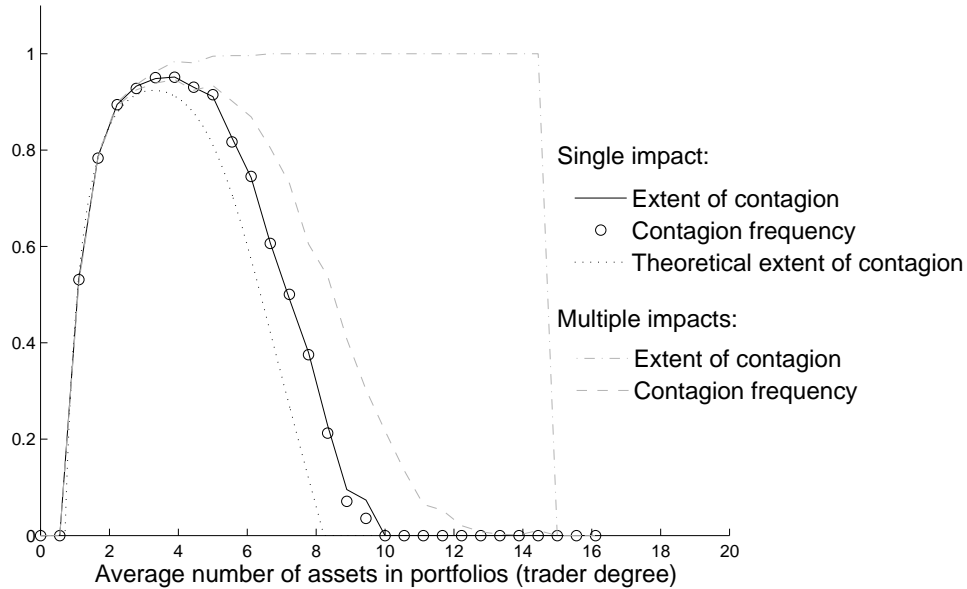


Figure 3.7: Contagion frequency, conditional extent of contagion and theoretical extent of contagion under single impact ($K/V = 0.01$, compared with multiple impacts)

Figure 3.7 also shows that the simulated extent of the contagions is now consistent with the probability of contagion under the single impact assumption.¹⁹ The extent of the contagions (the size of giant vulnerable cluster) in this case is limited and gradually decreases when the network gets better connected. The divergence (under the multiple impacts assumption) now disappears and the two curves are now almost indistinguishable. This confirms our earlier conjecture that the multiple impacts are the reason that underlies robust-yet-fragile phenomenon.

Our explanation to the difference behaviors under the single and multiple-impact assumptions in the higher range of the contagion window suggests that most damages here are done after the first wave of impacts. This implies that early government

¹⁹In simulations any further mark-to-market loss to a trader after her first impact is ignored.

intervention can potentially be highly effective in restricting the contagion. After the initial bankruptcy breaks out, there could still be time and chances for the government to bail out those who survive but are weakened before they incur further losses. This could prevent the contagion from spreading to the entire network.

Figure 3.7 also shows that the theoretical sizes of the giant vulnerable clusters calculated from equation (3.14) and (3.15) are largely consistent to the simulated extent of contagions under the single-impact assumption.

Figure 3.8 shows how changes in the capital buffer and the average degree jointly affect the frequency of extensive contagions. When the capital buffer is high, the system risk is low for all average degrees. When the capital buffer starts to decline, the probability of contagion is significant within a small window in the low average degree range. As the capital buffer continues to drop, this window expands to high average degrees. This shows that increasing the capital buffer can effectively decrease the probability of a widespread contagion. If there is no restriction on the leverage ratios, even a very well connected network faces huge systemic risks.

Gleeson and Cahalane (2007), Gleeson (2008), Gleeson et al. (2011) develop a mean-field method to calculate the extent of contagion without Monte-Carlo simulations. Their methods allow multiple rounds of hits and the results are quite consistent with those from simulations where multiple hits are allowed.

3.4 Discussion: Front Running and Contagion

In this section we examine the effect of a predation on the contagion by allowing some shareholders front-running the distressed shareholders who are liquidating.

Front running, or predatory trading, is in line with Brunnermeier and Pedersen

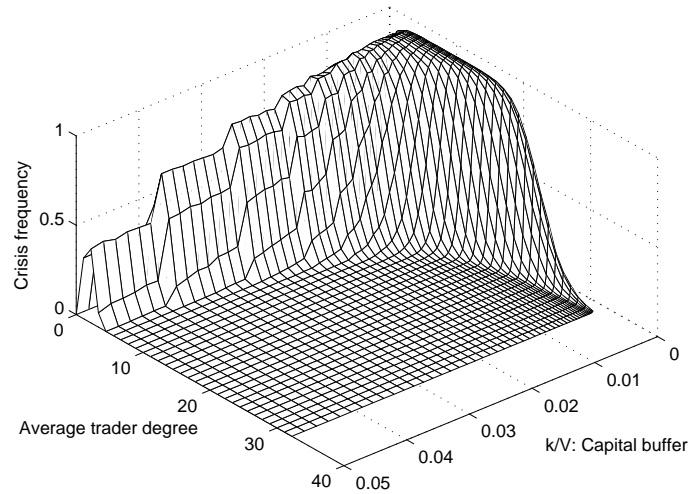


Figure 3.8: Contagion with various capital buffers

(2005a): when some traders are in distress and forced to liquidate their assets, other shareholders who are aware of this may take the advantage by selling before the distressed and then later buy back the asset to make a profit. Front running, though generally illegal, has long been suspected on Wall Street. Investigations and convictions appear in press from time to time.²⁰ There are evidences suggesting that during the 1998 LTCM collapse several market participants front-ran LTCM.²¹

Figure 3.9 illustrates a simplified version of the price dynamics of a liquidation with front running. In Figure 3.9a some distressed agents are forced to liquidate their positions. Without a predation the price of an illiquid asset will decline permanently after the forced liquidations. In Figure 3.9b, if another shareholder (predator) knows about the oncoming liquidations, she will try to sell before the liquidations and then buy back to her original position after the liquidations. By selling at a higher price and

²⁰See Khan and Lu (2009)

²¹See Cai (2003)

then buying back at a lower price, the predator can make a profit. Notice that because

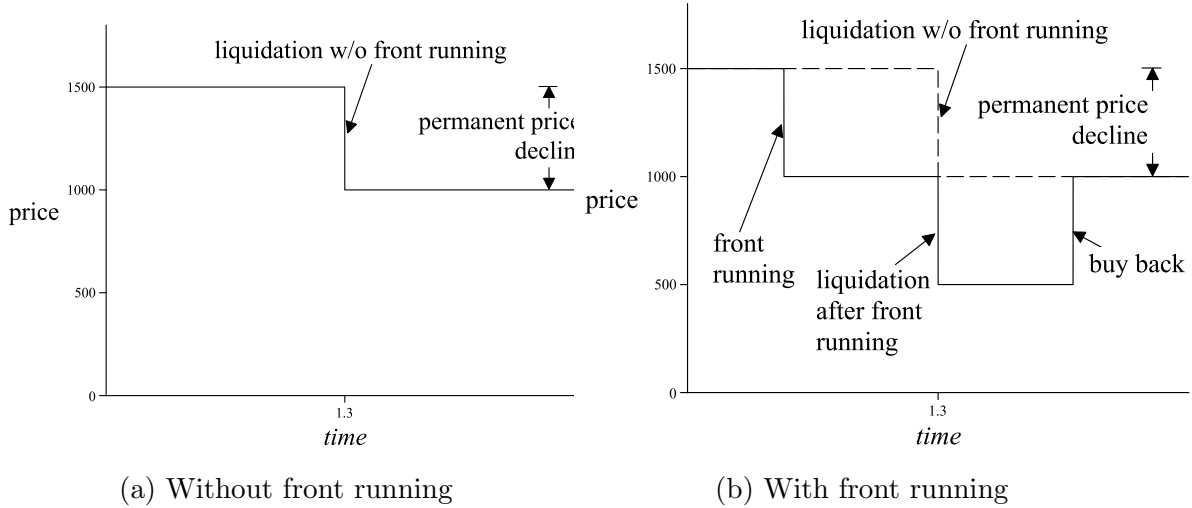


Figure 3.9: Price dynamics in liquidation

of the front running, there is an excessive price decline during the liquidation. At the bottom of this price overshooting where the forced liquidation is about to finish, the price is lower than that without the predation. This price overshooting is the reason why we are concerned about the predatory behavior in a network, because it further writes down the asset price during the liquidation, aggravates the adverse price impact and facilitates the spread of the contagion.

Consider a model where the financial network is the same as before but with a very simple front running. Whenever there is a distressed trader forced to liquidate, there is exactly one non-distressed shareholder preys on this distressed trader. Though stylized, this setting provides a benchmark to show the potential damages that front runnings could cause. This is also consistent with the fact that a liquidation is usually highly sensitive information and front running is illegal so that both the distressed

and the predator want to keep it secret.²² The predator sells all her shares of this asset before the distressed trader starts to sell and buys back and restores her initial position after the forced liquidation finishes. The impact of the front running on shareholders is measured at the bottom of the price overshooting and all shareholders whose capital buffers are exhausted at this lowest price become bankrupt and have to liquidate all their portfolios. The predators are myopic in the sense that they do not forecast whether or not they will be made bankrupt because of the excessive price declines caused by their own predations. When they do find themselves in trouble, they will have to liquidate, just as other shareholders.²³

In the theoretical model where it is assumed that no closed loop exists in the network, the contagion arrives at an asset via exactly one of its shareholders, i.e. there is exactly one distressed shareholder in an asset. Again we assume the simple asset price in the linear form: $\rho(x) = \gamma x$. Since shares are evenly distributed among shareholders, after the predator and distressed both have sold, the price declines by twice that without front running and the mark-to-market loss to each remaining shareholder doubles. Recall that without predator this loss is $\frac{V}{d^2}$. With one predator, it increases to $\frac{2V}{d^2}$. Consequently, the assets with degree $2 \leq d_s < \sqrt{\frac{2V}{K}}$ are vulnerable, i.e. the vulnerability range expands by $\sqrt{2}$. With more vulnerable assets, it is expected that both the probability and the extent of the contagion will increase.

The results from simulations are shown in Figure 3.10. We see that at the lower end of the contagion window the two models do not differ much. When the average degree is beyond 3, the front running makes a difference and outweighs the case without front

²²In addition, Brunnermeier and Pedersen (2005a) show that, the more predators, the less profit for each predator can be made out of the predation.

²³This can be interpreted as predators' incomplete information about the market participants and others' positions, etc.

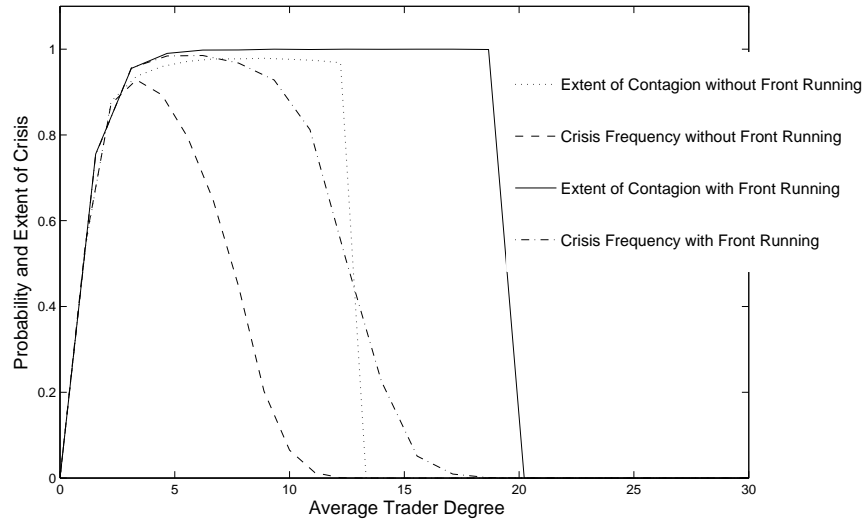


Figure 3.10: Contagion with front running vs. without front running ($k/V = 0.01$)

running. With the front running the contagion window expands significantly and both the high contagion frequency and the high extent of contagion extend significantly. This reminds us that when the predatory behavior is common, the asset markets are exposed to much larger risks and the robust-yet-fragile feature is even more evident.

3.5 Conclusion

In this paper we develop a model of contagion of bankruptcy and liquidations over the complex financial networks. The model applies to any systems where investors are connected by their overlapping portfolios and the liquidations of one shareholder could adversely affect the portfolios values of the others. This includes markets of stocks, currencies, securities and commodities. Portfolio diversification may reduce the likelihood of contagion, but it also increases the extent of contagion when liquidation spread beyond its initial neighborhood. In this case, the failures of the majority

of the investors are primarily caused by the multiple rounds of adverse price impacts, rather than the first wave. This model can also incorporate behaviors such as front running and examine their effects on the contagion.

Our results suggest that when the contagion of liquidation spreads out and has affected many investors considerably, there is still time and opportunities for a government to intervene before they suffer from further rounds of liquidations. If the government can bail out weakened key investors in the network, the damage could be contained in some small area rather than system wide.

This model provides a preliminary method to evaluate the sizes and the probabilities of forced liquidation over the asset markets. It would be useful to extend the model by assuming a fixed leverage rather than a fixed capital buffer so that an investor's capital is proportional to his portfolio value. It will be interesting to examine the losses and the rates of failure for investors with different level of diversification and identify an optimal level. It would also be highly desirable to use actual data on shareholding structures of stock markets to simulate the contagion.

Chapter 4

A comparative analysis between asset bubbles and auctions

4.1 Introduction

An asset bubble is usually defined as a large price deviation from its fundamental or intrinsic value, which can last for extended periods. Historical examples of bubbles include the Dutch tulip mania of the 1630s, the South Sea bubble of 1720 in England, the Mississippi bubble in France, the Great Recession of 1929 in the United States, the DotCom bubble in the late 1990s, and the most recent housing bubble and crash since 2008. Such phenomena have long been intriguing to the economists because they not only affect the financial sector, but also have huge impacts on the real economy. In addition, bubbles have been difficult to generate and explain theoretically. One major hurdle is the no trade theorem. One version of this theorem states that if the initial allocation is efficient, there is common knowledge that all traders are rational and agents have a common prior belief about the distribution of the asset

fundamental value, then the agents have no incentive to trade.¹ In the dynamic models the standard neoclassical theory precludes the existence of bubbles by a backward induction.²

Facing these impossibilities, some economists choose to deny that the bubbles exist.³ On the other hand, many explanations to the bubbles have been suggested (see the brief literature review at the end of section 4.1). But even among those who believe the existence of bubbles, the cause remains disputed.

In this paper we provide an alternative view on why bubbles arise by comparing an asset bubble model to an auction model. We first present a simple model where a bubble arises from the rational traders' asymmetric information and the interaction between the rational and the behavioral traders. The asset price keeps growing due to behavioral traders' excessive optimism. All the rational traders have a common prior belief about the asset fundamental value, but each trader receives a private signal. As the price continues to rise, rational traders gradually unload their positions. The bubble will burst when a certain fraction of traders have exited the market. A trader understands that by selling early she can make a small profit; by selling late she might be able capture a large price appreciation but also have higher risk of being caught in the crash. The key ingredient in the model is that a trader does not know how many others have beliefs higher or lower than hers. Therefore, every trader has a different posterior belief and this belief dispersion induces a dispersion of exit strategies, which is what allows the bubble to arise and grow. It is worth mentioning that three frictions are at work allowing the bubble to exist: 1. limited short selling, 2. the lack of common knowledge of the population's belief, and 3. the unobservability

¹See Grossman and Stiglitz (1980) and Milgrom and Stokey (1982).

²See Santos and Woodford (1997).

³See Garber (1990)

of individual's trade.

Then we show that this model is equivalent to the model of the discriminatory-price (first-price) common value auction. The auction is one where traders simultaneously bid for contracts (this is called a reverse or procurement auction), with the lowest bids winning and receiving their respective bidding prices and the rest bidders receiving an unknown common outside option. The recent literature on auction models shows that the equilibrium marginal bidding price⁴ does not converge to the true value of the object in this auction.⁵ This deviation parallels the price deviation from the asset fundamental in the asset bubble and implies that the bubble arises due to the same reason as the failure of the price convergence in this auction. As a trading mechanism this auction fails to aggregate the information dispersed in the bidders and bidders' strategies systematically deviate from the true value. Equivalently, the market often overreacts to changes in fundamentals because the privately informed investors are unable to reach a consensus and are instead optimally riding bubbles. This equivalence may be interpreted as a "bad news" for the efficient market hypothesis because it demonstrates that bubbles are rooted in the information asymmetry and the failure of information aggregation, which is an intrinsic feature of the market. In contrast in a uniform-price (second-price) auction the equilibrium price does not deviate from the true value.

Next we identify two incentives in traders' strategies that shape the bubbles. In the discriminatory-price common value auction there exist two "bid shading"⁶ incentives:

⁴The marginal bidding price is the the highest winning bid or lowest losing bid.

⁵A detailed review of this literature is postponed to Section 4.3.2.

⁶Bid shading is the practice of a bidder placing a bid below what she believes a good is worth. In this paper bidder adjust their bids, not necessarily reduce their bids.

1. traders try to avoid both the winner's and the loser's curses⁷ and 2. traders' price-setting incentive (because they get what they bid upon winning) in a first-price auction compared to price-taking behavior in a second-price auction. We show that the size of the bubble equals the price-setting component in the equilibrium strategies.

Based on the above analysis we discuss several experiments of tax and subsidy on our model. In one of the experiments we find that if a transaction tax and subsidy are imposed such that the traders' payoff structure is transformed from a discriminatory-price to a uniform-price auction, then there is no bubble at all. This suggests that taxes and subsidies can be used to affect the payoff structures and reduce the bubble. In another experiment we halve the size of the bubble and avoid the budget deficit in the previous one.

In addition the two incentives have opposite responses to a change in the winning percentage in the auction (or the triggering sale threshold in the bubble model), which can explain the differences between several models' results in the literature.

Our model is related to a vast literature on bubbles. For surveys on bubbles, refer to Brunnermeier (2009), Brunnermeier (2001), Brunnermeier and Oehmke (2012) and Scherbina (2013). One strand of this literature allows heterogeneous priors or rational agents to "agree to disagree". Harrison and Kreps (1978) show that when agents disagree about the probability distribution of the dividend stream, an agent may buy an asset at a price that exceeds her valuation since she believes that she can find a more optimistic buyer. Scheinkman and Xiong (2003) justify this behavior by an overconfidence as a source of the belief disagreement. Allen et al. (1993) and Conlon (2004) allow agents to have heterogeneous priors and hold worthless assets in

⁷Simply put, the loser curse is that a bidder realizes that she has underbid upon losing. See Section 4.3.3 for details.

the hope of selling it to greater fools. One advantage of these models is that all agents are rational. But to some extent they lack a price path with a clear run-up with a sudden crash. Our model adopts the more conventional common prior belief setting which is parsimonious and the upturn and the crash in the price path are evident and clear.

Another strand of this literature, including our model, studies bubbles that arise from the interaction between rational traders and behavioral traders. In De Long et al. (1990) the rational traders buy the asset and entice positive-feedback traders to enter the market. After the price is pushed up, the rational traders can profitably unload. The bubble exists because time is discrete in their model. If we allow continuous time and assume that some of rational traders will not receive the full price when they sell together,⁸ then the bubble will disappear. This is because the rational traders will then have an incentive to preempt each other, which diminishes the bubble until it disappears. Our model is robust in this sense and the lack of common knowledge prevents the rational traders from preempting each other.

In the literature of rational herding bubbles also arise. See Brunnermeier (2001) and Chamley (2004) for surveys on this topic. Avery and Zemsky (1998) introduce a sequential trade model, where the asset value is either 0 or 1 and a trader is uninformed with probability $1 - \mu$ and informed with probability μ . Informed traders also have two private types: poorly and perfectly informed. The market maker adjusts the price after observing each round of transaction. A “bubble” arises when many poorly informed traders are herding and the market maker keeps raising the price. When a perfectly informed trader shows up and behave differently, the bubble bursts.

⁸This is a realistic assumption in that it takes time to sell a chunk of shares, especially when the market is near the brink of a crash.

Lee (1998) studies an “information avalanche”, where a trader must pay a fee but can decide when to trade and the market maker adjusts the price after every round of trade. All traders are informed but they differ in the precision of their information. A partial informational cascade can occur if the traders “wait and see” and then an extreme signal triggers all previous inactive traders to sell. In both models, the price run-up arises from the uncertainty about the value, while in our model the upturn is exogenous though the bubble component is endogenous. In their models the event of a bubble followed by a crash happens with very small probabilities and traders are only given one opportunity to trade, while in our model the bubble and the crash happen with probability one and traders can buy and sell for many times.

Abreu and Brunnermeier (2003) (henceforth AB2003) challenge the efficient market perspective by showing that it is optimal for rational traders to ride a bubble. In their model, the price grows exogenously and at some random moment the growth rate of the fundamental value falls behind that of the price, hence a bubble emerges. Rational traders become aware of this sequentially. We transform their uncertainty from the dimension of time to value/price. This transformation abstracts time away and removes the assumption of sequential awareness, with the price the only concern and the model now comparable to an auction.

Results from the recent empirical studies of stock markets are consistent with AB2003. Temin and Voth (2004) show that a major investor in the South Sea Bubble knew that a bubble was in progress and nonetheless invested in the stock and rode the bubble profitably. Brunnermeier and Nagel (2004) and Griffin et al. (2011) both study the Tech Bubble in the late 1990s. They show that, instead of correcting the bubble, hedge funds turned out to be the most aggressive investors. They profited in

the upturn and unloaded their positions before the downturn.

Doblas-Madrid (2012) (henceforth DM2012) constructs a discrete-time version of AB2003. DM2012 removes the behavioral agents from the model, partially endogenizes the upward price path and makes the price responsive to sales.

AB2003 and DM2012 provide a convincing framework of bubbles, but why a bubble exists is not entirely clear. AB2003 explains that the lack of common knowledge prevents traders from perfectly coordinating, hence the backward induction has no bite on the bubble. However, this is only a necessary condition for a bubble to exist. Brunnermeier and Morgan (2010) show that a discrete-trader version of AB2003 can be recast as an auction, but they do not further pursue this direction. I preserve the backbone of AB2003's framework and provide a comparative analysis between bubbles and auctions by focusing on the incentives in traders' strategy that generate the bubble.

The remainder of the paper is organized as follows. Section 4.2 introduces the model and characterizes the equilibrium. Section 4.3.1 shows the equivalence of the bubble model to a discriminatory-price common value auction. Section 4.3.2 reviews the auction literature and shows that the price generally does not converge to the true value in a common value auctions. Section 4.3.3 identifies the two reverse bid shading incentives which generate the bubble, Section 4.3.4 devises several simple tax/subsidy experiments to show how to reduce the size of the bubble and Section 4.3.5 shows that the two incentives respond differently to changes in model parameters. Section 4.4 concludes.

4.2 The model

Time is continuous and there is only one asset. The asset's fundamental value θ is unobservable. There is a unit mass of risk neutral rational traders (henceforth the *traders*), each holding 1 unit of the asset at the beginning. Limited short selling is allowed and without loss of generality the asset position is restricted and normalized to $[0, 1]$ for each trader. A trader can buy and sell shares at any time, which is unobservable to others. The price can be publicly observed and increases continuously and deterministically. Without loss of generality assume that the initial price is zero.⁹ At any time if the price rises above θ , we say there is a *bubble*. There is no discounting.

As in AB2003, the backdrop is that the asset price increases exogenously, which can be interpreted as there are behavioral agents buying the asset. These behavioral agents are overly optimistic and believe that a technology shock has permanently raised the productivity. They keep buying the asset and push up the price, as was the case during the Tech Bubble in the late 1990s. The rational traders start to sell when they gradually believe that the price is too high. We assume that when an exogenous fraction κ ($0 < \kappa < 1$) of the rational traders has sold, the price stops increasing and jumps instantly to its fundamental value and stays there thereafter, i.e., the bubble bursts. In line with AB2003 we call this an endogenous crash. This threshold can be interpreted as a point at which the selling pressure cannot be concealed by the price noise and the price is too high becomes common knowledge among all rational traders and behavioral agents. Behavioral agents are not explicitly modeled here and a rational trader is only concerned about how many other rational traders have sold. An example of the price path is depicted in Figure 4.1, where the size of the bubble

⁹As long as the initial price equals the lowest possible θ , results will not change (up to a shift).

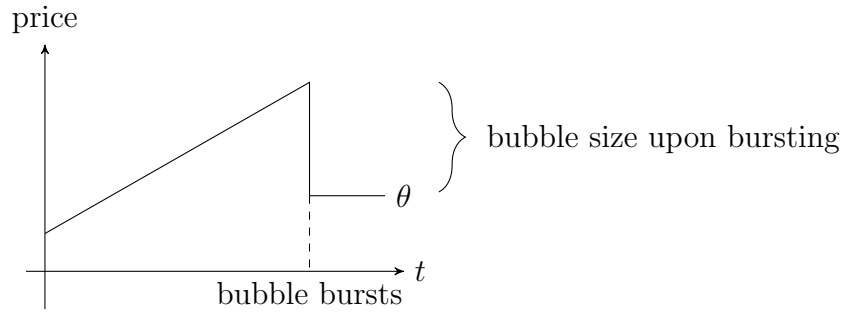


Figure 4.1: An example of the price path

is the gap between the current price and θ . To rule out the nuance equilibrium where all traders hold the asset forever and never sell, we assume there is an exogenous upper bound \bar{B} for the bubble size and the bubble will burst when it is larger than \bar{B} , even if the selling threshold κ has not been reached. This is the exogenous crash in AB2003. Imposing such an exogenous upper bound is reasonable because a bubble cannot grow forever. We are only interested in the endogenous crash so we assume that \bar{B} is large enough.¹⁰

We assume that θ is uniformly distributed over $[0, \infty)$.¹¹ Each trader receives a private signal v at the beginning, which is uniformly distributed on $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$. v can be regarded as a trader's type. Let $\Phi(\theta|v)$ be the posterior CDF about θ of a trader with signal v , and $\phi(\theta|v)$ be the corresponding PDF. Given the signal v , the

¹⁰We will show that in equilibrium \bar{B} is never binding and the bubble always bursts due to the threshold κ .

¹¹The improper uniform distribution on $[0, \infty)$ has well defined posterior belief when we specify how signals are distributed. It is chosen because it gives closed-form solutions. The model with an exponential prior belief also gives closed-form solutions. See Appendix C.2 for details. The uniform prior case is adapted from Li and Milne (2014) and the exponential prior case is adapted from AB2003.

support of the posterior is $[v - \frac{\eta}{2}, v + \frac{\eta}{2}]$ and the conditional belief is $\Phi(\theta|v) = \frac{\theta - (v - \frac{\eta}{2})}{\eta}$.¹²

Figure 4.2 depicts the posterior beliefs about θ for trader v , v' and v'' . These different

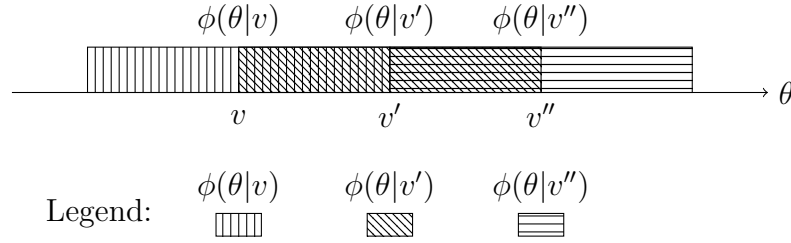


Figure 4.2: Posterior beliefs

posterior beliefs reflect different opinions about the asset fundamental value. A trader is not sure about her signal's position within $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$, i.e., she does not know how many others' signals are lower or higher than hers. This is an important element in the model because this lack of common knowledge of the distribution of the population prevents traders from perfectly coordinating each other. In contrast, in the standard literature with a common posterior belief, perfect coordination leads to a backward induction that rules out the existence of the bubble. The above specifications are such that all traders' posterior belief have exactly the same shape except a horizontal shift. Hence, traders will behave the same relative to their respective signals (except those close to the lower boundary), and everyone sells at a different price because they each has a different signal.

Since our model is essentially equivalent to AB2003, we use the results from their Lemma 3, Lemma 4 and Proposition 1 without proof.

¹²When $\theta < \frac{\eta}{2}$, some traders will receive negative signals, which is perfectly compatible with the assumption that θ is non-negative. When $\theta < \eta$, those with $v < \frac{\eta}{2}$ will have a truncated belief support because θ cannot be below zero. This cause traders with very low signals behave differently, which will be clarified in Proposition 4.2.1 where we characterize the equilibrium. The rest of this section will ignore this special case.

Lemma 4.2.1. (*Proposition 1 in AB2003*) *A trader uses a trigger strategy: she sells only once, whereby she sells all her shares and will never buy back.*

Note that we allow traders to buy and sell for many times in the model and the above lemma is a result derived from the model, instead of an assumption imposed. Given all traders' signal profile, let $P^*(v)$ denote the selling price of a trader v .

Lemma 4.2.2. (*Lemma 3 and 4 in AB2003*) *$P^*(\cdot)$ is continuous and strictly increasing on $(-\frac{\eta}{2} + \eta\kappa, \infty)$.*

It follows that traders with higher signals must sell at higher prices and that $P^{*-1}(\cdot)$ is well defined. Let p_T denote the highest price before the crash. Since θ is a random variable, $p_T = P^*(\theta - \frac{\eta}{2} + \eta\kappa)$ is also a random variable, given $P^*(\cdot)$. Suppose that a trader v decides to sell at price p , then if $p < p_T$, she will be able to flee the market before the crash; otherwise, she will be caught in the crash. By inverting this relationship we know that $\theta = P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa$. Then she will flee the market before the crash if $\theta \in [P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa, v + \frac{\eta}{2}]$ but will get caught if $\theta \in [v - \frac{\eta}{2}, P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa]$. Denote $\omega(p)$ the expected payoff from selling at price p . Given that all others use strategy $P^*(\cdot)$, the expected payoff for trader v is

$$\omega(p) = \int_{\theta=v-\frac{\eta}{2}}^{P^{*-1}(p)+\frac{\eta}{2}-\eta\kappa} \theta\phi(\theta|v)d\theta + \int_{P^{*-1}(p)+\frac{\eta}{2}-\eta\kappa}^{v+\frac{\eta}{2}} p\phi(\theta|v)d\theta \quad (4.1)$$

Note that the belief in this game is static. Although the lower boundary of the

posterior belief may shrink upwards over time when the price increases,¹³ this change is expected. Pre-crash sale price p in (4.1) is conditional on selling before the crash, and any price in the process up to p is expected and already considered in (4.1). Hence there is no need to update the belief for a trader. This invariability is similar to the situation where a common value Dutch auction is equivalent to a common value first-price sealed bid auction so that a belief updating is unnecessary.

A trader's problem is

$$\max_p \omega(p)$$

We first describe how to solve for the equilibrium strategy $P^*(\cdot)$, then we characterize the unique equilibrium in Proposition 4.2.1.

Before we proceed, we need a technical assumption.

Assumption 4.2.1. $P^*(\cdot)$ is differentiable on $(-\frac{\eta}{2} + \eta\kappa, \infty)$.

Differentiating $\omega(p)$ w.r.t p , imposing $P^*(v) = p$ and setting $\frac{dE[R|v]}{dp} = 0$, we have the first order condition

$$1 = \left[P^* - \left(v + \frac{\eta}{2} - \eta\kappa \right) \right] \frac{1}{P^{*'}} \frac{\phi\left(v + \frac{\eta}{2} - \eta\kappa | v\right)}{1 - \Phi\left(v + \frac{\eta}{2} - \eta\kappa | v\right)} \quad (4.2)$$

This FOC can be interpreted in terms of a marginal benefit (MB) and a marginal

¹³The initial belief $\phi(\theta|v)$ is uniform over $[v - \frac{\eta}{2}, v + \frac{\eta}{2}]$. Given that all others use equilibrium strategy $P^*(\cdot)$, the bubble will burst at $p_T = P^*\left(\theta - \frac{\eta}{2} + \eta\kappa\right)$ and hence $\theta = P^{*-1}(p_T) + \frac{\eta}{2} - \eta\kappa$. If the bubble bursts at any price p_e , then it must be that $\theta = P^{*-1}(p_e) + \frac{\eta}{2} - \eta\kappa$. When the current price p_e is such that $P^{*-1}(p_e) + \frac{\eta}{2} - \eta\kappa < v - \frac{\eta}{2}$, where $v - \frac{\eta}{2}$ is the lower bound of θ , the bubble will certainly not burst for trader v . When price p_e has increased such that $P^{*-1}(p_e) + \frac{\eta}{2} - \eta\kappa > v - \frac{\eta}{2}$, from trader v 's point of view, the bubble can burst any moment. The fact that the bubble has not burst below p_e implies that θ cannot be below $P^{*-1}(p_e) + \frac{\eta}{2} - \eta\kappa$, which shrinks the support of θ from below and the new support of θ is now $[P^{*-1}(p_e) + \frac{\eta}{2} - \eta\kappa, v + \frac{\eta}{2}]$.

cost (MC). For a trader who evaluates selling at p vs. $p + \Delta$ (Δ is small), the relevant marginal event is that the bubble bursts between p and $p + \Delta$, which implies that $\theta = v + \frac{\eta}{2} - \eta\kappa$. The MB of selling at $p + \Delta$ instead of p is Δ (the price appreciation). The MC is that she could get caught in the crash if the bubble bursts in between p and $p + \Delta$, which equals $p - \theta$ (the loss due to the bursting) multiplied by $\frac{\Phi(P^{*-1}(p+\Delta)+\frac{\eta}{2}-\eta\kappa|v)-\Phi(P^{*-1}(p)+\frac{\eta}{2}-\eta\kappa|v)}{1-\Phi(P^{*-1}(p)+\frac{\eta}{2}-\eta\kappa|v)}$ (the probability of bursting between p and $p + \Delta$). Dividing both sides by Δ and letting $\Delta \rightarrow 0$, we have $MB = 1$ and $MC = (p - \theta) \frac{1}{P^*} \frac{\phi(v+\frac{\eta}{2}-\eta\kappa|v)}{1-\Phi(v+\frac{\eta}{2}-\eta\kappa|v)}$ ($\theta = v + \frac{\eta}{2} - \eta\kappa$ if the bubble bursts between p and $p + \Delta$).

4.2.1 Equilibrium

Proposition 4.2.1. *There is a unique equilibrium with a bubble of size B . A trader v sells at price*

$$\begin{cases} B, & \text{if } v < -\frac{\eta}{2} + \eta\kappa \\ P^*(v) = v + \frac{\eta}{2} - \eta\kappa + B, & \text{if } v \geq -\frac{\eta}{2} + \eta\kappa \end{cases}$$

This gives rise to a bubble of the size B , where $B = \eta\kappa$.

See Appendix C.1 for proof.¹⁴ The equilibrium strategy is depicted in Figure 4.3.

¹⁴The proof of uniqueness of the strategies in the lower boundary $[-\frac{\eta}{2}, -\frac{\eta}{2} + \eta\kappa]$ requires an additional technical assumption: when any positive mass of traders sell at the same price and the bubble bursts right at that price, only some of them (random draw) can sell at the pre-crash price, while others have to sell at the post-crash price θ .

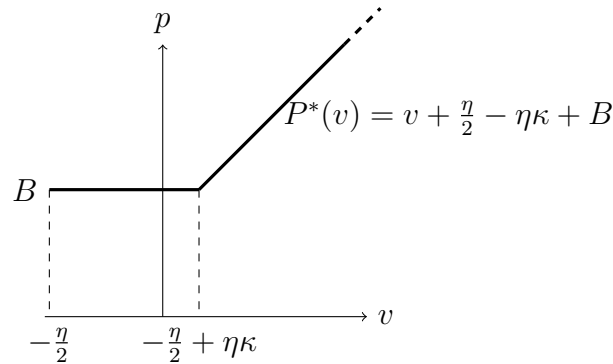


Figure 4.3: The equilibrium strategy

4.3 A comparative study between bubbles and auctions

4.3.1 Relationship to an auction

The model in Section 4.2 can be recast as an auction. Specifically, this is a reverse discriminatory-price (first-price) sealed-bid multi-unit auction with a single unit demand and a common value outside option, and bidders are continuous instead of discrete. We will explain these terms step by step. First, all the traders participate in this single auction. It is a reverse auction (also called a procurement auction) because bidders (traders) are selling instead of buying. In this reverse auction the auctioneer has continuous (identical) projects/contracts of mass κ ($\kappa < 1$) and would like to buy (labor or service) from bidders of a mass of one who want to provide/sell (their labor or service). A mass κ of the bidders who bid the lowest (selling at lowest prices) win and each winner receives a contract value equal to their respective bidding prices (selling prices).¹⁵ In contrast, in a normal auction bidders are buyers and winners are those who bid the highest. It is a discriminatory-price auction because each winner gets her own bidding price, which corresponds to the first-price in a single

¹⁵Winners need not exert any effort to fulfill the contract.

object auction. The losers receive a common value outside option θ . Figure 4.4 depicts the bidding strategies. Winners in the auction are those whose bidding (selling)

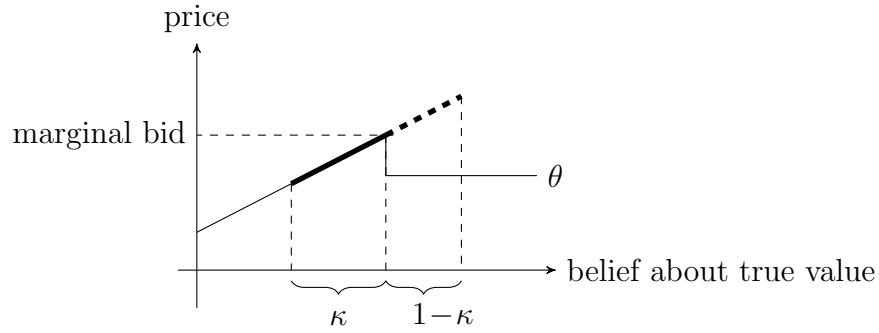


Figure 4.4: The bidding strategy

prices are low enough to sell before the crash, which is the thick solid line Figure 4.4. The losers are those who are caught in the crash, and the thick dashed line represents their planned, though never realized, selling prices, because the bubble bursts before they have a chance to sell. All traders are involved in this single auction.

In order to win, a trader necessarily bids low enough to be in the lower fraction κ of all traders. But the lower she bids, the lower the payoff upon winning. Since a trader receives either p (her bidding price) or θ , she only compares these two alternatives and her bidding strategy only depends on her belief about θ : the lower her belief about θ , the lower she bids. This is why it is a common value auction. In this case the expected payoff conditioned on signal v is exactly the equation (4.1).

We can also recast the model as a slightly different auction without the outside option. Winners now need to exert a common but uncertain effort θ to fulfill the

contract.¹⁶ Then the expected payoff of trader v changes to

$$\omega(p) = \int_{P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa}^{v + \frac{\eta}{2}} (p - \theta)\phi(\theta|v)d\theta$$

This expected payoff is different from that in the previous auction (equation (4.1)), but the first order condition is the same, so the equilibrium strategy is the same.

In what follows we will show that the above auction (as well as some other common value auctions) fails to aggregate information and the bidding prices deviate from the true value of the objects in the auction.

4.3.2 A review of common value auction literature: price convergence

In Figure 4.4 the marginal bid, which is the highest winner’s bid and also the lowest loser’s bid, is the bid that equals the bursting price. In our bubble model this equilibrium marginal bidding price is higher than the fundamental value (i.e. $P^*(\theta - \frac{\eta}{2} + \eta\kappa) = \theta + \eta\kappa > \theta$). If somehow the marginal bidding price equals the true value θ , then there will be no bubble. Similarly, in the literature of uniform-price (second-price) common value auctions, if the bidding price converges to the true value of the object, then it is said that the information aggregation holds. The price convergence, if holds, demonstrates that, while no one knows the true value of the objects with certainty, the auction as a price formation process can aggregate the information diffused in the economy and reveal the true value.

Early literature on the common value auctions shows that the bidding prices converge to the true value of the object and that the auction is a trading mechanism

¹⁶The common effort can be thought of as each winning bidder to build an identical house with the same design.

that achieves the market efficiency.¹⁷ However, the assumptions adopted in the early literature turn out to be quite strong. It not only requires the monotone likelihood ratio property (MLRP), but also that the likelihood ratio approaches to zero.¹⁸

As shown by the recent literature in common value auctions,¹⁹ when the requirement that the likelihood ratio converges to zero is not satisfied, the price generally fails to converge to the true value. Kremer (2002) shows that both the first- and the second-price single object common value auctions fail to aggregate information. Jackson and Kremer (2007) show that in the discriminatory-price common value auction with n traders, k identical objects for sale and each bidder desiring only one item, the information aggregation also fails and the price does not converge to asset value even when both $k \rightarrow \infty$ and $n \rightarrow \infty$. In particular, they show that the marginal (or called pivotal) bid in the auction is lower than the true value of the asset. The only situation where the price does converge to the true value is Pesendorfer and Swinkels (1997), who show that the price converges to the true value in a uniform-price common value auction when both $k \rightarrow \infty$ and $n \rightarrow \infty$.

To apply these results, we extend our model (a reverse discriminatory-price common value auction) to a uniform-price-setting and compare its results with the discriminatory case. Under the uniform-price, all winners receive the marginal bidder's

¹⁷Wilson (1977) and Milgrom (1979) showed that, in a first-price common value single unit auction, the winning bid converges in probability to the value of the object as the number of bidders n becomes large. Milgrom (1981) showed that, in a uniform-price (second-price) common value auction with k identical objects for sale and each bidder only desiring one item, where all winning bidders pay the (same) $k + 1$ th bid (which corresponds to the second price in single object auction), if we fix k and let the total number of bidders $n \rightarrow \infty$, then the price (the highest loser's bid, which is also the $k + 1$ th highest bid) also converges to the true value.

¹⁸Let $f(v|\theta)$ be the density distribution of a bidder's estimate when true value is θ . MLRP requires that, if $\theta_1 < \theta_2$, $\frac{f(v|\theta_1)}{f(v|\theta_2)}$ decreases in v , which most distributions satisfy. But the price convergence also requires that $\frac{f(v|\theta_1)}{f(v|\theta_2)} \rightarrow 0$ as $v \rightarrow \bar{v}$, where \bar{v} is the upper bound of the support of v .

¹⁹Bidders and objects are all discrete in these models.

bidding price. We also compare the above models (continuous bidder and object, reverse auction) with the standard auction models (discrete n bidders and k objects, normal auction) under the framework of common value auctions. The price convergence are summarized in Table 4.1.

	Uniform-price	Discriminatory-price
Standard auctions (normal, $k \rightarrow \infty$, $n \rightarrow \infty$)	$P = V$	marginal $P < V$
Our models (reverse, continuous)	$p_T = \theta$	$p_T > \theta$

Table 4.1: Price convergence (P is the trading price, V is the true value and p_T is the bursting price)

Under the discriminatory-price, the standard auctions (Jackson and Kremer (2007)) have a marginal bidding price that is lower than the true value. Since our model is a reverse auction, this means that the marginal price in our reverse auction should be higher than the true value, which is true. So why bubbles arise in our model has the same root as why the marginal bidding price does not equal the true value in the discriminatory-price common value auctions. This connection shows that bubbles arise are not due to any peculiar assumption in our model and the price deviations exist in more general settings. When the private information is dispersed in the market participants and no one is sure about the asset fundamental value, the market generally fails to aggregate information and the price fails to reflect average belief. Put differently, riding the bubble becomes optimal to each investor. Compared to the previous literature in asset bubbles this interpretation highlights and further clarifies that bubbles and crashes can arise from the information asymmetry, which is an intrinsic characteristic of the market. This is a bad news for the market efficiency hypothesis in the sense that, every time a news arrives and opinions vary, there could potentially be a coordination failure and a bubble arises.

Under the uniform-price, the bidding price converges to the true value in the stand auctions (as Pesendorfer and Swinkels (1997)), which implies that no bubble will arise in this case. This result suggests that recasting the payoff structure towards a uniform-price auction could potentially reduce a bubble. We will discuss them in Section 4.3.4.

4.3.3 Two bid shading incentives

Now we discuss why the marginal bidding price is higher (lower) than the true value in the reverse (normal) discriminatory-price common value auctions by decomposing the equilibrium strategy into two components: the *marginal bidder incentive* and the *price-setting incentive*. We then show how they affect the marginal bid so that the bid deviates upwards from true value and generates a bubbles. The equilibrium selling price $P^*(v)$ in Proposition 4.2.1 can be decomposed as follows.

$$\begin{aligned}
 P^*(v) &= v + \text{marginal bidder incentive} + \text{price-setting incentive} \\
 &= v + \frac{\eta}{2} - \eta\kappa + \frac{1 - \Phi(v + \frac{\eta}{2} - \eta\kappa|v)}{\phi(v + \frac{\eta}{2} - \eta\kappa|v)}
 \end{aligned}$$

Marginal bidder incentive

The marginal bidder incentive is that every bidder tries to avoid both the winner’s and the loser’s curses by assuming that she is the marginal bidder. As a result the marginal bidder bids exactly at the true value. This incentive exists in all common value auctions and in common value auctions only.

This incentive is best illustrated in a uniform-price common value (normal) auction, where there is no interference of the other incentive. The winner’s curse is that, if a bidder turns out to be the winner, then her signal is one of the highest among all

bidders and biased upward. Hence, if she has bid naively, it is very likely that she has overestimated and overbid. This bias is more striking in the classic auction models where there are n bidders but only one object for auction. Sophisticated bidders in this case should shade their bids conditioning on winning. The *loser's curse*²⁰ is much less well-known because in most auction models there is only one object, while the loser's curse matters only when the number of objects is comparable to the number of bidders.²¹ The loser's curse is that upon losing a bidder finds that her signal is among the lowest and hence she may have underestimated the true value and underbid. This is more striking when there are n bidders with $n - 1$ objects for auction. To avoid this curse, sophisticated bidders should shade their bids conditioning on losing.

In a general case where there are n bidders and $k < n$ objects, the equilibrium bidding strategy in a uniform-price common value auction is to bid conditioning on being the marginal bidder.²² This is because, if the marginal bidder's bid is higher than the true value, then all winners will be paying a price higher than the true value due to the uniform-price-setting. Then all bidder will revise their bids downwards. If the marginal bid is lower than the true value, then a bidder has the incentive to raise her bid to have a higher probability of winning while (essentially) not affecting the price she pays upon winning. Ex ante, no one knows whether she will be the marginal bidder. But in equilibrium everyone behaves as if she is the marginal bidder.²³

When our model is extended to a uniform-price-setting, the equilibrium bidding/selling strategy is $v + \frac{\eta}{2} - \eta\kappa$. Conditional on being the marginal bidder, i.e. the

²⁰Pesendorfer and Swinkels (1997) discuss this point in detail.

²¹Symmetrically, the winner's curse does not matter when there are n bidders and $n - 1$ objects.

²²In fact, in both of the single object and $n - 1$ objects cases, we have already conditioned on being the marginal bidder.

²³This is why in the normal discrete bidder uniform-price common value auctions, the equilibrium strategy is $E[v|X_1 = v, Y_k = v]$, where X_1 is my signal, and Y_k is the k th highest signal among all other bidders.

signal is $\theta - \frac{\eta}{2} + \eta\kappa$, a trader's bid will be exactly the true value θ . Therefore $\frac{\eta}{2} - \eta\kappa$ is a bidder's effort (on top of v) to avoid both the winner's and the loser's curses and to bid exactly at θ if being marginal bidder (See Table 4.2 for a summary).

Price-setting incentive

The price-setting incentive is that traders try to set the price to seize extra surplus in a discriminatory-price auction, compared to the price-taking behavior in a uniform-price auction. This incentive exists in all discriminatory-price auctions and in the discriminatory-price auctions only.

In a uniform-price auction, a bidder does not pay what she bids and her bid essentially has no impact on her payment. Hence she behaves like a price-taker. In contrast a bidder in a normal (reverse) discriminatory-price auction pays (gets) exactly what she bids upon winning. Hence, in a normal (reverse) auction she has an incentive to lower (raise) her bid.

This incentive is best illustrated in a private value auction. In a normal first-price private value auction if a bidder bids exactly at her private value, then obviously she always has a zero surplus. To extract a positive surplus, she shades her bid such that the equilibrium bidding strategy is higher than her private value.²⁴ To illustrate the price-setting incentive, we extend our model along a second dimension to the private value setting. In the private value setting a winner is to exert an effort equal to her signal v instead of a common value θ and the price-setting incentive is $\eta\kappa$ (See Table 4.2 for a summary). This term is actually the inverse of the hazard rate in the FOC

²⁴In normal first-price common value auctions, this incentive still exists. But when she lowers her bid, she also lowers her winning probability. So the equilibrium strategy has to balance these two forces.

(recall that the hazard rate is $\frac{1}{\eta\kappa}$). If we re-write Equation (4.2), we have²⁵

$$P^*(v) = (v + \frac{\eta}{2} - \eta\kappa) + \frac{1 - \Phi(v + \frac{\eta}{2} - \eta\kappa|v)}{\phi(v + \frac{\eta}{2} - \eta\kappa|v)}$$

where the term $\frac{1 - \Phi(v + \frac{\eta}{2} - \eta\kappa|v)}{\phi(v + \frac{\eta}{2} - \eta\kappa|v)}$ is a general form of the price-setting incentive. In comparison, the FOC in the uniform-price case does not have this price-setting term. In a discriminatory-price (reverse) auction, since a bidder gets what she bids, she would bid infinitely high if she is guaranteed to win. When she has to take into account the possibility of losing, then the situation where she is the marginal winner matters. In this case, $\frac{1 - \Phi(v + \frac{\eta}{2} - \eta\kappa|v)}{\phi(v + \frac{\eta}{2} - \eta\kappa|v)}$ is exactly what she can add to her bid to balance between seizing extra value and not forgoing too much opportunity of winning. Lastly, notice that the price-setting incentive $\eta\kappa$ increases in κ , which means that a higher κ (more winning slots) relieves the competition and results in a more aggressive bidding strategy (higher selling prices).

A comparison

The two incentives described above are well-known in the auction literature (may be under different names) but are entangled in the bidding strategies. Our model provides a unique setting where we can separate them explicitly, which is due to

²⁵For simplicity, we assume that $P^{*'} = 1$.

our information structure and distribution specifications.²⁶ Table 4.2 summarizes the selling/bidding strategies in our model, which is extended along the two dimensions and hence have four cases. One can observe that the marginal bidder incentive ex-

	Uniform prior	
	Uniform-price	Discriminatory-price
Private value	v	$v + \eta\kappa$
Common value	$v + \frac{\eta}{2} - \eta\kappa$	$v + \frac{\eta}{2} - \eta\kappa + \eta\kappa$

Table 4.2: Bidding strategies in continuous bidder/object reverse auctions

ists in all common value auctions only and the price-setting incentive exists in all discriminatory-price auctions only.

In our original model (the discriminatory-price common value case) both incentives exist in the selling strategy. So the bubble bursts at

$$p_T = P^*\left(\theta - \frac{\eta}{2} + \eta\kappa\right) = \underbrace{\theta - \frac{\eta}{2} + \eta\kappa}_{\text{marginal bidder's signal}} + \underbrace{\frac{\eta}{2} - \eta\kappa}_{\text{marginal bidding incentive}} + \underbrace{\frac{1 - \Phi}{\phi}}_{\text{price-setting incentive}} = \theta + \frac{1 - \Phi}{\phi}$$

By definition the marginal bidding incentive is to ensure the marginal bidder to bid at the true value θ , hence the marginal bidding incentive is canceled out by the gap

²⁶The reason why our model has a simple solution form and always has bubbles is due to the lack of common knowledge adopted from AB2003. Specifically, except at the lower boundary, there is no common knowledge about the lower or upper bound for current population's signals. In contrast, Jackson and Kremer (2007) assume that signals are always distributed within $[0, 1]$, irrespective of the realization of object value. The object value only affects the shape of the signal distributions. This specification introduces an extra complication: for those whose signals are high enough, they know for sure that they are among the highest types and they will win, hence they have incentive to further shade their bids; symmetrically, for those whose signals are low enough, they have incentive to bid even higher. As a result, bids are somewhat concentrated in the middle. When the realized object value is high, the average transaction price is below the true value; when the object value is very low, the average transaction price is higher than the true value. However, on average, the expected price is lower than the expected value, and in particular, the marginal bid is always lower than the true value.

between marginal bidder’s signal and true value θ . Therefore, both the marginal bidder and the price-setting incentive affect a trader’s selling strategies, but the size of the bubble is determined by price-setting incentive alone, i.e. the size of the bubble is

$$B = \frac{1 - \Phi(v + \frac{\eta}{2} - \eta\kappa|v)}{\underbrace{\phi(v + \frac{\eta}{2} - \eta\kappa|v)}_{\text{price-setting incentive}}}$$

Table 4.3 shows the bursting prices in the uniform and the discriminatory-price under the common value framework. Since the price-setting incentive increases in κ

	Uniform prior	
	Uniform-price	Discriminatory-price
Marginal bids	θ	$\theta + \eta\kappa$

Table 4.3: Marginal bids (bursting prices) in continuous bidders/objects reverse common value auctions

(discussed in the next subsection), the bubble size also increases in κ .

4.3.4 Reduce the size of the bubble: transaction tax and subsidy

Now we explorer several experiments where we alter the traders’ incentives by imposing a proportional transaction tax (Tobin tax) and a subsidy to our original model and evaluate their effects on the bubble. The tax and the subsidy are imposed on all the sales, on sales that are executed before the crash only and on sales executed after the crash only, respectively. The results provide some theoretical grounds and warnings to tax policies.

Write the expected payoff for trader v in a general form:

$$\omega(p) = \int_{\theta=v-\frac{\eta}{2}}^{P^{*-1}(p)+\frac{\eta}{2}-\eta\kappa} B\phi(\theta|v)d\theta + \int_{P^{*-1}(p)+\frac{\eta}{2}-\eta\kappa}^{v+\frac{\eta}{2}} A\phi(\theta|v)d\theta$$

where A is the trader's pre-crash payoff and B is her post-crash payoff. Without a tax or a subsidy, $A = p$ and $B = \theta$. Let τ denote the tax rate. We consider the following five experiments: 1. taxing on all sales, 2. taxing on the pre-crash sales only, 3. taxing on the post-crash sales only, 4. subsidizing the pre-crash sales only and 5. taxing and subsidizing the pre-crash sales.

Experiment 1: taxing on all sales

If we levy a tax of rate τ on both pre and post-crash sale revenues, then $A = (1 - \tau)p$ and $B = (1 - \tau)\theta$. Solving a trader's problem, we find that the tax does not change a trader's strategies relative to the original model and the bubble size is unchanged. This is because the FOC is now $1 - \tau = [(1 - \tau)P^* - (1 - \tau)\theta] \frac{1}{P^*} \frac{\phi}{1 - \Phi}$. Normalizing the FOC by dividing $(1 - \tau)$ on both sides, we see that the FOC is the same as the one without tax, hence the tax does not distort a trader's incentive. This result is consistent with Scheinkman and Xiong (2003) and the empirical evidences (see the literature review in Chapter 2) which show that transaction taxes generally failed to reduce the price volatilities.²⁷

²⁷The ineffectiveness of the transaction tax on bubbles in this model does not conflict with the result in Chapter 2 that a transaction cost can reduce bubbles. This model does not have purchase cost and these simple experiments aim to illustrate the basic effects of the tax on trading incentives. In Chapter 2 where purchase costs are taken into account, such a tax should be able to reduce the bubble.

Experiment 2: taxing on the pre-crash sales only

If we levy the tax on the pre-crash sales only, then $A = (1 - \tau)p$ and $B = \theta$ and we have a surprising result that the bubble becomes larger. This scenario is relevant when it is expected that there will be a significant tax cut after the market crashes to fight the recession and stimulate the economy. This gives an equilibrium strategy $P^*(v) = \frac{v + \frac{\eta}{2}}{1 - \tau}$ and the bubble size becomes $\frac{\tau\theta + \eta\kappa}{1 - \tau}$ which is larger than $\eta\kappa$ in the original model. In this case the tax works against its intention to deflate the bubble. The intuition for this result is that the FOC is now $1 - \tau = [(1 - \tau)P^* - \theta] \frac{1}{P^{*'}} \frac{\phi}{1 - \Phi}$. After the normalization by dividing $(1 - \tau)$ on both sides the FOC becomes $1 = [P^* - \frac{\theta}{1 - \tau}] \frac{1}{P^{*'}} \frac{\phi}{1 - \Phi}$, where the difference between the pre- and the post-crash sales (in the brackets) is smaller than that without the tax. A smaller payoff difference makes traders care less about being caught and encourages them to sell more aggressively at higher prices,²⁸ which inflates the bubble. It also shows that the bubble size actually grows with θ : the higher the realization of θ , the larger the bubble. Therefore policy makers should be wary of this counterproductive effect of the transaction taxes when dealing with bubbles, especially when it is foreseeable that there will be a tax cut after the crash.

²⁸More formally, when the gap between pre and post-crash sales becomes smaller due to the tax, the marginal cost of holding the asset is smaller than its marginal benefit at this price, and a trader's best response is to hold a bit longer and sell at a higher price. Since each trader is infinitesimal, one trader's deviate does not change the loss (the bubble size), but will change her individual hazard rate $\frac{1}{P^{*'}} \frac{\phi}{1 - \Phi}$. This reasoning correctly prescribes the direction of how selling strategies should adjust in response to the tax. But since this game is not strategic complementary, new equilibrium may not be found by having all traders make the same deviation and then find the new best response and iterating the above steps in hope that it converges.

Experiment 3: taxing on the post-crash sales only

If we levy the tax on the post-crash sales only, the bubble will be smaller. In this case $A = p$ and $B = (1 - \tau)\theta$. The equilibrium strategy becomes $P^*(v) = (1 - \tau)(v + \frac{\eta}{2})$ and the bubble size becomes $(1 - \tau)\eta\kappa - \tau\theta$, which is smaller than the original bubble size $\eta\kappa$. The reason is that with the tax only imposed on θ , the payoff difference between fleeing and being caught becomes larger in the normalized FOC, which advises traders to sell more cautiously and conservatively, i.e. they forego some price appreciation and sell earlier to secure the realized appreciation. Although the bubble becomes smaller, punishing only the distressed investors may be politically unacceptable.

Experiment 4: subsidizing only the pre-crash sales

Now we consider a scenario with only a subsidy on the pre-crash sales that can entirely neutralize the price-setting incentive and hence completely eliminate the bubble. Instead of being proportional, the subsidy is a “complement” in the form of a lump sum: the government guarantees that the final payoff of all pre-crash sellers (but not post-crash sellers) equals that of the marginal seller. See Figure 4.5. In terms of implementability, the subsidy can be dispensed only after the crash when the marginal selling price is identified, which requires that the government be able to pull out transaction records and know who have sold before the crash and at what prices. In this experiment, $A = P^*(\theta - \frac{\eta}{2} + \eta\kappa)$ and $B = \theta$, and the equilibrium selling strategy is $P^*(v) = v + \frac{\eta}{2} - \eta\kappa$. The lowest type will sell at $P^*(\theta - \frac{\eta}{2}) = \theta - \eta\kappa$, and the pivotal type $v = \theta - \frac{\eta}{2} + \eta\kappa$ will sell at θ . Hence, there is no bubble at all. The intuition for this result is that the subsidy neutralizes a trader’s price-setting incentive so that she does not have to worry about selling too early. In equilibrium all traders will

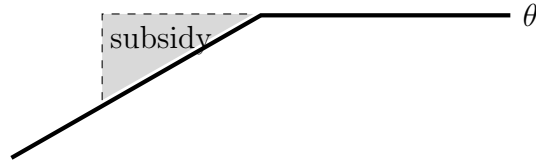


Figure 4.5: No bubble scheme with unbalanced budget

adjust their selling strategies such that the bubble simply disappears. Notice that the subsidy guarantees that every pre-crash seller gets the pivotal bidding price, not θ .²⁹ Such a plan, however, transfers huge public funds to private investors, which may not be economically or politically acceptable.

Experiment 5: taxing and subsidizing pre-crash sales

Now we look at a scenario with both lump sum tax and subsidy that can weaken the price-setting incentive and partially deflate the bubble while maintaining a balanced budget. Consider the situation where the government announces that all pre-crash sellers will get the selling price of the median pre-crash sellers after the lump sum adjustment, i.e. we subsidize the lower half of the pre-crash sellers and tax the higher half of the pre-crash sellers (both in lump sum). See Figure 4.6. Again the tax and subsidy cannot be applied before the crash. Since pre-crash sellers are uniformly distributed in $[\theta - \frac{\eta}{2}, \theta - \frac{\eta}{2} + \eta\kappa]$, the median pre-crash seller is $\theta - \frac{\eta}{2} + \frac{\eta\kappa}{2}$ and every pre-crash seller gets $P^*(\theta - \frac{\eta}{2} + \frac{\eta\kappa}{2})$ after the tax or subsidy. Then $A = P^*(\theta - \frac{\eta}{2} + \frac{\eta\kappa}{2})$ and $B = \theta$, which gives the equilibrium strategy $P^*(v) = v + \frac{\eta}{2} - \frac{\eta\kappa}{2}$. As a result the bubble reduces to $\frac{\eta\kappa}{2}$, which is half of the original size and we can maintain a balanced budget. There is still a bubble because some pre-crash trades are taxed (as

²⁹This scenario and the next one are closely related to turning a discriminatory-price common value auction to a uniform-price one, which is discussed in the early version of this paper.

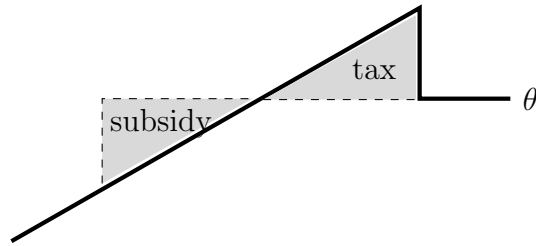


Figure 4.6: Half bubble scheme with balanced budget

in the Experiment 2), and the bubble becomes smaller because some pre-crash trades are subsidized (as in the Experiment 4), i.e. we have combined the Experiment 2 and 4. The selling strategies before the crash are uniformly distributed in $[\theta - \frac{\eta\kappa}{2}, \theta + \frac{\eta\kappa}{2}]$. Since everyone effectively gets θ , taxing a trader $v \in [\theta - \frac{\eta}{2}, \theta - \frac{\eta}{2} + \eta\kappa]$ of a lump sum amount $v + \frac{\eta}{2} - \frac{\eta\kappa}{2} - \theta$ will balance the budget (a negative amount means subsidizing instead of taxing).

4.3.5 Opposite responses of the two incentives to κ

Now we show that the two incentives have opposite responses to a change in κ , which can explain different results of several models in the literature. In our original model (a reverse auction),

- When κ increases, the marginal bidder incentive decreases the bidding prices. When κ becomes larger, the winner's curse becomes smaller (or equivalently the loser's curse becomes larger) because the winners now constitute a larger portion of the population and hence their signals are less biased compared to the population. Sophisticated bidders respond to this change by easing their effort to offset the winner's curse and the bidding prices become lower.
- When κ increases, the price-setting incentive increases the bidding prices.

When the winning slots are scarce, the competition is intense. Bidders undercut each other and the price-setting incentive is thwarted. When κ becomes larger, the competition becomes mild and bidders can ask for higher prices without losing much winning opportunities.

The responses of the two incentives to a change in κ in a reverse auction are summarized in Table 4.4. The responses in a normal auction are symmetric.

	Reverse auction	
	$\kappa \nearrow$	$\kappa \searrow$
Marginal bidder incentive	WC $\searrow \Rightarrow$ bid lower	WC $\nearrow \Rightarrow$ bid higher
Price-setting incentive	CP $\searrow \Rightarrow$ bid higher	CP $\nearrow \Rightarrow$ bid lower

WC: Winner’s curse
 CP: Competition

Table 4.4: Opposite responses of the two incentives to a change in κ

When κ changes, the overall response of the strategy is determined by which incentive dominates, which in turn depends on the specification of a model. The equilibrium strategy $P^*(\cdot)$ in our model under an exponential prior decreases in κ (see Appendix C.2). That is, when more traders are allowed to sell before the crash, they become more cautious and sell earlier. This seemingly counterintuitive result emerges because the marginal bidder incentive dominates the price-setting incentive, hence the overall response of a trader’s strategy to a rise in κ is negative. Under a uniform prior belief the strategy does not depend on κ , which suggests that the two incentives perfectly offset each other. In Brunnermeier and Morgan (2010), which is a discrete agent version of AB2003, the strategies increase in κ . This is because a trader’s payoff in their model is an exponential function of the bidding/selling price, while a trader’s effort to fulfill the contract is not. When κ changes, the coefficient that multiplies the inverse hazard rate in the price-setting incentive is larger, hence

the price-setting incentive dominates the marginal bidder incentive.

4.4 Conclusion

In this paper we construct a simple model of asset bubbles, where the rational traders optimally ride the bubble. We show that this model is equivalent to a reverse discriminatory-price common value auction, which implies that bubbles arise due to the same reason as why the bidding prices fail to reveal the true value in the auction. By decomposing the bidding strategies into two components, we clarify the traders' incentives to "shade their bids" and their roles in the formation of bubbles. In particular, the size of the bubble equals the price-setting component. The failure of the discriminatory-price common value auctions as a trading mechanism to reveal the true value of objects demonstrates that bubbles and crashes are rooted in the inability of the market to aggregate private information and are intrinsic features of the market.

In addition we devise simple tax/subsidy schemes that could (partially) offset traders' incentives and reduce the size of bubbles. These experiments provide directions for future policy studies.

Chapter 5

Summary and Conclusions

In this thesis I explore the non-stable side of financial markets from two perspectives: bubbles and contagions.

Asset bubbles and crashes have huge effects on both financial and real sectors, and governments are currently implementing new tax policies in response to potential housing bubbles. I study the effects of taxes, especially capital gains tax, on asset bubbles, and find that capital gains tax has no effect on the size of the bubble when there is a perfect capital loss tax credit, and the tax can deflate the bubble when there is no tax credit. Therefore in dealing with bubbles, governments should not only impose the tax, but also tighten tax credits. In addition, it is recommended that a transaction cost at a very low rate be retained due to its large marginal deflating effect on bubbles. The model also shows that central banks' low interest rate policies may encourage arbitrageurs to invest aggressively and hence could induce bubbles.

In a related but simpler model of asset bubbles, I show that the model is equivalent to a reverse discriminatory-price common value auction, which demonstrates that bubbles arise for the same reason that bidding prices fail to reveal the true value in that type of auction. The equilibrium trading strategies can be decomposed into

two components, which shows traders' incentives to "shade their bids". I find that influencing these incentives by changing the payoff structure can affect the size of the bubble. These findings demonstrate that bubbles and crashes arise from the inability of the market to aggregate private information dispersed among participants. Several tax/subsidy experiments are explored to offset traders' incentives and reduce the size of bubbles.

It would be highly desirable to empirically verify the effectiveness of the suggested policies and test the predictions from the two models about asset bubbles. In the first essay the belief is largely static and market entry decisions are essentially exogenous. Therefore introducing dynamic beliefs such as herding would probably generate more interesting and richer implications. It is also possible to apply the payoff and information structures on issues such as mortgage insurance and the relationship between payoff uncertainty and research qualities to explain why agents with unbiased private signals might collectively make biased aggregate choice.

I then study financial stability through the phenomenon of contagion. On a network that consists of investors connected with overlapping portfolios, liquidation can become contagious if the downward price pressure is large. Portfolio diversification may reduce the probability of contagion, but it also helps transmit liquidation extensively when a contagion spreads beyond its initial area. Results from our model imply that early government bailouts may be crucial in containing the contagion within its initial neighborhood, since the failure of most investors is primarily due to the erosion from the gradual negative price impact of liquidations. It would be highly beneficial to simulate the responses of markets to shocks by using data from actual shareholder portfolios in stock markets.

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Appendix A

The Impact of Capital Gains Tax and Transaction Cost on Asset Bubbles

A.1 Proof of Lemma 2.3.1

We first prove the following lemma.

Lemma A.1.1. $\lim_{v \rightarrow \infty} P_s^*(v, P_p) = \infty, \forall P_p.$

Proof. Assume there exists \bar{P} such that $\bar{P} > P_s^*(v, P_p), \forall v$. For a trader with signal $v > \bar{P} + \frac{\eta}{2}$, she knows for sure that $\theta > \bar{P}$. Then selling at price $P_s^*(v, P_p) < \bar{P} < \theta$ cannot be an equilibrium strategy, because she would rather sell at price v , for instance, if the bubble has not burst, or at θ , if the it has burst. A contradiction. \square

Lemma A.1.2. *There exists v' (and P_p) such that $P_s^*(v', P_p) = P_b^*(v), \forall v \geq \frac{\eta}{2}$.*

Proof. The lowest possible θ is zero, hence the bubble cannot burst below $P_b^*(\frac{\eta}{2})$, and no trader should sell below $P_b^*(\frac{\eta}{2})$. For trader $v = -\frac{\eta}{2}$, she know for sure that $\theta = 0$ and the bubble will burst at $P_b^*(\frac{\eta}{2})$. Hence she will sell exactly at $P_b^*(\frac{\eta}{2})$. Because both $P_b^*(\cdot)$ and $P_s^*(\cdot, \cdot)$ are continuous, and $P_s^*(\cdot, \cdot)$ does no have an upper bound, the lemma holds. \square

Corollary 2.3.1 implies that $P_b^*(\cdot)$ is weakly increasing. Suppose that there exists $\frac{\eta}{2} \leq \underline{v} < \bar{v}$ such that $P_b^*(\underline{v}) = P_b^*(\bar{v})$. Then the probability of bursting at $P_b^*(\underline{v})$ is strictly positive. By Lemma A.1.2, there exists v such that $P_s^*(v, P_p) = P_b^*(\underline{v}) + \varepsilon$, where $\varepsilon > 0$. Consider trader v . When ε is small enough, the extra benefit, ε , from selling at $P_b^*(\underline{v}) + \varepsilon$ instead of at $P_b^*(\underline{v})$ will be smaller than the loss from being caught in the crash at $P_b^*(\underline{v})$. Then trader v is strictly better off by lowering her selling price from $P_b^*(\underline{v}) + \varepsilon$ to $P_b^*(\underline{v})$. A contradiction.

A.2 Proof of Proposition 2.3.1

Suppose trader v who does not hold the asset stops buying for the first time at $P_b^*(v)$ and then switches back to *buy*. Since price is continuously and strictly rising and $P_b^*(\cdot)$ is continuous, there must be trader $v + \varepsilon$ who does not hold the asset and has stopped buying before v could restart buying, by Assumption 2.3.2. By Corollary 2.3.1, trader v cannot restart buying before trader $v + \varepsilon$ stops and then restarts buying, and trader $v + \varepsilon$ cannot do so before trader $v + 2\varepsilon$ does so. Proceeding this way, trader v is not buying until trader $\theta + \frac{\eta}{2}$ stops and then restarts buying. That trader $\theta + \frac{\eta}{2}$ stops buying is exactly what triggers the burst.

If a trader v has previously sold the asset at $P_s^*(v, P_p)$ and now tries to buy again, it contradicts the belief that all lower types have stopped buying, since $P_b^*(v) \leq P_s^*(v, P_p)$.

A.3 Proof of Proposition 2.4.1 and A.4.1

Part 1: Equilibrium verification for $v > \frac{\eta}{2}$

Suppose that all other traders use strategies prescribed in Proposition 2.4.1 or A.4.1.

Sale decision: Since $\theta = P_b^{*-1}(P_s) - \frac{\eta}{2} = P_s - \theta - D_{NT \text{ or } T}$, then $\Phi(\theta|v) = \frac{\theta - (v - \frac{\eta}{2})}{\eta}$ and $\phi(\theta|v) = \frac{1}{\theta}$. Substitute into Equation (2.6), we have $\frac{d\omega}{dP_s} = \frac{1}{\eta}(-P_s + \frac{\eta}{2} + v)$, and SOC is $\frac{d^2\omega}{dP_s^2} = -1 < 0$. Set $\frac{d\omega}{dP_s} = 0$, we can solve for $P_s^*(v, P_p)$. Therefore, given all traders stop buying at $P_b^*(v)$, selling at $P_s^*(v)$ is an equilibrium.

Purchase decision: Because $\omega(P_p, P_s)$ (in both equation 2.4 and 2.5) is decreasing in P_p , $P_b^*(v) = v + D_{NT \text{ or } T}$ is uniquely pinned down by (2.3). Under the strategy $P_b^*(\cdot)$, we can use the condition that $P_b + 2c \leq P_b^{*-1}(P_s) - \frac{\eta}{2}$ and $B > 0$ to derive the restrictions for parameters $\tau_c, \tau, \frac{R}{\eta}$ and $\frac{c}{\eta}$. Therefore $B > 0$ when $\frac{c}{\eta} < \frac{1}{4}$.

Part 2: Equilibrium uniqueness for $v > \frac{\eta}{2}$

We prove the uniqueness by four lemmas. Let $p_b^*(v) \equiv P_b^*(v) - v$. Suppose that $B > 0$ upon burst.

Lemma A.3.1. *Any equilibrium strategy must be that $p_b^*(v)$ is bounded.*

Proof. The continuity of $P^*(\cdot)$ implies that $P^*(\cdot)$ is finite for finite v . Recall that $P^*(\cdot)$ is an increasing function. If $\lim_{v \rightarrow \infty} p_b^*(v) = \infty$, then $\sqrt{B} > 0$, there exists θ such that $P_b^*(\theta + \frac{\eta}{2}) - \theta > \sqrt{B}$. Hence $P^*(v)$ cannot be an equilibrium strategy. \square

Lemma A.3.2. *In any equilibrium, FOC = 0 holds for all traders in $(\frac{\eta}{2}, \infty)$.*

Proof. Assume in an equilibrium strategy $P_b^*(\cdot)$ and $P_s^*(\cdot)$, there exists a trader v such that her $\frac{\partial\omega}{\partial P_s} \neq 0$. If her $\frac{\partial\omega}{\partial P_s} < 0$, that means she can increase her expected payoff by decreasing her selling price p . The only lower boundary for price is zero. But selling at zero is never optimal because $\theta > 0$ for sure. Hence for all trader $v > \frac{\eta}{2}$, selling at the corner solution zero cannot be optimal. Therefore in any equilibrium a trader v 's $\frac{\partial\omega}{\partial P_s}$ cannot be strictly negative. If her $\frac{\partial\omega}{\partial P_s} > 0$, that means she can increase her expected payoff by increasing her selling price p . But there is no upper boundary

for price, so $\frac{\partial \omega}{\partial P_s} > 0$ itself implies this strategy is not optimal. Therefore in any equilibrium a trader v 's FOC cannot be strictly positive. In addition, those P_s at which $\omega(P_b, P_s)$ is non-differentiable ($\frac{\partial \omega}{\partial P_s}$ does not exist) cannot be in the equilibrium for the following reason. $\frac{\partial \omega}{\partial P_s}$ does not exist at two points: $P_b^{*-1}(P_s) - \frac{\eta}{2} = v \pm \frac{\eta}{2}$. But P_s at neither point can be equilibrium strategy, because $P_b^{*-1}(P_s) - \frac{\eta}{2} = v + \frac{\eta}{2} \implies P_s = P_b^*(v + \eta)$, which is not rational when bubble size is strictly positive, and $P_b^{*-1}(P_s) - \frac{\eta}{2} = v - \frac{\eta}{2} \implies P_s = P_b^*(v)$, which is not rational when transaction cost $c > 0$. Hence FOC=0 holds for all traders $v > \frac{\eta}{2}$ in equilibrium. \square

From $\omega(P_b^*(v), P_s^*(v)) = (1 - \tau)R$ we have $\frac{d\omega(P_b^*(v), P_s^*(v), v)}{dv} = \frac{\partial \omega}{\partial P_b} \frac{dP_b}{dv} + \frac{\partial \omega}{\partial P_s} \frac{dP_s}{dv} + \frac{\partial \omega}{\partial v} = 0$. Since $\frac{\partial \omega}{\partial P_s} = 0$ in EQ, we have $\frac{\partial \omega}{\partial P_b} P_b' + \frac{\partial \omega}{\partial v} = 0 \implies -P_b' - (v - \frac{\eta}{2})\phi(v - \frac{\eta}{2}|v) + P_s\phi(v + \frac{\eta}{2}|v) = 0$. For uniform distribution, this means

$$P_s^*(v) = \eta P_b^{*'}(v) + v - \frac{\eta}{2} \quad (\text{A.1})$$

Substitute (A.1) into FOC: $\frac{\partial \omega(P_b, P_s)}{\partial P_s} = 0$ (partially) and for uniform distribution, we have

$$P_b^{*'}(P_b^{*-1}(P_s^*(v))) - 1 = \frac{\eta}{\eta + v - P_b^{*-1}(P_s^*(v))} [P_b^{*'}(v) - 1] \quad (\text{A.2})$$

From $P_s^*(v) > P_b^*(v) \implies P_b^{*-1}(P_s^*(v)) > v$. From $P_s^*(v) \leq P_b^*(\theta + \frac{\eta}{2}) < P_b^*(v + \frac{\eta}{2} + \frac{\eta}{2}) = P_b^*(v + \eta) \implies P_b^{*-1}(P_s^*(v)) < v + \eta$. Hence, $0 < \eta + v - P_b^{*-1}(P_s^*(v)) < \eta \implies$

$\frac{\eta}{\eta + v - P_b^{*-1}(P_s^*(v))} > 1$. Therefore, from (A.2) we know $\begin{cases} P_b^{*'}(P_b^{*-1}(P_s^*(v))) > P_b^{*'}(v), & \text{if } P_b^{*'}(v) > 1 \\ P_b^{*'}(P_b^{*-1}(P_s^*(v))) < P_b^{*'}(v), & \text{if } P_b^{*'}(v) < 1 \end{cases}$.

For $\forall v_1$, its "images" v_2, v_3, \dots are defined as $v_2 \equiv P_b^{*-1}(P_s^*(v_1))$, $v_3 \equiv P_b^{*-1}(P_s^*(v_2))$,

etc. Let $x_i \equiv \frac{\eta}{\eta + v_i - P_b^{*-1}(P_s^*(v_i))} = \frac{\eta}{\eta + v_i - v_{i+1}}$ and $X_i \equiv \prod_{j=1}^i x_j$. Then given any v_1 , we

have $P_b^{*'}(v_i) - 1 = x_{i-1}[P_b^{*'}(v_{i-1}) - 1] = X_i[P_b^{*'}(v_1) - 1]$.

Lemma A.3.3. $\lim_{i \rightarrow \infty} X_i = \infty$

Proof. Suppose $\lim_{i \rightarrow \infty} X_i$ converges to a finite value. This implies that $\lim_{i \rightarrow \infty} x_i = 1$, which in turn implies that $v_{i+1} \rightarrow v_i$, i.e. $P_b^{*-1}(P_s^*(v_i)) \rightarrow v_i$. Recall that $P_s^*(v_i) - P_b^*(v_i) \geq 2c$. This means that the slope of the line segment $(v_{i+1}, P_b^*(v_{i+1})) - (v_i, P_b^*(v_i))$ can be arbitrarily large, and this slope goes to infinity as $i \rightarrow \infty$. Then $P_b^*(\cdot)$ must diverge upward from 45° line, i.e. $P_b^*(v) - v$ is not bounded. A contradiction. \square

Lemma A.3.4. $P_b^{*'}(v) = 1, \forall v > \frac{1}{2}$

Proof. Suppose $\exists v_1$ such that $P_b^{*'}(v_1) < 1$. Since $x_i > 1, \forall i$, if $X_i \rightarrow \infty$, then there must exist v_i such that $P_b^{*'}(v_i) - 1 < -1 \implies P_b^{*'}(v_i) < 0$, which violates the assumption that $P_b^{*'}(\cdot) > 0$. Suppose $\exists v_1$ such that $P_b^{*'}(v_1) > 1$, we have $\lim_{i \rightarrow \infty} P_b^{*'}(v_i) = \infty$. We know that in equilibrium $P_s^*(v_i) = \eta P_b^{*'}(v_i) + v_i - \frac{\eta}{2} \leq P_b^*(v_i + \eta)$. Since $P_b^{*'}(v_i)$ will grow unboundedly, we see that $\eta P_b^{*'}(v_i) - \frac{3}{2}\eta \leq P_b^*(v_i + \eta) - (v_i + \eta)$, which implies that $P_b^*(v_i + \eta) - (v_i + \eta)$ will also grow unboundedly, which contradicts Lemma A.3.1 that $p_b^*(v_i) = P_b^*(v_i) - (v_i)$ must be bounded. \square

Let $P_b^*(v) = v + g$, where g is a constant. We can solve for $P_s^*(\cdot)$ through standard procedure. Because $\omega(P_p, P_s)$ is decreasing in P_b in both NT and T equilibria, $P_b^*(v) = v + D_{NT \text{ or } T}$ is uniquely pinned down by (2.3), i.e. $g = D_{NT \text{ or } T}$. The belief that $B \leq 0$ will lead to an entirely different strategy profile (Proposition A.5.1), which cannot be equilibrium strategy when parameters satisfy Proposition 2.4.1 or A.4.1.

A.4 Equilibrium with the T bubble

Proposition A.4.1. (*Equilibrium with T bubble*) When $\begin{cases} \tau_c \leq \tau_c^1 \text{ and } \tau^{TNT1} < \tau < \tau^B, \text{ or} \\ \tau_c^B \leq \tau_c \leq \tau_c^1 \text{ and } \tau^B < \tau \end{cases}$

and $\frac{R}{\eta} < \frac{1}{8}(1 - \frac{2c}{\eta})^2 - \frac{c}{\eta}$ and $\frac{c}{\eta} < \frac{3-2\sqrt{2}}{2}$, or when $\begin{cases} \tau_c \leq \tau \text{ and } \tau < \tau^B, \text{ or} \\ \tau_c^B \leq \tau_c \leq \tau \text{ and } \tau^B < \tau \end{cases}$ and

$\frac{1}{8}(1 - \frac{2c}{\eta})^2 - \frac{c}{\eta} < \frac{R}{\eta} < \frac{1}{2}(1 - \frac{4c}{\eta})$ and $\frac{c}{\eta} < \frac{1}{4}$, there exists a unique trading equilibrium

in which bubble size is $B = \frac{\eta}{2} + D_T > 0$, the bubble bursts at $\theta + B$, all the trader are

T traders and a trader $v > \frac{\eta}{2}$ will

$\begin{cases} \text{buy, if price} < P_b^*(v) = v + D_T; \\ \text{sell, if price} \geq P_s^*(v, P_p) = v + \frac{\eta}{2}; \\ \text{hold, if } P_b^*(v) \leq \text{price} < P_s^*(v, P_p). \end{cases}$, where $D_T \equiv \frac{1+4\tau\frac{c}{\eta}-\tau_c(1+4\frac{c}{\eta})-2d_T}{2(1-2\tau+\tau_c)}\eta$,

$d_T \equiv \sqrt{(1-\tau) \left[(\tau-\tau_c)(1-\frac{2c}{\eta})^2 + \frac{2R}{\eta}(1-2\tau+\tau_c) + \frac{4c}{\eta}(1-\tau_c) \right]}$, $\tau_c^B \equiv \tau - \frac{\eta^2}{4c^2}(1-\tau)(1 - \frac{2R}{\eta} - \frac{4c}{\eta})$ and $\tau^B \equiv \frac{1-\frac{2R}{\eta}-\frac{4c}{\eta}}{(1-\frac{2c}{\eta})^2-\frac{2R}{\eta}}$.

The equilibrium strategies with a T bubble in Proposition A.4.1 is depicted in Figure A.1. Now all traders (including break-even traders) are T traders¹ and they all sell at $v + \frac{\eta}{2}$ (undistorted), irrespective of their purchase prices. There is no NT trader in this case. It can be verified that the conclusions on an NT bubble (i.e. $\frac{\partial B}{\partial \tau_c} > 0$, $\frac{\partial B}{\partial \tau} < 0$, $\frac{\partial B}{\partial c} < 0$ and $\frac{\partial B}{\partial R} < 0$) extend to T bubble, and that an NT bubble is always larger than a T bubble. See Figure 2.5 and 2.6.

¹Their the stop-buy strategy are all below the dividing line $v - 2c - D_T$, because the tax and transaction cost are higher now.

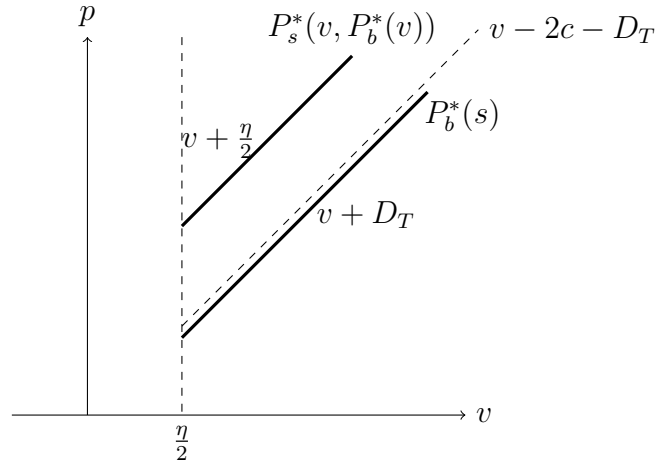


Figure A.1: Equilibrium strategies without tax credit and with positive bubble in case T

A.5 Equilibrium without bubble

The equilibrium where there is no bubble (bubble size < 0) corresponds to a corner solution of (2.2).

Proposition A.5.1. (*Equilibrium without bubble*) When $\tau_c \leq \tau_c^B$ and $\frac{1}{8}(1 - \frac{2c}{\eta})^2 - \frac{c}{\eta} < \frac{R}{\eta} < \frac{1}{2}(1 - \frac{4c}{\eta})$ and $\frac{c}{\eta} < \frac{1}{4}$, or when $\frac{R}{\eta} \geq \frac{1}{2}(1 - \frac{4c}{\eta})$, or $\frac{c}{\eta} \geq \frac{1}{4}$, there exists a “unique” trading equilibrium in which bubble size $B = \eta + D_N \leq 0$, and a trader

- $v \leq -D_N - \frac{\eta}{2}$ will never buy the asset;

$$\bullet v > \max(\frac{\eta}{2}, -D_N - \frac{\eta}{2}) \text{ will } \left\{ \begin{array}{l} \text{buy, if price} < P_b^N(v) = \begin{cases} v + \frac{\eta}{2} + D_N, & \text{if } 2\frac{R}{\eta} \leq 1; \\ v - 2c - R, & \text{if } 2\frac{R}{\eta} > 1 \end{cases} \\ \text{hold, if } P_b^N(v) \leq \text{price} < v + \frac{\eta}{2}. \\ \text{sell, if price} \geq v + \frac{\eta}{2} \end{array} \right.$$

where $D_N \equiv -\frac{\eta}{\tau} - 2c + \frac{\eta}{\tau} \sqrt{(1 - \tau)(1 - 2\tau \frac{R}{\eta})}$

Proof. We first verify that this is an equilibrium. When everyone else stops buying at $P_b^N(v)$,² bubble bursts at $p_T = \theta + \frac{\eta}{2} - 2c - R$, which is smaller than θ when $\frac{2c+R}{\eta} \geq \frac{1}{2}$. As the bubble size is non-positive, it is optimal for traders to sell after the burst, i.e. they will try selling as late as possible. For trader v , the largest possible θ is $v + \frac{\eta}{2}$, so selling at any price $\geq v + \frac{\eta}{2}$ is justified and is equivalent. We assume that in this case traders sell at $v + \frac{\eta}{2}$. Then everyone simply gets the after burst price θ , which means their expected sale revenue is $E[\theta|v]$. Then the best response in purchase stage is to stop buying at $P_b^*(v) = E[\theta|v] - 2c - R = v - 2c - R$. Then for traders with $v \leq 2c + R$, their stop-buy price is zero or negative, so they will never buy the asset.

Uniqueness: If $B < 0$, then selling below $P_b^*(v + \frac{\eta}{2} + \frac{\eta}{2}) = v + \eta + g$ is strictly dominated by at or above $v + \eta + g$, and the latter means that the trader will get θ for sure. Knowing that the expected sale price is $E[\theta|v] = v$, a trader will stop buying at $v - 2c - R$ to maintain a non-negative expected payoff. Then $B \leq 0$ requires that $\frac{2c+R}{\eta} \geq \frac{1}{2}$. The belief that $B > 0$ will lead to an entirely different strategy profile (Proposition 2.4.1 or A.4.1), which cannot be equilibrium strategy when $\frac{2c+R}{\eta} > \frac{1}{2}$. \square

In this equilibrium, when the highest type stops buying, the price is still lower than θ . When the ‘‘bubble’’ bursts, the price jumps *up* to θ . This is because the fixed transaction cost and the return from outside option are too large compared to η , and the stop-buy strategy is pushed so low that the bubble becomes negative. With a

²The stop-buy price $P_b^N(v)$ is pinned down by $\int_{\theta=v-\frac{\eta}{2}}^{P_b^N+2c} (\theta - P_b^N - 2c)\phi(\theta|v)d\theta + (1-\tau) \int_{\theta=P_b^N+2c}^{v+\frac{\eta}{2}} (\theta - P_b^N - 2c)\phi(\theta|v)d\theta = (1-\tau)R$ and we have $P_b^N(v) = v + \frac{\eta}{2} - \frac{\eta}{\tau} - 2c \pm \frac{\eta}{\tau} \sqrt{(1-\tau)(1-2\tau\frac{R}{\eta})}$. Since stopping below the lowest possible θ is not optimal, it must be that $P_b^N(v) + 2c \geq v - \frac{\eta}{2}$, we then have $P_b^N(v) = v + \frac{\eta}{2} - \frac{\eta}{\tau} - 2c + \frac{\eta}{\tau} \sqrt{(1-\tau)(1-2\tau\frac{R}{\eta})}$. Furthermore the square root requires that $2\frac{R}{\eta} \leq 1$. If $2\frac{R}{\eta} > 1$, then $P_b^N(v) = v - 2c - R$

negative “bubble”, if a trader sells before the “crash”, she forgoes the price appreciation that would have certainly realized had she waited till the “bubble” bursts. As a result, everyone has an incentive to hold the asset until the uncertainty is resolved. Obviously the strategy profile in Proposition 2.4.1 or A.4.1 is no longer an equilibrium. Knowing that she will be selling at θ after the “crash” and the expected sale price is $E[\theta|v]$, a trader v should buy the asset if the current price is lower than $E[\theta|v] - 2c - R$.

This equilibrium is unique in the sense that the stop-buy strategy and “bubble” size are unique, but selling strategy is not unique. For trader v , since the bubble will burst below $v + \frac{\eta}{2}$ for sure and the price will be fixed at θ thereafter, price will never rise above $v + \frac{\eta}{2}$. So all strategies selling at any price above $v + \frac{\eta}{2}$ are optimal, although they will never be implemented.

A.6 Equilibrium with *NT* bubble and $R > 0$

With $R > 0$, Proposition 2.4.1 also has to be modified as follows. $\tau^{TNT1} \equiv \frac{1}{2} - \frac{4c+2R}{(1-\frac{2c}{\eta})^2}$ and $\tau_c^1 \equiv 2\tau - 1 + 8(1 - \tau) \frac{\frac{2c+R}{\eta}}{(1+\frac{2c}{\eta})^2}$.

Appendix B

Contagion of Liquidation on Asset-Trader Network

B.1 Generating Functions

Let D be a discrete random variable taking values $0, 1, 2, \dots$, and let $p_i = \text{Prob}[D = j]$ for $j=0, 1, 2, \dots$

The *probability generating function* of the distribution, p_i , of the random variable D is

$$f(x) = E(x^D) = \sum_{j=0}^{\infty} P(D = j)x^j = \sum_{j=0}^{\infty} p_j x^j$$

Note that

$$f(1) = \sum_{j=0}^{\infty} p_j = 1$$

The probability distribution p_i can be uniquely determined by the generating function $f(x)$ in the following sense:

$$p_i = \frac{1}{j!} \left. \frac{d^j f(x)}{dx^j} \right|_{x=0} = \frac{1}{j!} f^{(j)}(0)$$

Moments. The average over the probability distribution is given by

$$\mu = \langle D \rangle = \sum_{j=0}^{\infty} j p_j = f'(1)$$

and higher moments are given by

$$\langle D^n \rangle = \sum_{j=0}^{\infty} \left[\left(x \frac{d}{dx} \right)^n f(x) \right]_{x=1}$$

Distribution of sum. If D_1, D_2, \dots, D_n are independent discrete random variables with generating functions $f_1(x), f_2(x), \dots, f_n(x)$, then the generating function of $D_1 + D_2 + \dots + D_n$ is $f_1(x) \cdot f_2(x) \cdot \dots \cdot f_n(x)$. For example, if D_1 and D_2 are i.i.d. random variables from distribution p_i , then the distribution of $D_1 + D_2$ is generated by

$$\begin{aligned} [f(x)]^2 &= \left[\sum_i p_i x^i \right]^2 \\ &= \sum_j \sum_k p_j p_k x^{j+k} \\ &= p_0 p_0 x^0 + (p_0 p_1 + p_1 p_0) x^1 + (p_0 p_2 + p_1 p_1 + p_2 p_0) x^2 \\ &\quad + (p_0 p_3 + p_1 p_2 + p_2 p_1 + p_3 p_0) x^3 + \dots \end{aligned}$$

Appendix C

The origin of bubbles

C.1 Proof of Proposition 4.2.1

Part 1: Equilibrium verification for $v > \frac{\eta}{2}$

It is straightforward to substitute the equilibrium $P^*(\cdot)$ into (4.2) and show that it satisfies the equation, and verify that the $\text{SOC} < 0$ in equilibrium.

Part 2: Equilibrium uniqueness for $v > \frac{\eta}{2}$

We prove the uniqueness by four lemmas. Let $P^*(v)$ be the equilibrium strategy for an equilibrium, and $p^*(v) \equiv P^*(v) - v$.

Lemma C.1.1. *Any equilibrium strategy $P^*(\cdot)$ must be such that $p^*(v)$ is bounded.*

Proof. The continuity of $P^*(\cdot)$ implies that $P^*(\cdot)$ is finite for finite v . Recall that $P^*(\cdot)$ is an increasing function. If $\lim_{v \rightarrow \infty} p^*(v) = \infty$, then $\forall \bar{B} > 0$, there exists θ such that $P^*(\theta - \frac{\eta}{2} + \eta\kappa) - \theta > \bar{B}$. Hence $P^*(v)$ cannot be an equilibrium strategy. \square

Lemma C.1.2. *In any equilibrium, $\text{FOC} = 0$ holds for all traders in $(\frac{\eta}{2}, \infty)$.*

Proof. Assume in an equilibrium strategy $P^*(\cdot)$, there exists a trader v such that her $\frac{d\omega}{dp} \neq 0$. If her $\frac{d\omega}{dp} < 0$, that means she can increase her expected payoff by decreasing

her selling price p . The only lower boundary for price is zero. But selling at zero is never optimal because $\theta > 0$ for sure. Hence for all trader $v > \frac{\eta}{2}$, selling at the corner solution zero cannot be optimal. Therefore in any equilibrium a trader v 's $\frac{d\omega}{dp}$ cannot be strictly negative. If her $\frac{d\omega}{dp} > 0$, that means she can increase her expected payoff by increasing her selling price p . But there is no upper boundary for price, so $\frac{d\omega}{dp} > 0$ itself implies this strategy is not optimal. Therefore in any equilibrium a trader v 's FOC cannot be strictly positive. In addition, those p at which $\omega(p)$ is non-differentiable cannot be in the equilibrium for the following reason. $\frac{d\omega}{dp} = \frac{1}{P^{*'}(P^{*-1}(p))} \phi(P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa|v) [P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa - p] + 1 - \Phi(P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa|v)$. We know that $\frac{d\omega}{dp} = 1$ when p is such that $P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa < v - \frac{\eta}{2}$ and $\frac{d\omega}{dp} = 0$ when p is such that $P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa > v + \frac{\eta}{2}$. When $v - \frac{\eta}{2} < P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa < v + \frac{\eta}{2}$, $\omega(\cdot)$ is differentiable ($\frac{d\omega}{dp}$ is continuous), because $P^{*'}$ and ϕ are both and continuous in that range. $\omega(\cdot)$ is not differentiable ($\frac{d\omega}{dp}$ not continuous) at two points: $P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa = v \pm \frac{\eta}{2}$. But p at neither point can be equilibrium strategy, because $P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa = v - \frac{\eta}{2} \implies p = P^*(v - \eta + \eta\kappa)$ and $P^{*-1}(p) + \frac{\eta}{2} - \eta\kappa = v + \frac{\eta}{2} \implies p = P^*(v + \eta\kappa)$, while equilibrium requires that $p = P^*(v)$. Hence FOC=0 holds for all traders $v > \frac{\eta}{2}$ in equilibrium. \square

Lemma C.1.3. *If $p^*(\cdot)$ is not a constant, then there exist \underline{v} and \bar{v} such that $p^*(\bar{v}) > p^*(\underline{v})$ and $p^{*'}(\bar{v}) \leq p^{*'}(\underline{v})$.*

Proof. Suppose otherwise. Then $\forall \underline{v}$ and \bar{v} , $p^*(\bar{v}) > p^*(\underline{v}) \implies p^{*'}(\bar{v}) > p^{*'}(\underline{v})$. But the continuity and differentiability of $p^*(\cdot)$ implies that $p^*(v)$ will diverge when $v \rightarrow \infty$, which contradicts Lemma C.1.1. \square

Lemma C.1.4. *$p^*(\cdot)$ is a constant.*

Proof. Suppose otherwise, and let \underline{v} and \bar{v} be such that $p^*(\bar{v}) > p^*(\underline{v})$ and $p^{*'}(\bar{v}) \leq p^{*'}(\underline{v})$. Consider the FOC in Equation (4.2). The hazard rate of bursting at $P^*(\underline{v})$ is

$\frac{1}{P^*(\underline{v})} \frac{\phi(\underline{v} + \frac{\eta}{2} - \eta\kappa|\underline{v})}{1 - \Phi(\underline{v} + \frac{\eta}{2} - \eta\kappa|\underline{v})}$ and the hazard rate of bursting at $P^*(\bar{v})$ is $\frac{1}{P^*(\bar{v})} \frac{\phi(\bar{v} + \frac{\eta}{2} - \eta\kappa|\bar{v})}{1 - \Phi(\bar{v} + \frac{\eta}{2} - \eta\kappa|\bar{v})}$. For both uniform and exponential prior cases, $\frac{\phi(\underline{v} + \frac{\eta}{2} - \eta\kappa|\underline{v})}{1 - \Phi(\underline{v} + \frac{\eta}{2} - \eta\kappa|\underline{v})} = \frac{\phi(\bar{v} + \frac{\eta}{2} - \eta\kappa|\bar{v})}{1 - \Phi(\bar{v} + \frac{\eta}{2} - \eta\kappa|\bar{v})}$. However, $P^*(\underline{v}) - (\underline{v} + \frac{\eta}{2} - \eta\kappa) = p^*(\underline{v}) - \frac{\eta}{2} + \eta\kappa$ and $P^*(\bar{v}) - (\bar{v} + \frac{\eta}{2} - \eta\kappa) = p^*(\bar{v}) - \frac{\eta}{2} + \eta\kappa$, with $p^*(\bar{v}) - \frac{\eta}{2} + \eta\kappa > p^*(\underline{v}) - \frac{\eta}{2} + \eta\kappa$. Hence the FOC in Equation (4.2) cannot be satisfied for both trader \underline{v} and \bar{v} , a contradiction. \square

Part 3: Equilibrium verification and uniqueness for $v \leq \frac{\eta}{2}$

For a trader with signal $v \in [-\frac{\eta}{2}, \frac{\eta}{2}]$, the support of her posterior belief about θ is $[0, v + \frac{\eta}{2}]$, which is different than $[v - \frac{\eta}{2}, v + \frac{\eta}{2}]$ (the support of a trader with $v > \frac{\eta}{2}$). We show that, for a trader in $[-\frac{\eta}{2} + \eta\kappa, \frac{\eta}{2}]$, her strategy $P_s^*(v) = v + \frac{\eta}{2} - \eta\kappa + B$ can still be derived from FOC of the expected payoff. Denote $\Phi_s(\theta|v)$ the CDF of her posterior belief and $\phi_s(\theta|v)$ the corresponding p.d.f. In particular, $\Phi_s(\theta|v) = \frac{\Phi(\theta) - \Phi(0)}{\Phi(v + \frac{\eta}{2}) - \Phi(0)} = \frac{\Phi(\theta)}{\Phi(v + \frac{\eta}{2})}$ and $\phi_s(\theta|v) = \frac{\phi(\theta)}{\Phi(v + \frac{\eta}{2})}$, then $\frac{\phi_s(\theta|v)}{1 - \Phi_s(\theta|v)} = \frac{\phi(\theta)}{\Phi(v + \frac{\eta}{2}) - \Phi(\theta)} = \frac{\phi(\theta|v)}{1 - \Phi(\theta|v)}$. Hence the first order condition is exactly the same, which gives the same strategy. The proof of uniqueness in Step 2 goes through in this case as well so the uniqueness follows. For a trader in $[-\frac{\eta}{2}, -\frac{\eta}{2} + \eta\kappa)$, the bubble cannot burst before trader $v = -\frac{\eta}{2} + \eta\kappa$ sells (recall that $P_s^*(\cdot)$ is continuous and strictly increasing), hence she will sell together with trader $-\frac{\eta}{2} + \eta\kappa$. Actually all traders within $[-\frac{\eta}{2}, -\frac{\eta}{2} + \eta\kappa)$ will sell together with trader $-\frac{\eta}{2} + \eta\kappa$. To see why traders within $[-\frac{\eta}{2}, -\frac{\eta}{2} + \eta\kappa + \varepsilon)$ will not sell together at a price higher than $P_s^*(-\frac{\eta}{2} + \eta\kappa)$, notice that trader $v = -\frac{\eta}{2}$ knows for sure that $\theta = 0$ and she will not join the rest, since more than $\eta\kappa$ mass of traders selling together will make her worse off, so she has incentive to preempt.

C.2 Solutions under the exponential prior belief

The exponential distribution has a density $\phi(\theta) = \lambda e^{-\lambda\theta}$ and a conditional CDF $\Phi(\theta|v) = \frac{e^{\lambda\eta} - e^{\lambda(v + \frac{\eta}{2} - \theta)}}{e^{\lambda\eta} - 1}$. The hazard rate is $\frac{\lambda}{1 - e^{-\lambda\eta\kappa}}$. The marginal bidder incentive is $\frac{\eta}{2} - \eta\kappa$ and the price-setting incentive is $\frac{1 - e^{-\lambda\eta\kappa}}{\lambda}$. The equilibrium bidding strategies in the 4 types of auctions are summarized in Table C.1. The marginal bidding price

	Exponential prior	
	Uniform-price	Discriminatory-price
Private value	v	$v + \frac{1 - e^{-\lambda\eta\kappa}}{\lambda}$
Common value	$v + \frac{\eta}{2} - \eta\kappa$	$v + \frac{\eta}{2} - \eta\kappa + \frac{1 - e^{-\lambda\eta\kappa}}{\lambda}$

Table C.1: Bidding strategies in continuous bidder/object reverse auctions under exponential prior belief

in the uniform-price and discriminatory-price common value auctions are summarized in Table C.2.

	Exponential prior	
	Uniform-price	Discriminatory-price
Marginal bids	θ	$\theta + \frac{1 - e^{-\lambda\eta\kappa}}{\lambda}$

Table C.2: Marginal bids (bursting prices) in continuous bidder/object reverse auctions